Physics 218 C

Lecture 4b

→ Phenomenology and Basic Physics of ITG / η_i Modes and Turbulence

Contents

→ Review:
- → energy flow in pellet experiments
- → physics of negative compressibility and reactive instabilities
- → Regular and Flat n.

[redacted]

→ Estimates: ITG transport. with ZF effects. ⇒ hint of intrinsic rotation.

→ Related Dynamics:
- PVG / PSFI
- Impurity Modes

→ Particle Transport, Pinch and IOC

- Particle transport by ITG
- Pinch → "Ion Mixing Mode" / TEP → TBC

Mechanisms

↔ Chemotaxis

Foundations

- IOC - a comment

→ Further details of ITG Modes.

→ TBC.

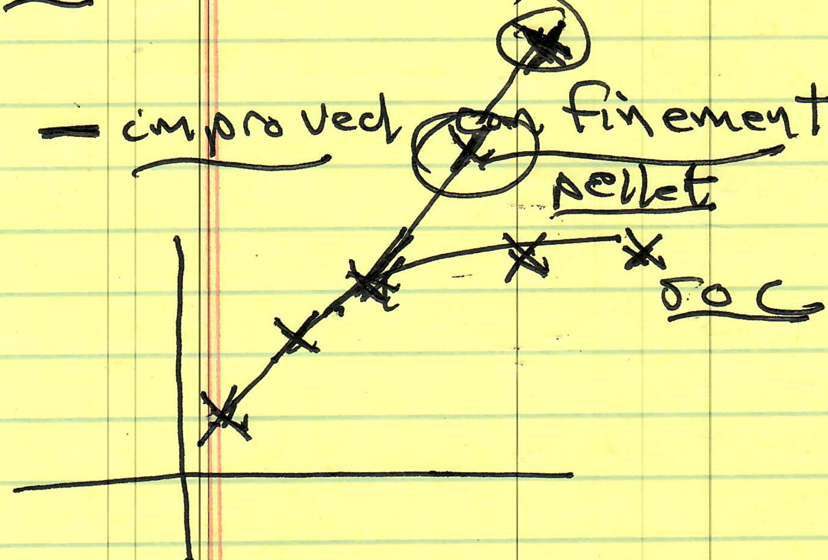
Surprise:

— peaked n

and

— improved confinement of energy

("an ITB")



