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Study of ion turbulent transport and profile formations using global gyrokinetic full- $f$ Vlasov simulation

Y. Idomura$^1$, H. Urano$^2$, N. Aiba$^2$ and S. Tokuda$^2$

$^1$ Japan Atomic Energy Agency, Higashi-Ueno 6-9-3, Taitou, Tokyo 110-0015, Japan
$^2$ Japan Atomic Energy Agency, Mukouyama 801-1, Naka, Ibaraki 311-0193, Japan

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Abstract

A global gyrokinetic toroidal full- $f$ five-dimensional Vlasov simulation GT5D (Idomura et al 2008 Comput. Phys. Commun. 179 391) is extended including sources and collisions. Long time tokamak micro-turbulence simulations in open system tokamak plasmas are enabled for the first time based on a full- $f$ gyrokinetic approach with self-consistent evolutions of turbulent transport and equilibrium profiles. The neoclassical physics is implemented using the linear Fokker–Planck collision operator, and the equilibrium radial electric field $E_r$ is determined self-consistently by evolving equilibrium profiles. In ion temperature gradient driven turbulence simulations in a normal shear tokamak with on-axis heating, key features of ion turbulent transport are clarified. It is found that stiff ion temperature $T_i$ profiles are sustained with globally constant $L_{\parallel i} \equiv |T_i'/T_i|$ near a critical value, and a significant part of the heat flux is carried by avalanches with $1/f$ type spectra, which suggest a self-organized criticality. The $E_r$ shear strongly affects the directions of avalanche propagation and the momentum flux. Non-diffusive momentum transport due to the $E_r$ shear stress is observed and a non-zero (intrinsic) toroidal rotation is formed without momentum input near the axis.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Five-dimensional (5D) gyrokinetic simulations are essential tools to study anomalous turbulent transport in tokamak plasmas [2–6]. Although a number of gyrokinetic simulations have been developed so far, most of the existing simulations are $\delta f$ simulations in an isolated system without sources and collisions or in an open system with fixed gradients. In a $\delta f$ approach, only the perturbed distribution function $\delta f$ is solved by forcing the equilibrium distribution $f_0$ to be fixed. $\delta f$ gyrokinetic simulations with fixed gradients have been successful in estimating steady transport levels for profiles observed in experiment (see e.g. [6]). However, it still has difficulty in addressing open issues such as the profile stiffness, transient transport properties and the formation of transport barriers [6]. In particular, it is very difficult to simulate turbulent transport with stiff profiles, because the experimental data contain finite errors and a slight change in fixed gradients leads to a large increase in turbulent transport [7]. To resolve this issue, a recent advanced gyrokinetic $\delta f$ simulation adopts an approach in which equilibrium profiles are adjusted until turbulent transport levels reach the experimental values, where the balance conditions of particle and heat fluxes are satisfied [4]. By using this approach, ion and electron heat fluxes in the experiment were successfully reproduced [8]. However, when one intends to reproduce the particle transport, the momentum transport and the heat transport for each particle species, simultaneously, one has to optimize all the related equilibrium profiles such as the density, the toroidal rotation, the temperature, the radial electric field and so on at each radial position. This may be a formidable task because turbulent transport often has non-local properties, and several transport channels have off-diagonal terms, which lead to indirect interaction among different transport channels. In addition, global $\delta f$ simulations normally use an adaptive source model to fix the equilibrium profiles on average [9], and its particle, momentum and heat source profiles also have to match the experiment. On the other hand, in a full- $f$ approach, such flux balance conditions are automatically satisfied by simply imposing experimentally relevant source profiles. Then, one observes turbulent transport and evolving equilibrium profiles, and if these profiles deviate from the experiment, one proceeds to improve physical models. This is the validation process of a full- $f$ approach, which may have less ambiguity.

