

Justify all your answers to all 3 problems. Write clearly.

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$ $\gamma = 1 / \sqrt{1 - v^2 / c^2}$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron: $m_e = 0.511 \text{ MeV}/c^2$ Proton: $m_p = 938.26 \text{ MeV}/c^2$ Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit: $1 u = 931.5 \text{ MeV}/c^2$; electron volt: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law: $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = C e^{-E/(k_B T)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, ϕ = work function;]

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$ $ke^2 = 14.4 \text{ eV \AA}$

Coulomb force: $F = \frac{kq_1 q_2}{r^2}$; Coulomb energy: $U = \frac{kq_1 q_2}{r}$; Coulomb potential: $V = \frac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n(\theta) = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\theta/2)}$; $b = \frac{kq_\alpha Q}{2K_\alpha} \cot(\theta/2)$

Hydrogen spectrum: $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = 1.09678 \times 10^7 \text{ m}^{-1} = \frac{1}{911.8 \text{ \AA}}$

Bohr atom: $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$; $L = m_e v r = n\hbar$ angular momentum