N.B.: Electron
→ HPP ~ $1/n$
in post pellet

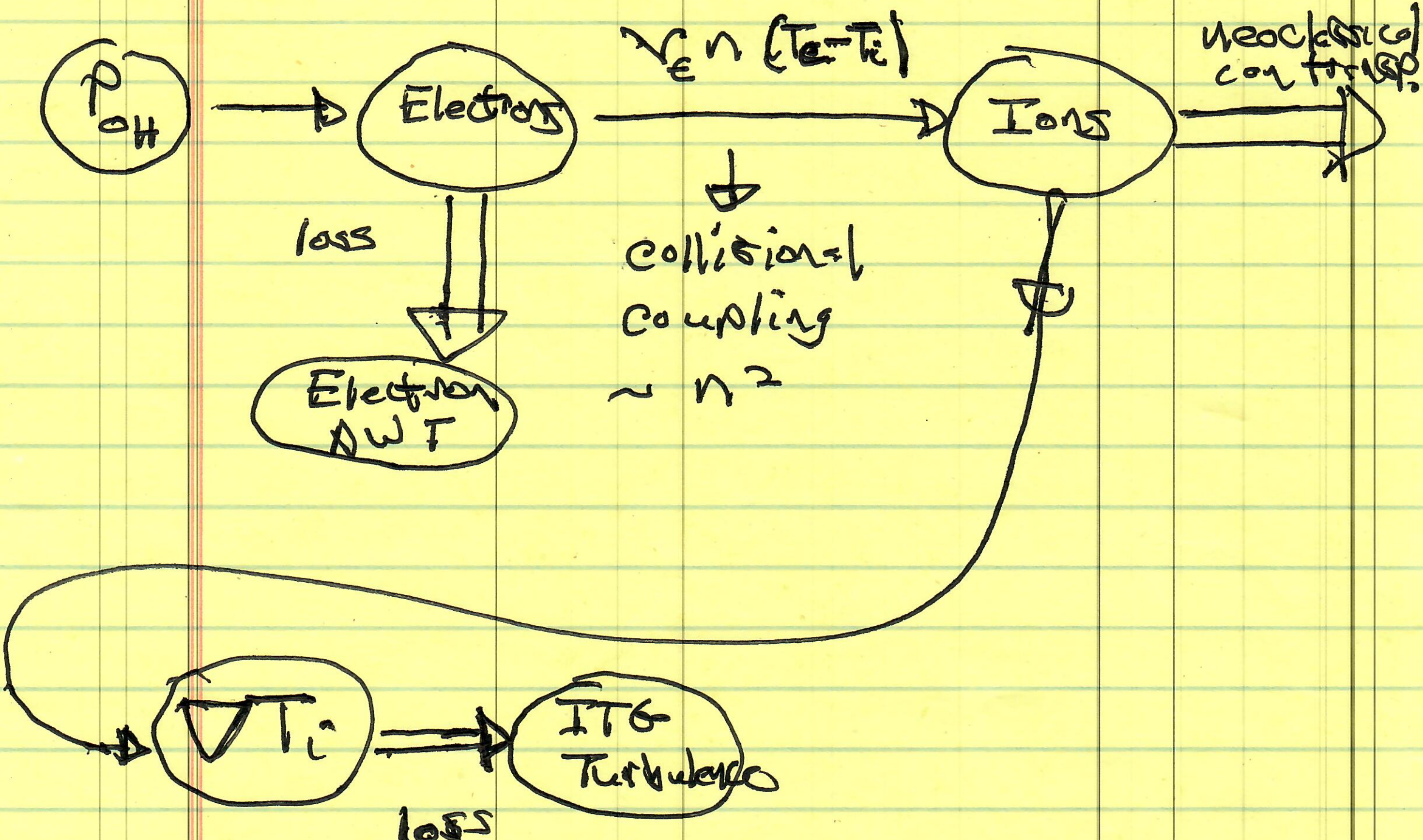
i.e. pellet + peaked n ⇒

~ restored LOC trend.

see also: IOC (Saturated → Improved Ohmic Confinement)

⇒ Confinement experiments indicated that peaked profiles are good → Can improve confinement.

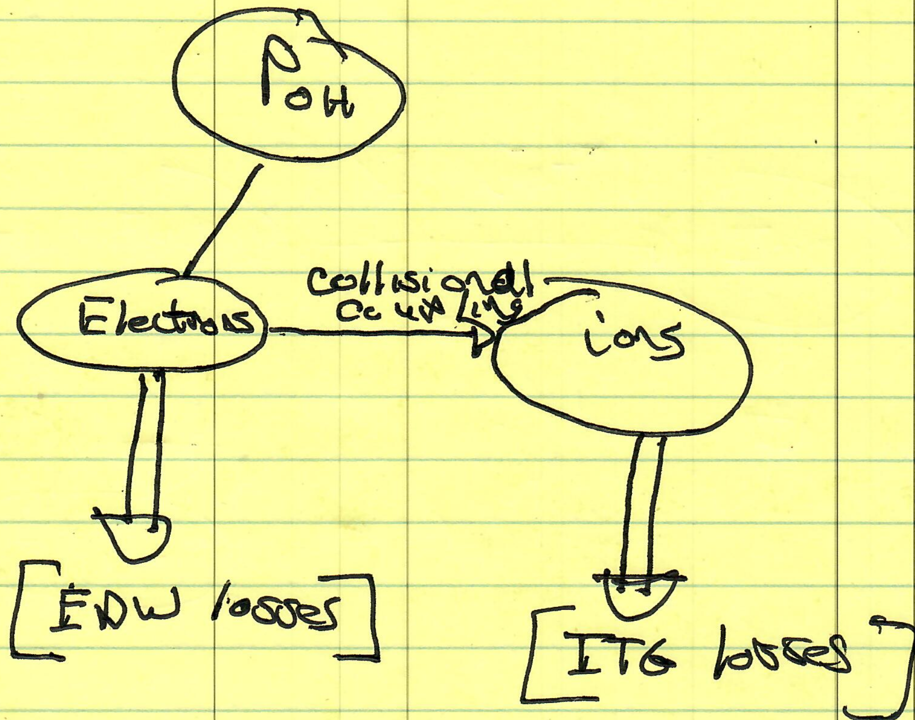
Note energetics:



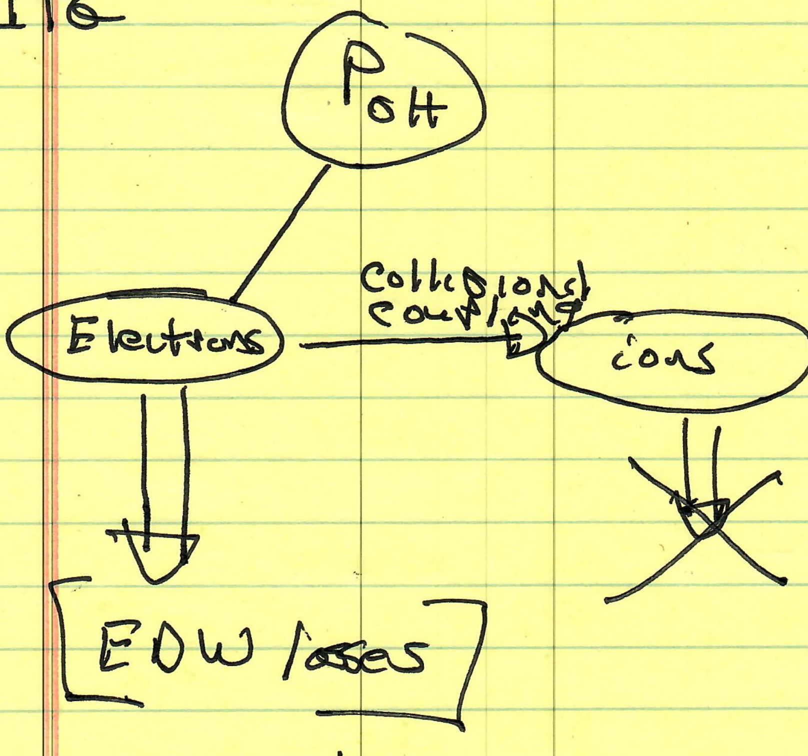
Note: Collisional coupling converts electron heating to ion ∇T_i free energy, and can trigger ITG loss channel.

i.e. ECH + high n $\Rightarrow$ ITG.

Now



with peaked n-profile → eliminate ITG



c.f. $\chi_{HPP} \sim 1/n$ in peaked density plasmas

⇒ 'Flat density' ITG

one reason why η_i "is" [redacted] becomes "ITG":

(Can also have "flat density" drift/TEM → ∇T_i ⇒ wave
$\boxed{\nabla T_e \Rightarrow \text{drives}}$)

→ Accuracy ?

- Fluid theory not accurate for threshold, useful for physical understanding

- $\omega^2 = k_\parallel^2 v_{Ti}^2 \left(\frac{2}{3} - \eta_i\right)$

$\omega^2 \lesssim 0$ ⇒ $\omega - k_\parallel v_\parallel$ easily resonant

∴ ion Landau damping enters.

⇒ $\boxed{\text{Kinetics}}$

→ PV ? shear flow can modify significantly → Dimits shift !

- $\underbrace{\nabla_\parallel \tilde{v}_\parallel}_{H\text{-}M}$ breaks PV conservation of H-M.

⇒ can formulate PV budget, Potential enstrophy budget. Useful.

→ Related ? — $\begin{cases} \text{PSFI} \\ \text{PVG} \\ \vdots \end{cases}$ { Parallel shear flow instability }

$\underline{\nabla \langle V_\parallel \rangle}$ driven instability ! (S)

So what ? — shear flow ! ... (R)

Negative compression, not KH ! (S)
↓ Inf. Rt
acoustic coupling !

⇒ $\boxed{\text{'relative' of ITG}/\eta_i \Rightarrow \text{PSFI}}$

⇒ couples linearly to ITG/η_i.

⇒ rather delicate, but $\underline{\text{active}}$.