In addition, former fluid simulations with fixed fluxes or fixed sources [10, 11] revealed an interaction between turbulent transport and equilibrium profiles, where gradients are
not fixed but fluctuate near their critical values via avalanche like transport processes with $1/f$ type spectra. This kind of power law in the frequency spectrum is a typical feature of self-organized criticality (SOC) like phenomena [12] and suggest non-local transport properties related to Bohm like features in the tokamak micro-turbulence [13]. Fluid simulations in [11] also showed qualitative and quantitative differences between fixed gradient and fixed flux approaches, which may correspond to $\delta f$ and full-$f$ approaches in the context of a gyrokinetic simulation. In general, SOC phenomena have a scale free character in space and time. This raises a serious question about the validity of the former approach, in which an evolutionary equation of $\delta f$ is derived by assuming a separation of characteristic spatio-temporal scales between $f_0$ and $\delta f$. Another important issue is to simulate the edge turbulence where the perturbation amplitude reaches $\delta f/f_0 \sim \mathcal{O}(1)$. From these backgrounds, recently, developments of global gyrokinetic full-$f$ Vlasov simulations have been started [1, 14–17].

In a full-$f$ approach, turbulent transport and equilibrium profiles are solved self-consistently following the same first principles. In core plasmas, amplitudes of the density fluctuation $bn$ are small compared with the equilibrium density $n_0$, $bn/n_0 < 1\%$, and the normalized collisionality $\nu^*$ is very small $\nu^* \ll 1$. In order to treat such small amplitude and weak dissipation phenomena in a full-$f$ approach, we need a numerical scheme which satisfies high accuracy and numerical stability simultaneously. To resolve this requirement, we developed a non-dissipative conservative finite difference scheme (NDCFD) [18], which keeps the numerical stability by conserving the phase space volume, $f$, and $f^2$, which are conservation properties inherent to the gyrokinetic Poisson bracket operator. The NDCFD was successfully applied to a global gyrokinetic toroidal full-$f$ 3D Vlasov code (GT5D) [1], and the code was verified through linear and nonlinear benchmarks of the ion temperature gradient (ITG) driven turbulence against a global gyrokinetic $\delta f$ 3D particle-in-cell code (GT3D) [19]. In the benchmark, robustness and accuracy of NDCFD were examined, and a possibility of long time full-$f$ gyrokinetic simulations was demonstrated. At the moment, GT5D uses a gyrokinetic model for the ITG turbulence in core plasmas, and will be improved to an extended gyrokinetic model for edge plasmas in future works.

In this work, we extend GT5D including sources and collisions, and develop long time source driven ITG turbulence simulations based on a full-$f$ approach. Source and sink models are needed to simulate open system tokamak plasmas, in which a heat flux is imposed by auxiliary heating. A sink model is closely related to boundary conditions. Since the edge turbulence is out of the scope of this study, the boundary condition of core plasmas is imposed by a sink model reflecting conditions given by edge plasmas. Collisions work as a physical dissipation mechanism of fine scale velocity space structures, which are produced by mixing due to the ballistic mode. From the viewpoint of the entropy balance relation [20–22], such a dissipation mechanism is essential for reaching statistically steady states in long time gyrokinetic turbulence simulations [23]. In addition, the collisional effect or the neoclassical physics itself is an important physics ingredient in gyrokinetic simulations. The neoclassical transport gives a baseline of transport levels, when turbulent transport is quenched, e.g. in transport barriers. The neoclassical physics dictates relevant kinetic equilibrium. In our previous collisionless gyrokinetic simulations, $f_0$ was chosen as a gyrokinetic Vlasov equilibrium distribution defined by a function of three constants of motion, the canonical toroidal angular momentum $P_\phi$, the energy $c$ and the magnetic moment $\mu$ [19]. However, particle distributions given by such Vlasov equilibria do not coincide between ions and electrons because of their different orbit widths, and the charge neutrality is not exactly satisfied. This means that we need to determine relevant kinetic equilibria with equilibrium radial electric fields $E_r$, which are dictated by the neoclassical physics in a perturbative manner. The equilibrium $E_r$ and mean $E_r \times B$ flows play critical roles in turbulent transport. In order to keep the standard neoclassical physics in core plasmas, we implement ion–ion collisions using the linear Fokker–Planck collision operator, and verify it through comparisons against standard local neoclassical theories. In source driven ITG turbulence simulations, we trace long time evolutions of the ITG turbulence and equilibrium profiles in a normal shear tokamak. As a qualitative validation of GT5D, we study stiffness of the ion temperature $T_I$ profile, intermittent transport phenomena and momentum transport in the ITG turbulence, and discuss their correspondence with the experiment.

