[1] Consider the one-dimensional dynamical system given by

\[ \frac{dN}{dt} = rN - b(e^{N/K} - 1) \]

(a) Show that \( N \) and \( t \) can be rescaled to \( \nu \) and \( s \) such that

\[ \frac{d\nu}{ds} = c\nu - e^\nu + 1 \]

Find \( \nu, s, \) and \( c \) in terms of \( N, t, r, K, \) and \( b \).

[20 points]

(b) Show that a bifurcation occurs at a critical value \( c = c^* \). Find \( c^* \) and identify the type of bifurcation. Show what’s happening in a sketch.

[30 points]

[2] Consider the two-dimensional dynamical system given by

\[ \frac{dx}{dt} = x - y - x^3 \]
\[ \frac{dy}{dt} = rxy - y^2 \]

where \( r > 0 \).

(a) Assuming \( r > 1 \), how many fixed points are there? Find them. \textit{Hint: Start with the second equation.}

[10 points]

(b) Show that for \( r < 1 \) there are two more fixed points. Find them.

[10 points]

(c) Expanding about a fixed point \((x^*, y^*)\), with \( u_x \equiv x - x^* \) and \( u_y = y = y^* \), the linearized dynamics takes the form \( \dot{u} = M \dot{u} \), where \( M \) is a \( 2 \times 2 \) matrix. Find an expression for \( M \) at the fixed point \((x^*, y^*)\).

[20 points]

(d) What is the classification of the fixed point \((x^*, y^*) = (0, 0)\)?

[10 points]