To clarify, $\underline{\text{put}}$ $\begin{cases} \nabla T \to 0 \\ \nabla n \to 0 \end{cases}$, $\underline{T_i \to 0}$

$$\partial_t \frac{|e|\hat{\phi}}{T} + \nabla_\parallel \tilde{V}_\parallel = 0$$

PVG
↓

$$\partial_t \tilde{V}_\parallel = -V_{Er}\, \partial_r \langle V_\parallel \rangle - c_s^2 \nabla_\parallel \frac{|e|\phi}{T}$$

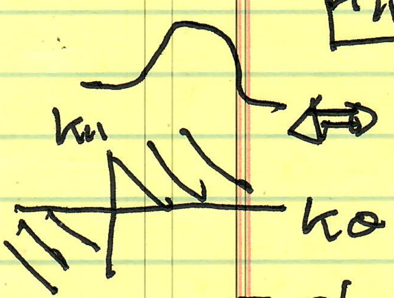
$$\partial_t^2 \frac{|e|\hat{\phi}}{T} + \nabla_\parallel \left(-V_{Er}\, \partial_r \langle V_\parallel \rangle - c_s^2 \nabla_\parallel \frac{|e|\hat{\phi}}{T} \right) = 0$$

$$-\omega^2 - k_\parallel k_\theta \rho_s c_s \partial_r \langle V_\parallel \rangle + k_\parallel^2 c_s^2 = 0$$

$$\boxed{\begin{aligned} \omega^2 &= -k_\parallel k_\theta \rho_s c_s \partial_r \langle V_\parallel \rangle + k_\parallel^2 c_s^2 \\ &= -k_\parallel c_s \left(k_\theta \rho_s \partial_r \langle V_\parallel \rangle - k_\parallel c_s \right) \end{aligned}}$$

Need $-k_\theta \rho_s \partial_r \langle V_\parallel \rangle > k_\parallel c_s$ { for instability
$\Rightarrow$ criterion on $\nabla_r \langle V_\parallel \rangle$

- $\boxed{k_\parallel k_\theta}$, [redacted] > 0 for $\partial_r \langle V_\parallel \rangle < 0$



$\Leftrightarrow$ spectral asymmetry { required / develops

- clearly negative compression ;
$\Rightarrow$ $\nabla \langle V_\parallel \rangle$ induces needed phase.
- ∇n stabilizing, (as M_i)

- couples to ITG. cf. N Mattor, PD.
  and ultimately $\chi_\phi \sim \chi_i$

PSFI **most** important in strong rotation, flat n regimes ⇒ core H-mode, with NBI, strong.

→ **Also**: Toroidal ITG

→ curvature driven

→ commonality with FLR stabilized interchange $\omega_D \omega_{*p_i}$ vs ω_{*n}^2

→ R/L_{Ti}, L_n/L_{Ti} are critical parameters.

→ Fluid, but mechanism different than negative compression.

→ (Zonal) Flows and Saturation

→ clearly $\underline{\nabla} \cdot \underline{V}_{,pol} \rightarrow \frac{d}{dt} \nabla_\perp^2 \tilde{\phi}$

$\rightarrow \partial_r \langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle$

⇒ ITG generates zonal flows !

⇒ additional coupling from diamagnetic contribution

⇒ shear feedback, as usual.

also Zonal eqns:

→ $\underbrace{\partial_r \langle \tilde{V}_r \tilde{T} \rangle + \partial_t \langle T \rangle}_{\text{Zonal}} + \ldots\ldots = 0$

⇒ Ion temperature profile corrugations formed.

$\left\{ \begin{array}{l} \phi_Z \\ \delta T_Z \end{array} \right.$

also

→ $\underbrace{\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle}$ $+ \cdots = 0$

⇒ (Zonal) acoustic flows generated.

Note:

$$\tilde{v}_{\parallel} = -\frac{i\, v_{Ti}^2 k_{\parallel} \tilde{V}_{Er}}{L_{Ti}\, \omega^2 (1 - \;)}$$

small

$$\omega_r^2 + 2i\gamma\omega_r - \gamma^2$$

so

$$\langle \tilde{v}_r \tilde{v}_{\parallel} \rangle = + \Pi_r \frac{\partial \langle T_i \rangle}{\partial r}$$

symmetry ?

$$\Pi_r = \mathrm{re}\left\{ \sum_n |V_{r_n}|^2 (-i)\, \frac{v_{Ti}^2 k_{\parallel}}{T_i (\omega_r + 2i\gamma\omega_r - \gamma^2)} \right\}$$

i.e. then:

$$\partial_t \langle \delta V_{\parallel} \rangle = -\partial_r \left(\Pi_r \frac{\partial \langle T_i \rangle}{\partial r} \right)$$

$\nabla \langle T_i \rangle$ drives parallel flow

$\Rightarrow$ welcome to intrinsic rotation!