This paper is organized as follows. In section 2, the gyrokinetic equations, the linear Fokker–Planck collision operator and source models are given, and their numerical methods are explained. In section 3, benchmark calculations of the neoclassical transport phenomena are shown. In section 4, source driven ITG turbulence simulations are presented, and long time behaviour of turbulent transport and profile formations is addressed. Finally, in summary, qualitative comparisons between the present simulation results and the experiment are discussed.

2. Calculation model

In this study, we consider the electrostatic ITG turbulence described by gyrokinetic ions and adiabatic electrons in an axisymmetric toroidal configuration. In the modern gyrokinetic theory [24], the gyrokinetic equation is written using the gyro-centre Hamiltonian,

$$ H = \frac{1}{2}m_i v_i^2 + \mu B + e\phi, \quad (1) $$

and the gyrokinetic Poisson bracket operator,

$$ \{F, G\} = \frac{\Omega_\alpha}{B} \left( \frac{\partial F}{\partial \alpha} \frac{\partial G}{\partial \mu} - \frac{\partial F}{\partial \mu} \frac{\partial G}{\partial \alpha} \right) $$

$$ + \frac{B^*}{m_i B^*_i} \left( \nabla F \frac{\partial G}{\partial v_i} - \frac{\partial F}{\partial v_i} \nabla G \right) $$

$$ - \frac{c}{e B^*_i} b \cdot \nabla F \times \nabla G, \quad (2) $$

in the gyro-centre coordinates $Z = (t; R, v_i, \mu, \alpha)$, where $R$ is a position of the guiding centre, $v_i$ is the parallel velocity, $\mu$ is the magnetic moment, $\alpha$ is the gyro-phase angle, $B = bb$ is the magnetic field, $b$ is the unit vector in the parallel direction, $m_i$ and $e$ are the mass and charge of ions, respectively, $c$ is the velocity of light, $\Omega_\alpha = (eB)/(mc)$ is the cyclotron frequency, $B^*_i = b \cdot B^*$ is a parallel component of $B^* = B + (Bv_i/\Omega_\alpha) \nabla \times b$, $\phi$ is the electrostatic potential and...
the gyro-averaging operator is defined as $\langle \cdot \rangle_a \equiv \int \cdot \, \text{d}a/2\pi$. By using equations (1) and (2), the gyrokinetic equation in the collisionless and sourceless limit is simply given as the Liouville equation,
\[
\frac{\text{D} f}{\text{D} t} = \frac{\partial f}{\partial t} + [f, H] = \frac{\partial f}{\partial t} + \{f, H\} = 0,
\]
where $f$ is the guiding centre distribution function, and the nonlinear characteristics are given as
\[
\dot{R} \equiv [R, H] = \frac{B^*}{m_1 B_1^2} \frac{\partial H}{\partial v_{1||}} + \frac{c}{e B_1^2} b \times \nabla H = v_{1||} b + \frac{c}{e B_1^2} b \times (e \nabla \langle \phi \rangle_a + m_1 v_{1||}^2 b \cdot \nabla b + \mu \nabla B),
\]
\[
\dot{v}_{1||} = \{v_{1||}, H\} = -\frac{B^*}{m_1 B_1^2} (e \nabla \langle \phi \rangle_a + \mu \nabla B).
\]
Equations (4) and (5) satisfy the phase space volume conservation, which is written as an incompressible condition of the Hamiltonian flow,
\[
\nabla \cdot (J\dot{R}) + \frac{\partial}{\partial v_{1||}} (J\dot{v}_{1||}) = 0,
\]
where $J = m_1^2 B_1^3$ is the Jacobian of the gyro-centre coordinates. From the phase space volume conservation (6), the gyrokinetic equation (3) yields its conservative form,
\[
\frac{\partial f}{\partial t} + \nabla \cdot (J\dot{R}f) + \frac{\partial}{\partial v_{1||}} (J\dot{v}_{1||} f) = 0.
\]
By adding a collision term $C(f)$, a source term $S_{\text{src}}$ and a sink term $S_{\text{sink}}$, a conservative gyrokinetic equation used in GT5D is written as
\[
\frac{\partial f}{\partial t} + \nabla \cdot (J\dot{R}f) + \frac{\partial}{\partial v_{1||}} (J\dot{v}_{1||} f) = JC(f) + JS_{\text{src}} + JS_{\text{sink}}.
\]
The self-consistency is imposed by the quasi-neutrality condition or the gyrokinetic Poisson equation,
\[
-\nabla_{||} \cdot \frac{\rho_0^2}{\lambda_{\text{Di}}^2} \nabla_{\perp} \phi + \frac{1}{\lambda_{\text{De}}^2} (\phi - \langle \phi \rangle_l) = 4\pi e \left[ \int f \delta([R + \rho] - x) \, \text{d}^6 Z - n_{\text{De}} \right],
\]
where $R + \rho$ is a particle position, $\text{d}^6 Z = m_1^2 B_1^3 \, \text{d}R \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a$ is the phase space volume of the gyro-centre coordinates, $\rho_0$ is the Larmor radius evaluated with the thermal velocity $v_{1||}, \lambda_{\text{Di}}$ and $\lambda_{\text{De}}$ are the ion and electron Debye lengths and $\langle \cdot \rangle_l$ is an integral flux surface average operator involving the geometry coupling effect [2]. In equation (9), the first term in the l.h.s. is an ion polarization effect coming from the first order term of the pull back transformation in the gyrokinetic ordering [24]. In this study, we use a linearized ion polarization term with a long wavelength approximation. This approximation may be valid for the core ITG turbulence, which is characterized by $k_i^2 \beta_i^2 \ll 1$ and $\beta_i n_0 / \lambda_i \ll 1$.

