

1.
$$\Delta y = y_{n+1} - y_n = \frac{\lambda D}{d} = \frac{(589.0 \text{ nm})(2.604 \text{ m})}{1.25 \text{ mm}} = 1.23 \text{ mm}$$

5. (a) $E = 10.0 \text{ MeV} = 1.60 \times 10^{-12} \text{ J}$

$$p = \frac{E}{c} = \frac{10.0 \text{ MeV}}{c} = 1.00 \times 10^7 \text{ eV}/c$$

$$p = \frac{1.60 \times 10^{-12} \text{ J}}{3.00 \times 10^8 \text{ m/s}} = 5.33 \times 10^{-21} \text{ kg} \cdot \text{m/s}$$

(b) $E = 25 \text{ keV} = 4.0 \times 10^{-15} \text{ J}$

$$p = \frac{E}{c} = \frac{25 \text{ keV}}{c} = 2.5 \times 10^4 \text{ eV}/c$$

$$p = \frac{4.0 \times 10^{-15} \text{ J}}{3.00 \times 10^8 \text{ m/s}} = 1.3 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

(c) $\lambda = 1.0 \mu\text{m} = 1.0 \times 10^3 \text{ nm}$

$$p = \frac{h}{\lambda} = \frac{1}{c} \frac{hc}{\lambda} = \frac{1}{c} \frac{1240 \text{ eV} \cdot \text{nm}}{1.0 \times 10^3 \text{ nm}} = 1.2 \text{ eV}/c$$

$$p = \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{s}}{1.0 \times 10^{-6} \text{ m}} = 6.6 \times 10^{-28} \text{ kg} \cdot \text{m/s}$$

(d) $E = hf = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(150 \times 10^6 \text{ Hz}) = 6.2 \times 10^{-7} \text{ eV} = 9.9 \times 10^{-26} \text{ J}$

$$p = \frac{E}{c} = \frac{6.2 \times 10^{-7} \text{ eV}}{c} = 6.2 \times 10^{-7} \text{ eV}/c$$

$$p = \frac{9.9 \times 10^{-26} \text{ J}}{3.00 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-34} \text{ kg} \cdot \text{m/s}$$

6. At $1 \text{ MHz} = 10^6 \text{ Hz}$, $E = hf = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(10^6 \text{ s}^{-1}) = 4 \times 10^{-9} \text{ eV}$

At $100 \text{ MHz} = 10^8 \text{ Hz}$, $E = hf = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(10^8 \text{ s}^{-1}) = 4 \times 10^{-7} \text{ eV}$

The range is from $4 \times 10^{-9} \text{ eV}$ to $4 \times 10^{-7} \text{ eV}$.

7. (a)
$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.00 \times 10^4 \text{ eV}} = 0.124 \text{ nm}$$

(b)
$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{1.00 \times 10^6 \text{ eV}} = 1.24 \times 10^{-3} \text{ nm}$$

(c) 350 nm:
$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{350 \text{ nm}} = 3.5 \text{ eV}$$

700 nm:
$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{700 \text{ nm}} = 1.8 \text{ eV}$$

The range is from 1.8 eV to 3.5 eV.

9.

$$\phi = \frac{hc}{\lambda_c} = \frac{1239.853 \text{ eV} \cdot \text{nm}}{352.8 \text{ nm}} = 3.514 \text{ eV}$$

$$eV_s = \frac{hc}{\lambda} - \phi = \frac{1239.853 \text{ eV} \cdot \text{nm}}{304.2 \text{ nm}} - 3.514 \text{ eV} = 0.561 \text{ eV}$$

$$V_s = 0.561 \text{ V}$$

11. (a)
$$\phi = \frac{hc}{\lambda_c} = \frac{1240 \text{ eV} \cdot \text{nm}}{254 \text{ nm}} = 4.88 \text{ eV}$$

(b) $\lambda < 254 \text{ nm}$

12. (a) With $\phi = 4.31 \text{ eV}$, $\lambda_c = \frac{hc}{\phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.31 \text{ eV}} = 288 \text{ nm}$

(b) $eV_s = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{252.0 \text{ nm}} - 4.31 \text{ eV} = 0.61 \text{ eV}$, so $V_s = 0.61 \text{ volts}$

13. (a) The total number of oscillators is

$$\int_0^{\infty} n(E) dE = \int_0^{\infty} \frac{N}{kT} e^{-E/kT} dE = \frac{N}{kT} (-kT) e^{-E/kT} \Bigg|_0^{\infty} = N$$

(b) The average energy is (from Equation 3.31)

$$E_{\text{av}} = \frac{1}{N} \int_0^{\infty} E n(E) dE = \frac{1}{kT} \int_0^{\infty} E e^{-E/kT} dE = kT \int_0^{\infty} x e^{-x} dx \quad \text{with } x = E / kT$$

The definite integral is a standard form that is equal to 1, so $E_{\text{av}} = kT$.

14. (a) The total number of oscillators at all energies is

$$\sum_{i=0}^{\infty} N_n = \sum_{n=0}^{\infty} A e^{-E_n/kT} = A \sum_{n=0}^{\infty} e^{-n\epsilon/kT} = A \frac{1}{1 - e^{-\epsilon/kT}}$$

Setting this result equal to N , we obtain $A = N(1 - e^{-\epsilon/kT})$.

(b) On the left side $\frac{d}{dx} \sum_{n=0}^{\infty} e^{nx} = \sum_{n=0}^{\infty} n e^{nx}$ and on the right side $\frac{d}{dx} \frac{1}{1 - e^x} = \frac{e^x}{(1 - e^x)^2}$. Setting

these equal to each other gives $\sum_{n=0}^{\infty} n e^{nx} = \frac{e^x}{(1 - e^x)^2}$.