→ Flows driven by :

$$Q_i \rightarrow \nabla T_i \rightarrow \langle \tilde{V}_r \tilde{V}_\parallel \rangle \rightarrow \langle V_\parallel \rangle$$

Plasma can rotate without torque.

→ symmetry breaking required.

What sets $\langle k_\parallel \rangle$?

aka α effect in dynamo.

TBC.

also : Impurity Modes

offshoot of ITG $\Rightarrow$

→ Particle Transport and "Pinch"

- recall basic theory of ITG/η_i

had Boltzmann electrons:

$$\frac{\tilde{n}}{n} = \frac{|e|\tilde{\phi}}{T}$$

$$\Rightarrow \underbrace{Q_i}_{\text{ion heat transport}} \neq 0 , \qquad \Gamma_n = 0$$

- if $\frac{\tilde{n}}{n}_k = \frac{|e|\tilde{\phi}}{T}_k (1 + i\delta_k)$

$$\delta_k = \delta n_k / \delta\phi_k$$

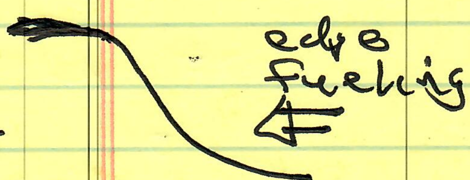
can induce Γ_n, [a rider]

typically, $\bar{\Gamma}_n < \bar{Q}_i$. — = with $\frac{\tilde{n}}{n}$, $\frac{\tilde{T}}{T}$

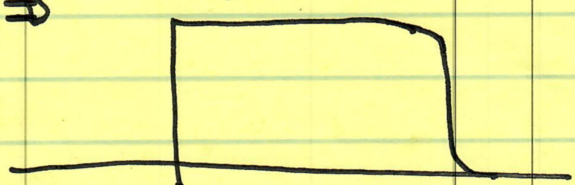
- some effect on stability, especially near marginal

- but ∇T_i relaxation driving Γ_n opens route to up-gradient transport i.e. pinch.

- Recall Lecture 1:



$\Rightarrow$ why not flat?



OH

(NBI - beam fueling)

$\boxed{\text{Fueling is key problem in ITER}}$

Answer: Flux includes convection

$$\Gamma_n = -D_n \nabla\langle n\rangle + V\langle n\rangle$$

with $V < 0$ "Pinch" $\Rightarrow$ up-gradient convection if $V<0$.

then
$$\frac{\nabla\langle n\rangle}{\langle n\rangle} = \frac{V}{D_n}$$

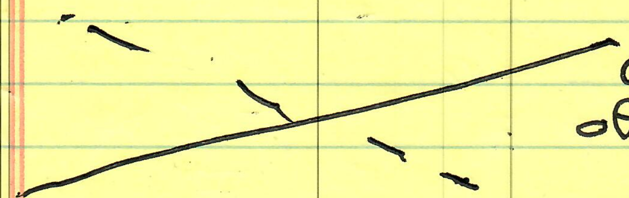
V is key to density profile formation.

Where does V originate from?

→ An Off-beat Perspective:

<u>Chemotaxis</u> ⇒ chemically induced motion

i.e.



c → concentration of nutrients

$n(\underline{x},t)$ → density of bacteria

$c(x,t)$ profile can induce up-gradient (in n) flux of bacteria.

More generally ⇒ c profile induces n flux.

⇒ simple, <u>statistical</u> theory of chemotactic response.

E.F. Keller, L.A. Segel (1971)

How does $n(x,t)$ evolve?

→ Keller-Segal Model

 — bacteria, with sensors

receptors

steps $\pm \Delta$

— length organism $\alpha \Delta$

— move by "pulling" (i.e. amebas) or "pushing" (flagella).

$$\begin{cases} C \equiv \text{food supply profile} \\ f \equiv \text{frequency of steps in given direction} \end{cases}$$

so

$$\text{Net Flux } J = \underbrace{\int_{x-\Delta}^{x} \overbrace{f\left(c\left(s+\tfrac{1}{2}\alpha\Delta\right)\right)}^{\#\text{ steps}} \overbrace{n(s)}^{\text{density bacteria}}\, ds}_{\text{steps} \to \text{right}}$$

$$- \underbrace{\int_{x}^{x+\Delta} F\left(c\left(s-\tfrac{1}{2}\alpha\Delta\right)\right) n(s)\, ds}_{\text{steps} \to \text{left}}$$

N.B.: Could re-write;

→ avg cell in $(x-\Delta, x)$ centered at $x - 1/2\,\Delta$

then first term in J:

$$\sim f\left[c\left(x + \tfrac{1}{2}\Delta(\alpha-1)\right)\right] n\left(x - \tfrac{1}{2}\Delta\right) \Delta$$

etc.