In order to implement the neoclassical physics in core plasmas, ion–ion collisions are modelled by the linear Fokker–Planck collision operator [25, 26], which is valid for $\delta f / f_0 \ll 1$.
\[
C(f) = \frac{\partial}{\partial s} (v_{1\perp} v^2 f) + \frac{\partial}{\partial u} (v_{1||} u f) + \frac{1}{2} \frac{\partial^2}{\partial s^2} (v_{1\perp} v^2 f) + \frac{1}{2} \frac{\partial^2}{\partial u^2} (v_{1\perp} v^2 f) + C_F,
\]
where $s = 2\mu B_i / m_i$ and $u = v_{1||} - U_{1||}$ are a moving frame with respect to the parallel flow velocity $U_1$ and $v^2 = u^2 + s$. The definitions of the collision frequencies, $v_{1||}, v_{1\perp}, v_{1||}, v_{1\perp}, v_{1\perp}, v_{1\perp}$, and the field particle operator $C_F$ are given in the appendix. The moving frame and collision frequencies are calculated at each position using evolving equilibrium profiles, which provide indirect nonlinear effects on $C(f)$. Operator (10) is annihilated by a shifted Maxwellian distribution with an arbitrary flow velocity. It is noted that equation (10) is given in the drift-kinetic limit, and we ignore corrections related to finite Larmor radius (FLR) effects to satisfy local conservation properties of the particle number, the momentum and the kinetic energy:
\[
\int C(f) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a = 0,
\]
\[
\int m_1 v_{1||} C(f) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a = 0,
\]
\[
\int \left( \frac{1}{2} m_1 v_{1\perp}^2 + \mu B \right) C(f) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a = 0,
\]
which are basic requirements used in the neoclassical theory [27, 28]. It should be noted that the gyrokinetic Vlasov–Poisson system, equations (8) and (9), naturally reduces to a physical model of the neoclassical theory in the axisymmetric limit with macroscopic radial electric fields $E_r$. Firstly, it is trivial that for macroscopic perturbations with $k_i L \sim O(1)$, the gyrokinetic equation (8) reduces to the drift-kinetic equation [27]. Here, $L$ is a characteristic scale length of equilibrium profiles. Secondly, by taking the time derivative and the flux-surface average of equation (9) and substituting equation (8), we derive a particle balance relation:
\[
\frac{\partial}{\partial t} \left[ \left( \frac{\rho_0^2}{4\pi e^2 \lambda_{\text{Di}}^2} \frac{\partial E_r}{\partial t} - \Gamma \right) \int C(f) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a \right]_t = 0,
\]
where the particle flux is defined as
\[
\Gamma = \left( \int f (\dot{R}_0 \cdot \nabla r) \delta([R + \rho] - x) \, \text{d}^6 Z \right)_t \approx \left( \int f (\dot{R}_0 \cdot \nabla r) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a \right)_t = \left( \delta f (v_B \cdot \nabla r) m_1^2 B_1^3 \, \text{d}v_{1||} \, \text{d}\mu \, \text{d}a \right)_t.
\]
Here, \( \mathbf{R}_0 = \{ \mathbf{R}, \langle H \rangle_c \} \) is a particle orbit given by an axisymmetric part of the Hamiltonian \( \langle H \rangle_c \), \( \mathbf{v}_b \) is the magnetic drift terms in equation (4), and \( \delta f = f - f_{M2} \) is a deviation of \( f \) from a local Maxwellian distribution \( f_{M2} \) at each flux surface. When the collision operator conserves the particle number, equation (14) leads to the equation for \( E_c \):

