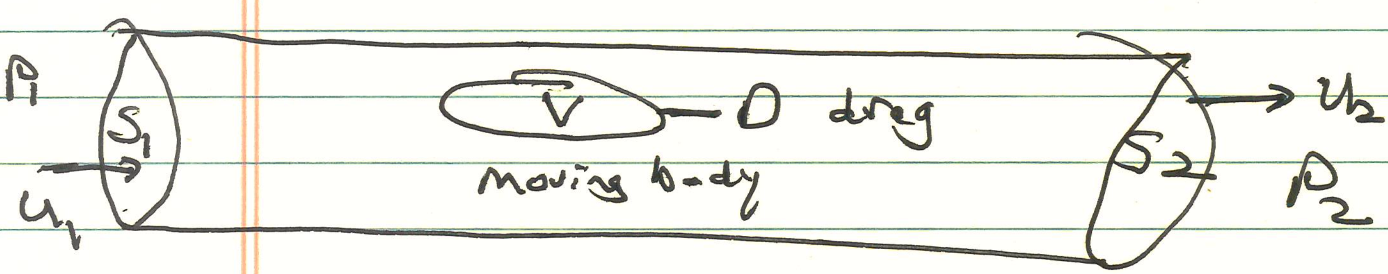
→ Notes 16: D'Alembert's Principle

→ "Drag on Ideal Fluids – D'Alembert's "Paradox" (Thm.)



Consider a body moving in an ideal fluid in a closed, frictionless container.

net Force, downstream:

$$F_{ds} = \int_{S_1} p_1 \, dS_1 - \int_{S_2} p_2 \, dS_2 - D$$

but this must be equal to difference in momentum fluxes thru bounding surfaces

$$F_{ds} = \int_{S_2} \rho u_2^2 \, dS_2 - \int \rho u_1^2 \, dS_1$$

so

$$\int_{S_1} P_1\, dS_1 - \int_{S_2} P_2\, dS_2 = \int_{S_2} \rho u_2^2\, dS_2 - \int \rho u_1^2\, dS_1$$
$$- D$$

so

$$D = \int_{S_2} (P_2 + \rho u_2^2)\, dS_2 - \int_{S_1} (P_1 + \rho u_1^2)\, dS_1$$

$$= \int dS_2\, \Pi_{zz}\Big|_2 - \int dS_1\, \Pi_{zz}\Big|_1$$

$$\underline{\underline{\Pi}} = \rho u_i u_k + \delta_{ij} P$$

$$\partial_t(\rho V_i) = -\partial_k \Pi_{ik}$$ (motion along z)

Now, in ideal fluid;

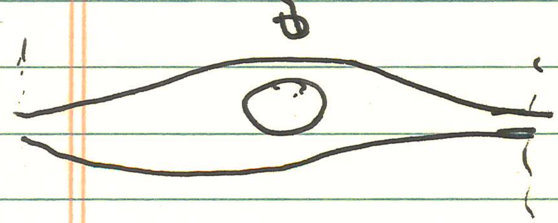
$$\underline{V}_T = \underline{U} + \underline{V}_{fluid}$$

$V_{fluid} \sim 1/r^3 \rightarrow$ dipole.

so, if take $2 \rightarrow +\infty$, $1 \rightarrow -\infty$ — move boundys to ∞.

$$\Pi_{zz} = \rho U^2 + P_0,$$ both

attached wake



i.e. asymptotically, $\Pi_{zz} \rightarrow \rho U^2 + P_0$
(no far field footprint of motion)

So

$$D = 0 \; !$$

→ No drag on body in ideal fluid.

→ Consequence of asymptotic behavior of flow and wake
i.e. attached wake.

→ exception: body near surface radiates.