Expanding J, as in Fokker-Planck Theory:

$$\frac{\partial n}{\partial t} = -\underline{\nabla} \cdot \underline{J}$$

$$\underline{J} \cong \Delta^2 \left\{ -f(c(x))\, n'(x) + (\alpha-1) f'(c(x))\, n(x)\, c'(x) \right\}$$

drift ~ c'

⇓

$$= -D \frac{\partial \langle n \rangle}{\partial x} + T \langle n(x) \rangle \left(\frac{\partial c(x)}{\partial x} \right)$$

f(c) is frequency

diffusivity $\dfrac{\Delta^2}{\Delta t} \sim f(c)\Delta^2$

convection drift.

$\sigma \equiv$ chemo-tactic coefficient

$$\sigma = (\alpha - 1)\, f'(c)\, \Delta^2$$

$$= (\alpha - 1)\,(\partial_x D)$$

for point particles / heaters

Blobs

$$\sigma = -\partial_x D$$

Thus, have:

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x} J \; ; \qquad J = -D \frac{\partial n}{\partial x} + \sigma \frac{\partial c}{\partial x} n$$

$$\underline{J} = -D \underline{\nabla} n + \sigma n \underline{\nabla} c$$

food concentration gradient drives flux (non-diffusive)

N.B.:

Directly from Fokker-Planck theory:

$$\frac{\partial n}{\partial t} = -\underline{\nabla}\cdot\underline{\Gamma}$$

diffn

$$\Gamma = \left\langle \frac{\Delta \underline{x}}{\Delta t} \right\rangle n - \underline{\nabla}\cdot\left(\frac{\langle \Delta \underline{x}\, \Delta \underline{x} \rangle}{2\Delta t} n \right)$$

$$V = \left\langle \frac{\Delta \underline{x}}{\Delta t} \right\rangle \Rightarrow \tau\, \underline{\nabla} c\, n$$

drift

$\underline{\nabla} c$ determines drift

$$\boxed{\text{Point: Concentration gradient of food drives flux of bacteria}}$$

(N.B.: What is the "food" for up-gradient particle flux in tokamak ?)

Steady profile:

$$\frac{\underline{\nabla} n}{n} = \frac{\tau\, \underline{\nabla} C}{D}$$

$$\frac{\underline{\nabla} n}{n} < 0 \Rightarrow \text{need } \tau\, \underline{\nabla} C < 0.$$

→ Back to tokamaks: $\quad \delta_{\underline{k}} = \frac{\delta n}{\delta \frac{|e|\phi}{T}}$

$$\Gamma = \langle \tilde{v}_r \frac{\tilde{n}}{n_0} \rangle = +\sum_{\underline{k}} \frac{ic}{B} k_\theta \hat{\phi}_{-k}\, i\delta_{\underline{k}} \frac{|e|\hat{\phi}_k}{T}$$

$$= -\sum_{\underline{k}} c_s (k_\theta \rho_s)\, \delta_{\underline{k}} \left|\frac{e\phi_k}{T}\right|^2$$

$\delta_{\underline{k}} < 0$ → outward flux. (instability for DW)

Now:

- $\delta_{\underline{k}}$ set by electrons

  (i.e. non-adiabatic electron response!)

- $\delta_{\underline{k}}$ encodes the "food".

- $\left|\frac{e\phi_k}{T}\right|^2$ accounts for the energy for steps.

- $\delta \delta_{\underline{k}} / \delta \nabla n > \quad$ so: $\Gamma \sim -D \nabla n$

i.e. particle diffusion must be positive

$\leftrightarrow$ entropy production !

$$\frac{\partial S}{\partial t} \sim -\Gamma \nabla n$$

$$= D \nabla n^2 - \nabla n \, \underline{\nabla n} \cdot \underline{\nabla} C$$

$S \to -\infty$ if $D < 0$.