\[
- \left( \frac{\rho_i^2}{4 \pi e^2 m_i} \right) \frac{\partial E_c}{\partial t} = \Gamma, \tag{16}
\]

which was used to determine neoclassical electric fields \([29, 30]\). Therefore, the present physical model is expected to recover the neoclassical theory in the axisymmetric limit. Equation (16) shows a balance between second order terms in the gyrokinetic ordering, when they are derived from the first order gyro-centre transform. In \([31]\), it was pointed out that the gyrokinetic equations derived from the first order transform in a so-called recursive approach cannot determine macroscopic electric fields. However, the above discussion shows that in the modern gyrokinetic theory, its flux-surface averaged form has high enough accuracy to determine \( E_c \) even with the first order transform.

A source term requires an empirical modelling. In this study, we simulate heat and momentum transport due to the ITG turbulence driven by on-axis heating with a given power input. On the other hand, at the boundary of core plasmas, we fix the ion temperature \( T_i \) and the parallel flow \( U_1 \) on average reflecting boundary conditions in H-mode plasmas, where the pedestal temperature is limited by edge localized modes and a no-slip boundary \( U_1 = 0 \) is imposed by the charge exchange with the neutrals. Following the above consideration, we use models for a heat source \( S_{src} \) near the axis and a heat sink \( S_{snk} \) in a boundary region:

\[
S_{src} = A_{src}(R) \tau_{src}^{-1}(f_{M1} - f_{M2}), \tag{17}
\]

\[
S_{snk} = A_{snk}(R) \tau_{snk}^{-1}(f_0 - f), \tag{18}
\]

where \( A_{src} \) and \( A_{snk} \) are deposition profiles, \( f_{M1} \) and \( f_{M2} \) are (shifted) Maxwellian distributions, \( \tau_{src} \) and \( \tau_{snk} \) are time constants for the energy sink and \( f_0 \) is the initial distribution. In equation (17), \( A_{src}, \tau_{src}, f_{M1} \) and \( f_{M2} \) are chosen to fix power input without particle and momentum input:

\[
\int S_{src} m_i^2 B_0^* d\mathbf{v}_i d\mu d\sigma = 0, \tag{19}
\]

\[
\int m_i v_i S_{src} m_i^2 B_0^* d\mathbf{v}_i d\mu d\sigma = 0, \tag{20}
\]

\[
\int \left( \frac{1}{2} m_i v_i^2 + \mu B \right) S_{src} d^6 Z = P_{in}, \tag{21}
\]

where \( P_{in} \) is power input. In equation (18), \( T_i \) and \( U_1 \) in the boundary region are modified towards their initial values by a Krook operator with the time constant \( \tau_{snk} \). This model works not only as a heat sink but also as a momentum source (sink) when a negative (positive) momentum flux is absorbed. This kind of momentum source at the boundary is considered as the origin of the intrinsic toroidal rotation.