(c) $E_{\text{av}} = \frac{1}{N} \sum_{n=0}^{\infty} N_n E_n = (1 - e^{-\epsilon/kT}) \sum_{n=0}^{\infty} (n\epsilon) e^{-n\epsilon/kT} = (1 - e^{-\epsilon/kT}) \epsilon \frac{e^{-\epsilon/kT}}{(1 - e^{-\epsilon/kT})^2} = \frac{\epsilon}{e^{\epsilon/kT} - 1}$

(d) For large λ , $e^{hc/\lambda kT} \approx 1 + hc/\lambda kT$ and thus $E_{\text{av}} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \approx \frac{hc/\lambda}{1 + hc/\lambda kT - 1} = kT$

As λ goes to 0, $e^{hc/\lambda kT} \rightarrow \infty$ and $E_{\text{av}} \rightarrow 0$.

15.
$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\frac{dI}{d\lambda} = 2\pi hc^2 \left[\left(\frac{-5}{\lambda^6} \right) \frac{1}{e^{hc/\lambda kT} - 1} + \left(\frac{1}{\lambda^5} \right) \frac{(-e^{hc/\lambda kT})(-hc/\lambda^2 kT)}{(e^{hc/\lambda kT} - 1)^2} \right]$$

Setting $dI/d\lambda$ equal to zero gives

$$-\frac{5}{\lambda} + \frac{(e^{hc/\lambda kT})(hc/\lambda^2 kT)}{e^{hc/\lambda kT} - 1} = 0$$

or, with $x = hc/\lambda kT$,

$$(x - 5)e^x + 5 = 0$$

This equation does not have an exact solution, but an approximate solution can be found by trial and error: $x = 4.9651 = hc/\lambda kT$, so

$$\lambda T = \frac{hc}{4.9651k} = \frac{1239.853 \text{ eV} \cdot \text{nm}}{4.9651(8.6174 \times 10^{-5} \text{ eV/K})} = 2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$$

16.
$$\int_0^{\infty} I(\lambda) d\lambda = \int_0^{\infty} \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$$

With $x = hc/\lambda kT$ and $dx = (-hc/\lambda^2 kT)d\lambda$,

$$\begin{aligned} \int_0^{\infty} I(\lambda) d\lambda &= 2\pi hc^2 \left(\frac{kT}{hc}\right)^3 \left(-\frac{kT}{hc}\right) \int_0^{\infty} \frac{x^3 dx}{e^x - 1} \\ &= 2\pi hc^2 \left(\frac{k}{hc}\right)^4 T^4 \int_0^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{2\pi k^4}{h^3 c^2} T^4 \frac{\pi^4}{15} = \sigma T^4 \end{aligned}$$

with $\sigma = 2\pi^5 k^4 / 15h^3 c^2$

$$18. \quad \lambda = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{T} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{6000 \text{ K}} = 483 \text{ nm}$$

This is in the middle of the visible spectrum, close to the peak sensitivity of the eye.

$$19. \quad \lambda = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{T} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{2.7 \text{ K}} = 1.1 \text{ mm (microwave region)}$$

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.1 \text{ mm}} = 1.1 \times 10^{-3} \text{ eV}$$

$$20. \quad (a) \quad \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{T} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{307 \text{ K}} = 9.4 \text{ } \mu\text{m (infrared)}$$

(b) Assume a person can be represented as a cylinder, about 6 feet (1.83 m) tall and 1 foot (0.30 m) in diameter. The surface area is $2\pi rL = 2\pi(0.15 \text{ m})(1.83 \text{ m}) = 1.72 \text{ m}^2$.

$$I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(307 \text{ K})^4 = 504 \text{ W/m}^2$$

$$P = IA = (504 \text{ W/m}^2)(1.72 \text{ m}^2) = 870 \text{ W}$$

(c) For $T = 20^\circ\text{C} = 293 \text{ K}$,

$$I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(293 \text{ K})^4 = 418 \text{ W/m}^2$$

$$P = IA = (418 \text{ W/m}^2)(1.72 \text{ m}^2) = 719 \text{ W}$$

Thus the net power radiated by a person is about 150 W.

$$21. \quad I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1750 \text{ K}) = 5.32 \times 10^5 \text{ W/m}^2$$

$$P = IA = I(\pi r^2) = (5.32 \times 10^5 \text{ W/m}^2)\pi(0.62 \times 10^{-3} \text{ m})^2 = 0.64 \text{ W}$$

23. (a) We consider an interval of width $d\lambda = 2.0 \text{ nm}$ at a central wavelength of 531.0 nm . At $T = 6000 \text{ K}$, $kT = (8.6174 \times 10^{-5} \text{ eV/K})(6000 \text{ K}) = 0.517 \text{ eV}$. The intensity in this interval is

$$\begin{aligned} dI &= I(\lambda) d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda \\ &= \frac{2\pi(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})^2 (2.0 \times 10^{-9} \text{ m})}{(531.0 \times 10^{-9} \text{ m})^5 (e^{(1240 \text{ eV}\cdot\text{nm})/(531.0 \text{ nm})(0.517 \text{ eV})} - 1)} = 1.96 \times 10^5 \text{ W/m}^2 \end{aligned}$$

- (b) The total radiant intensity emitted by the Sun is

$$I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(6000 \text{ K})^4 = 7.35 \times 10^7 \text{ W/m}^2$$

The fraction is then

$$\frac{1.96 \times 10^5 \text{ W/m}^2}{7.35 \times 10^7 \text{ W/m}^2} = 0.0027 = 0.27\%$$

$$I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(293 \text{ K})^4 = 418 \text{ W/m}^2$$

$$P = IA = (418 \text{ W/m}^2)(1.72 \text{ m}^2) = 719 \text{ W}$$