- then { "food" → electron temperature
$\underline{\nabla} C \sim \underline{\nabla} T_e$
random walk kicks → ITG turbulence.

expect, for pinch:

$$\frac{\delta \, \delta_n}{\delta \nabla n} > 0, \qquad \frac{\delta \, \delta_n}{\delta \nabla T_e} < 0$$

$\Downarrow$

need $\delta_{\underline{n}}$ phase opposite for ∇T_e, ∇n.

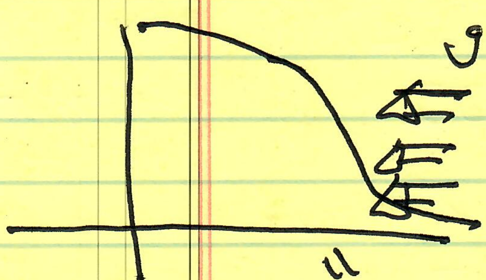
→ This brings us to:
"Ion Mixing Mode"

(Coppi and Spight '78)

→ η_i/ITG mode + collisional non-adiabatic electrons with ∇T_e.

Proto-type of Pinch

∇T_e → shifts sign δ_n → $V<0$



ν_{ee} → cooling → ITG + ∇T_e

⇒ influx particles.

"edge" version ITG

Analysis:

$$- \frac{k_\parallel^2 v_{Thi}^2}{\nu_i} < \hat{\omega} < \frac{k_\parallel^2 v_{The}^2}{\nu_{ei}}$$

ν_{ee}

$\alpha > 1$ → weak non-adiabatic electrons

For electrons:

$$-\frac{\partial \tilde{n}}{\partial t} + \tilde{V}_{r_E}\nabla\langle n\rangle + \langle n\rangle \nabla_{\parallel}\tilde{V}_{\parallel e} = 0$$

- neglect collisional transfer, inertia

$$0 \cong -\nabla_{\parallel}\tilde{P}_e + |e|n\tilde{\phi} - \alpha_T n \nabla_{\parallel}\tilde{T}_e$$

thermal force coeff (c.f. Braginsky)

$$\cong -\nabla_{\parallel}(\tilde{n}T_e + \tilde{T}_e n_0) + |e|n_0\tilde{\phi} - \alpha_T n\nabla_{\parallel}\tilde{T}_e$$

$$\Rightarrow \frac{\tilde{n}}{n} = \frac{|e|\tilde{\phi}}{T_e} - \frac{\tilde{T}_e}{T}(1+\alpha_T)$$

need $\tilde{T}_e/T$ for QN

$$-\frac{3}{2}n\left(\frac{\partial}{\partial t}\tilde{T}_e + \underline{\tilde{V}}_E\cdot\nabla\langle T_e\rangle\right)$$

$$+ nT_e\nabla_{\parallel}\tilde{V}_{\parallel e} = \nabla_{\parallel}\chi_{\parallel}\nabla_{\parallel}T_e$$

parallel conduction

$$\chi_{\parallel} = v_{The}^2/\nu_{ee}$$

$\alpha >> 1$

so

$$\frac{\tilde{T}_e}{T} = \frac{-i}{\left(\chi_\parallel k_\parallel^2 - i\frac{3\omega}{2}\right)}\left[\omega_{*e}\left(\frac{3}{2}\eta_e - 1\right)\frac{|e|\tilde{\phi}}{T_e} + \omega\frac{\tilde{n}_e}{n}\right]$$

$\Rightarrow$

$$\frac{\tilde{n}_e}{n} = \frac{|e|\tilde{\phi}}{T_e}\left[1 + (1+\alpha_T)\frac{i}{\chi_\parallel k_\parallel^2}\left(\omega - \omega_{*e} + \frac{3}{2}\omega_{*e}\eta_e\right)\right]$$

like CDW, usual dissipative phase shift

∇n inverse dissp.

∇T opposite sign phase shift $\Rightarrow$ dissipation will pinch

$$\left(\frac{\partial \delta_n}{\partial \nabla n}\right)\left(\frac{\partial \delta_n}{\partial \nabla T}\right) < 0.$$

$\Rightarrow$ ∇T_e feed.

need $\begin{cases} \nabla T_e < 0 \leftrightarrow \nabla C < 0 \\ \nabla > 0 \Leftrightarrow \delta_{\nabla T_e} > 0 \end{cases}$

Then:

– $\frac{\tilde{n}_e}{n_0} = \frac{\tilde{n}_i}{n_0}$, as before.

