(1) Consider the following vector field, described in 2D polar coordinates:
\[
\dot{r} = r(1 - r^2) + \lambda r \cos \theta \\
\dot{\theta} = 1,
\]
where \(0 < \lambda < 1\).

(a) Show that the radial component of the flow along the circle \(r = a \equiv \sqrt{1 - \lambda}\) is outward, and that the radial component along the circle \(r = b \equiv \sqrt{1 + \lambda}\) is inward.

(b) For \(\lambda = 0.8\), plot the vector field.

(c) Numerically integrate the dynamical system starting from the point \((r = 1, \theta = 0)\), with \(\lambda = 0.8\) and identify the stable limit cycle.

(2) E. E. Sel’kov, in 1968, developed a simple kinetic model for self-oscillations in glycolysis, which is a biochemical process by which living cells break down sugar. In some cases, such as yeast cell extracts, and in heart muscle cell extracts, periodic oscillations of reaction products have been experimentally observed. An abbreviated, adimensionalized version of Sel’kov’s model is given by the \(N = 2\) dynamical system
\[
\dot{x} = -x + ay + x^2 y \\
\dot{y} = b - ay - x^2 y,
\]
where \(x\) and \(y\) are the concentrations of ADP and F6P (fructose-6-phosphate), respectively. The kinetic parameters \(a\) and \(b\) are both positive.

(a) Find the fixed point.

(b) Linearize the dynamics in the vicinity of the fixed point and construct the matrix \(M\) of first partial derivatives.

(c) Identify regions in the quadrant \((a > 0, b > 0)\) by fixed point classification, and find the boundaries between different fixed point behaviors in terms of the kinetic parameters.

(d) Plot the vector field for some representative \((a, b)\) values corresponding to different fixed point types.

(e) Integrating the equation of motion (using \texttt{NDSolve} in \textit{Mathematica}), show that stable limit cycle behavior pertains when the fixed point is unstable.

(3) Consider the two-dimensional phase flow,
\[
\dot{x} = x + xy^2 - x^3 \\
\dot{y} = 2y + xy - y^2.
\]
(a) Find and classify all fixed points.

(b) Sketch the vector field.