A power balance in the above gyrokinetic equations is given as follows:

\[
\int H \frac{\partial f}{\partial t} d^6 Z = \frac{dE_{\text{kin}}}{dt} + \frac{dE_{\text{fld}}}{dt} + \frac{dE_{\text{coll}}}{dt} + \frac{dE_{\text{src}}}{dt} + \frac{dE_{\text{snk}}}{dt}, \tag{22}
\]
3. Benchmark test of neoclassical transport

In the neoclassical benchmark, we solve the gyrokinetic equations (3) and (4) in the axisymmetric and sourceless limit to estimate the ion neoclassical transport in relevant kinetic equilibria. Although GTSD can simulate steep profiles and low aspect ratio configurations with finite orbit width effects, in the present benchmark, we use a circular concentric tokamak configuration with a relatively large aspect ratio \( R_0/a = 5, \) \( a/\rho_i \approx 150, \) \( q(r) = 0.85 + 2.18(r/a)^2 \) and moderate density and temperature gradients \( (R_0/L_a = 1, \) \( R_0/L_i = 1) \) to make quantitative comparisons with standard local theories, where \( R_0 \) is the major radius, \( a \) is the minor radius, \( L_a = |n_0(\mathrm{d}n_0/\mathrm{d}r)|^{-1} \) and \( L_i = |T_i(\mathrm{d}T_i/\mathrm{d}r)|^{-1} \). In these parameters, the ratio of the ion banana orbit width \( \Delta_i \) to \( L_i \) is \( \Delta_i/L_i \approx 0.006, \) where a local approach is a good approximation. The normalized collisionality is chosen as \( n^* = 0.01-10, \) and the parallel flow is given as \( U_\parallel = -0.1v_{te}, 0, 0.1v_{te} \) corresponding to counter-, balance- and co-rotating tokamaks. From convergence tests, the time step width and the grid numbers are determined as \( \Delta t = 42\,\tau_i^{-1} \) and \( (N_R, N_\zeta, N_z, N_i, N_n) = (160,1,160,80,20) \).

We perform axisymmetric simulations starting from a local Maxwellian distribution \( f_{LM} \). Since \( f_{LM} \) does not annihilate the gyrokinetic Poisson bracket operator, the radial electric field \( E_r \) quickly develops in a transit time and the geodesic acoustic mode (GAM) is excited. Through the damping of GAMs, \( E_r \) develops to satisfy the ambipolar condition, and then, the system gradually approaches the neoclassical solution. Figure 1 shows the time histories of the particle flux \( \Gamma \) and the heat flux \( Q_{NC} \). As is noted in section 2, the physical model of the neoclassical simulation reduces to that of the neoclassical theory except for minor corrections such as remaining FLR effects on macroscopic perturbations. Therefore, in the present benchmark, we compare particle and heat fluxes defined following standard definitions in the neoclassical theory [27, 28]:

\[
\Gamma = \left\langle \int f(v_i \cdot \nabla_r) r^2 m_i^2 B_i^* \, \mathrm{d}v_i \, \mathrm{d}\mu \, \mathrm{d}r \right\rangle, \tag{28}
\]

\[
Q_{NC} = \left\langle \left( \frac{1}{2} m_i v_i^2 + \mu B \right) f(v_i \cdot \nabla_r) r^2 m_i^2 B_i^* \, \mathrm{d}v_i \, \mathrm{d}\mu \, \mathrm{d}r \right\rangle - \frac{3}{2} \Gamma T_i. \tag{29}
\]

Figure 1 shows transient evolutions of \( \Gamma \) and \( Q_{NC} \), respectively. Equation (16) leads to the ambipolar condition \( \Gamma = 0 \) in a steady state. In figure 1, this ambipolar condition is satisfied after the initial transient damping of GAMs. This result shows that the system follows the \( E_r \) equation (16). Then, the system enters a relaxation phase, where the heat flux coincides with the energy flux (the first term in the r.h.s. of equation (29)). It is noted that during GAM oscillations, \( Q_{NC} \) is positive on average. This heat flux is an order of magnitude larger than the neoclassical level, and is observed also in the collisionless limit. Since the present neoclassical simulation keeps only \( (m, n) = (0,0) \) component of the electric field, this flux is not produced by the \( E \times B \) drift but by a coupling between up–down asymmetric GAM perturbations and the magnetic drift. This collisionless transport mechanism may be important when GAMs are excited in a plasma. Figure 2 shows \( Q_{NC} \) observed in simulations with \( n^* \approx 0.1 \) and \( v^* \approx 1 \), which gives transport levels comparable to Chang–Hinton’s formula (C–H formula) [33]. In figure 3, the neoclassical heat diffusivity \( \chi_i = -Q_{NC}/(n_i \nabla T_i) \) observed in GTSD agrees with that estimated from the C–H formula over a wide \( v^* \) regime. Another important property of the ion neoclassical transport is the force balance relation among the parallel flow, the Pfirsch–Schlüter flow and the neoclassical poloidal flow. A force balance relation in the local neoclassical theory [27] is given as

