

Justify all your answers to all 3 problems. Write clearly.

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation : $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse : $v \rightarrow -v$

Velocity transformation : $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse : $v \rightarrow -v$

Spacetime interval : $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$; $\gamma = 1/\sqrt{1 - v^2/c^2}$

Relativistic Doppler shift : $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum : $\vec{p} = \gamma m \vec{u}$; Energy : $E = \gamma mc^2$; Kinetic energy : $K = (\gamma - 1)mc^2$

Rest energy : $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law : $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W}/\text{m}^2 \text{K}^4$

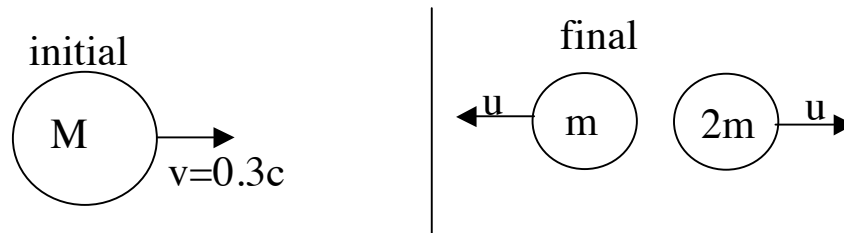
$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution : $P(E) = C e^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

$hc = 12,400 \text{ eV}\cdot\text{\AA}$; $k_B = (1/11,600) \text{ eV}/\text{K}$; $1 \text{ \AA} = 10^{-10} \text{ m}$

Problem 1 (10 points)



An object of mass M moving with speed $v=0.3c$ spontaneously splits into masses $2m$ and m , moving along the same line in the same and in the opposite direction with the same speed u , as shown in the figure.

- Find u (give your answer as u/c).
- (i) Find m in terms of M . (ii) Find the amount of mass converted into energy, in terms of M .
- Calculate the initial and final kinetic energies in terms of M , and verify that their difference accounts for the result found in (b)(ii).

Problem 2 (10 points)

An electron and a proton have the same momentum, $p=15\text{MeV}/c$.

- (a) Find their kinetic energies, K_e , K_p , in MeV.
- (b) Find their speeds relative to the speed of light, u_e/c , u_p/c .
- (c) Calculate their speeds and their kinetic energies assuming classical mechanics is valid instead of relativistic mechanics. Compare the results with the relativistic results obtained in (a) and (b), and explain why in one case they are close and in the other they are not.

Problem 3 (10 points)

A sphere emits electromagnetic radiation as a black body, with maximum power per unit wavelength for wavelength $10,000\text{\AA}$ ($1\text{\AA}=10^{-10}\text{m}$). The total power that it emits for all wavelengths is $5,000\text{W}$.

- (a) What is its temperature?
- (b) What is its radius, in cm?
- (c) Is the power per unit wavelength emitted at wavelength $100,000\text{\AA}$ smaller, equal or larger than that emitted at wavelength $1,000\text{\AA}$? Find approximately their ratio.