Physics 4E   Quiz 2 Solutions   Spring 2020

Problem 1

$$b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot\frac{\theta}{2}, \quad q_\alpha = 2e, \quad Q = Ze. \quad Z = 47.$$

$E_k = \frac{1}{2} m_\alpha v^2 = 6\text{ MeV}$ kinetic energy of α particle

(a) $f = \pi b^2 n t$ is fraction of α-particles scattered at angles larger than θ with $b = b(\theta)$

So:
$$\frac{f(\theta = 60°)}{f(\theta = 120°)} = \frac{b(60°)^2}{b(120°)^2} = \frac{\cot(30°)^2}{\cot(60°)^2} = \frac{(\sqrt{3})^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 9$$

So if 100 α particles are scattered at angle $> 120°$, 900 are scattered at angle $> 60°$

$\Rightarrow$ $\boxed{\text{800 } \alpha\text{-particles per second are scattered at angles between } 60° \text{ and } 120°}$

(b) The distance of closest approach, for impact parameter $b = 0$, is:

$$d_{min} = \frac{k q_\alpha Q}{E_k} = \frac{2ke^2 Z}{E_k} = \frac{2 \times 14.4 \times 47}{6 \times 10^6} \text{Å} = 2.26 \times 10^{-4} \text{Å}$$

Since Rutherford's theory holds for all angles $\Rightarrow$ radius of nucleus (R) is smaller than d_{min}, i.e. α particles don't get inside nucleus. $\boxed{R < 2.26 \times 10^{-4} \text{Å}}$

(c)
$$b(\theta) = \frac{k q_\alpha Q \cot(\theta/2)}{m_\alpha v^2} = \frac{1}{2}\frac{k q_\alpha Q}{E_k} \cot(\theta/2) = \frac{d_{min}}{2}\cot(30°) = \frac{d_{min}}{2} \cdot \sqrt{3}$$

$$\Rightarrow \boxed{b(\theta = 60°) = 1.95 \times 10^{-4} \text{Å}}$$

Problem 2

$$E_n - E_m = \frac{hc}{\lambda_{nm}} = E_0 Z^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right) \quad ; \; E_0 = 13.6\,\text{eV}$$

$$\Rightarrow \frac{1}{\lambda_{nm}} = \frac{Z^2}{911.76\,\text{Å}}\left(\frac{1}{m^2}-\frac{1}{n^2}\right) \Rightarrow \lambda_{nm} = \frac{911.76\,\text{Å}}{Z^2}\,\frac{1}{\left(\frac{1}{m^2}-\frac{1}{n^2}\right)}$$

We have $\frac{1}{m^2}-\frac{1}{n^2} < 1$. Therefore, minimum λ_{nm} for given Z is $\frac{911.76\,\text{Å}}{Z^2}$

So for $100\,\text{A} < \lambda < 130\,\text{A}$, we need $Z \geq 3$

(a) For $Z=3$, if we take $m=1, n=2 \Rightarrow \frac{1}{m^2}-\frac{1}{n^2} = \frac{3}{4}$

$$\Rightarrow \lambda = \frac{911.76}{9}\,\text{Å} \cdot \frac{4}{3} = 135\,\text{Å}.$$ That is too large.

So take $m=1, n=3 \Rightarrow \frac{1}{m^2}-\frac{1}{n^2} = 1-\frac{1}{9} = \frac{8}{9}$

$$\Rightarrow \lambda = \frac{911.76}{9} \times \frac{9}{8} = 113.97\,\text{Å}.$$ So

$\boxed{\begin{array}{l} Z=3 \\ \text{initial state}: m=1 \\ \text{final state}: n=3 \end{array}}$ (a)

(b) For $Z=4$:

$$\lambda = 57\,\text{Å}\,\frac{1}{\frac{1}{m^2}-\frac{1}{n^2}} = 57\,\text{Å} \times \frac{9}{5} = 102.6\,\text{Å} \leftarrow$$

Take e.g. $m=2, n=3 \Rightarrow \frac{1}{m^2}-\frac{1}{n^2} = \frac{1}{4}-\frac{1}{9} = \frac{5}{\cancel{9}\;36}$

$\boxed{\begin{array}{l} Z=\cancel{4}\;8 \\ \text{initial}: m=2 \\ \text{final}: n=3 \end{array}}$ (b)

(C) Ionize from ground state: $m=1, n=\infty \Rightarrow \frac{1}{m^2}-\frac{1}{n^2} = 1$

$\lambda \leq \frac{911.76}{Z^2}\,\text{Å}$ will ionize, extra energy goes into kinetic energy.

$Z=1$: $\lambda \leq 911.76\,\text{Å}$ yes

$Z=2$: $\lambda \leq 227.94\,\text{Å}$ yes

$Z=3$: $\lambda \leq 101.31\,\text{Å}$ yes

$Z=4$: $\lambda \leq 57\,\text{Å}$ NO

$\Rightarrow$ $\boxed{\text{For values } Z=1,2,3, \text{ this radiation will ionize the ions}}$

Problem 3

For Bohr atom, $mv^2 = \frac{ke^2}{r}$. If T is period of orbit,

$v = \frac{2\pi r}{T} \Rightarrow m \cdot \frac{4\pi^2 r^2}{T^2} = \frac{ke^2}{r} \Rightarrow r^3 = \frac{ke^2 T^2}{4\pi^2 m} \Rightarrow$

$\Rightarrow r = \left(\frac{ke^2 T^2 c^2}{4\pi^2 m c^2}\right)^{1/3} = \left(\frac{14.4 \times (7.77\times 10^{-15})^2 \times (3\times 10^{18})^2}{4\pi^2 \times 511{,}000}\right)^{1/3}$

$\Rightarrow \boxed{r = 33.85\,\text{Å}}$ from $r = a_0 n^2 \Rightarrow \boxed{n = 8}$, $\boxed{L = 8\hbar}$

(a) (b)

Alternative solution: from ang momentum quantization one finds

$v = \frac{\hbar}{m_e a_0 n}$. Here $v = \frac{2\pi r}{T} = \frac{2\pi a_0 n^2}{T} \Rightarrow$

$\frac{\hbar}{m_e a_0 n} = \frac{2\pi a_0 n^2}{T} \Rightarrow n^3 = \frac{\hbar T}{2\pi m_e a_0^2} = \frac{\hbar c}{2\pi m_e a_0^2} \cdot c \cdot T \Rightarrow$

$n^3 = \frac{1973 \times 3\times 10^{18} \times 7.77 \times 10^{-15}}{2\pi \times 511{,}000 \times 0.529^2} = 512 \Rightarrow \boxed{n = 8}$ etc

(c) Potential energy:

$U = -\frac{ke^2}{r} = -\frac{14.4\,\text{eV Å}}{33.85\,\text{Å}} \Rightarrow \boxed{U = -0.425\,\text{eV}}$ (c)

(d) De Broglie wavelength: $\lambda = \frac{h}{p}$

Angular momentum:

$L = n\hbar = mvr = pr = \frac{h}{\lambda} r \Rightarrow \lambda = \frac{hr}{\hbar n} = \frac{2\pi r}{n}$

so $\lambda = \frac{2\pi r}{n} = \frac{2\pi \times 33.85\,\text{Å}}{8} \Rightarrow \boxed{\lambda = 26.59\,\text{Å}}$