$\Rightarrow$

$\omega_{*Ti}/\omega_{*e} < 0$

$$-1 + iA\, k_\parallel^2 c_s^2 \frac{\omega_{*Ti}}{\omega^3} - \frac{\omega_{*e}}{\omega} = 0$$

$$A = \frac{3}{2} \mu_e \left(\omega_{*e}/\chi_\parallel k_\parallel^2\right)(1+\alpha_T) \sim O\left(\frac{\omega_{*e}}{\chi k_\parallel^2}\right) \ll 1$$

– $\omega \cong -\left(k_\parallel^2 c_s^2 \omega_{*Ti}/(1+A^2)\right)^{1/3} (1-iA)^{1/3}$

$\nabla n \to 0$ (no diffn)

ITG mode (usual)

correction

~ basically same as "flat n" case discussed previously

→ Finally, proceeding in Flat-n regime ($D_n \to 0$)!

$$\boxed{\Gamma_n = 3(1+\alpha_T)\langle n\rangle c_s \sum_k k_\theta^2 \rho_s^2 \left|\frac{e\phi_k}{T}\right|^2 \frac{c_s}{x k_\parallel^2} \frac{1}{\langle T\rangle} \frac{\partial\langle T\rangle}{\partial x}}$$

$$\frac{\partial\langle T\rangle}{\partial x} < 0 \Rightarrow \boxed{\begin{array}{c}\Gamma_n < 0 \\ \text{inward}\end{array}}$$

→ ITG / Ion Mixing Mode (inward) pinch.

→ Physics

$$\frac{\dot{S}}{nT} = -\frac{D\langle n\rangle}{n}\,\overline{\Gamma}_n - \frac{D\langle T\rangle_i}{T}\,\overline{Q}_{T_i}$$

↳ normalized entropy production:

growth rate correction 5.

$$\frac{\dot{S}}{nT} = \chi_T \left(\frac{\nabla \langle T_i \rangle}{T}\right)^2 + O\left(\omega / \chi_\parallel k_\parallel^3\right)$$

$$+ D \left(\frac{\nabla n}{n}\right)^2 - \frac{\nabla \langle n \rangle}{n} \Gamma_{n,na}$$

$$= \chi_T \left(\frac{\nabla \langle T_i \rangle}{T}\right)^2 + O\left(\omega / \chi_\parallel k_\parallel^2\right)$$

$$+ D_n \left(\frac{\nabla n}{n}\right)^2 - \underbrace{\left(\quad\right)}_{O\left(\frac{\omega}{\chi_\parallel k_\parallel^2}\right)} \frac{\nabla \langle n \rangle}{\langle n \rangle} \frac{\nabla \langle T_e \rangle}{T_e} \langle n \rangle$$

even for T_e fixed (i.e. fixed food supply),

→ ① → ∇T_i relaxation

– dominates entropy production

→ (4) - entropy destruction / production penalty due $V < 0$

$- \mathcal{O}\left(\omega / \chi_{\parallel} k_{\parallel}^2\right)$

→ (2) similar correction.

Thus, no issue with H-Thm / second Law ⇒ ITG relaxation produces more entropy than pinch destroys.

→ as ∇n steepens due pinch,
- flat n fails
⇓
- need retain ∇n in non-adiabatics

→ Physics is relaxation driven density transport and Chemotaxis.

→ Analogy with Chemotaxis:

Density	Bacterial Density
∇T_e	∇C (food supply)
$\nabla T_i / M_i$ $\langle \tilde{V}_r^2 \rangle$	Bacterial energy (i.e. ATP) → Δ^2
phases	sensing fctn.

→ IOC {Improved ohmic Confinement} (ASDEX '88)

- peaked n ⇒ good confinement

- how?: turn off fueling/gas valve ⇒ edge effect

→ why ⇝ Open question?!

⇝ speculate ∇T_i driven pinch which peaks n which turns off ITG.

→ No edge cooling / fueling to overload particles

→ Coupled heat / particle + instability evolution.

→ Dynamics ?