\[
\langle U_i \rangle = \frac{T_i I_m \Omega_i}{m_i \Omega_i} \left[ \frac{\mathrm{d}n_i}{\mathrm{d}r} - \frac{\mathrm{d} \ln n_i}{\mathrm{d}r} + \frac{\varepsilon}{T_i} E_i \right], \tag{30}
\]

where \( \psi \) is the poloidal flux, \( I = RB_i \), \( n_i \) is the ion density and \( k = k(v^*) \) is a coefficient of the neoclassical poloidal flow. In figure 4, \( E_i \) in a steady state is plotted for counter-, balance- and co-rotating tokamaks, and the results show good agreement with those estimated from equation (30). In figure 5, \( k(v^*) \) observed in GTSD is compared with equation (6.136) in [27] (H–H formula) and with a banana limit solution with a finite aspect ratio correction, 1.17 T_i (see equation (11.58) in [28]). It is noted that the former solution is obtained in the large aspect ratio limit \( e = r/R_0 \to 0 \), while the latter
solution takes account of a finite aspect ratio effect through a factor \( F_c = f_p/(f_p + 0.462 f_i) \), where the effective fraction of trapped particles is \( f_i = 1.46\sqrt{\epsilon} \) and \( f_p = 1 - f_i \). Over a wide \( \nu^* \) regime, \( k(\nu^*) \) shows qualitatively similar behaviour as the H–H formula, and a flip of the poloidal rotation in the collisional regime is captured. However, in the banana regime, \( k \) is close to \( 1.17F_c \) rather than the H–H formula. A similar discrepancy in \( k \) between neoclassical simulations and the H–H formula in the banana regime was reported also in [34]. These benchmark results show that GT5D can determine \( E_{ri} \) self-consistently by evolving equilibrium profiles. It is noted that in the neoclassical simulation, \( \chi_i \) and \( k \) reach steady state values at \( \tau = \tau_{0i} \). This gives a minimum time duration required to have the neoclassical physics in gyrokinetic simulations, where \( \tau_{0i} \) is the ion–ion collision time.

In the present neoclassical benchmarks, standard local neoclassical theories are recovered in the steady state, and the collision operator is verified quantitatively. In addition, the present axisymmetric simulations clarified transient behaviour of collisional and collisionless transport processes, which cannot be explained in the framework of conventional neoclassical theory.

4. Source driven ITG turbulence simulation

In the present study, we use a circular concentric tokamak configuration with \( R_0/\alpha = 2.79, a/\rho_i = \rho^\nu \sim 150, 1/3 \) wedge torus and \( q(r) = 0.85 + 2.18(r/a)^2 \), which gives cyclone like parameters [35], \( r_*/R_0 \sim 0.18, q(r_*) \sim 1.4 \) and \( \delta(r_*) \sim 0.78 \) at \( r_0 = 0.5a \). In this configuration, simulation parameters are chosen as follows: the time step width is \( \Delta t = 2\Omega_{ni}^{-1} \), grid numbers used in the gyrokinetic solver and the Fokker–Planck solver are \( (N_R, N_z, N_\phi, N_v_i, N_v_e) = (160, 32, 160, 80, 20) \), finite elements used in the field solver are \( (N_x, N_y) = (150, 150) \) and the system size is \( R = R_0 = -1.07a-1.07a (\Delta R = 2\rho_i), Z = -1.07a-1.07a (\Delta Z = 2\rho_i) \), \( \zeta = 0 \sim 2\pi/3 \).
The heating region. It is noted that the heating process does not produce negative values of \( n_i \) in a full torus configuration, and therefore, practical power input \( \dot{P}_{\text{in}} \) is determined by the present collisional dissipation of fine scale velocity space structures. Figure 6 shows the initial \( n_i, U_i, T_i \) profiles and the deposition profiles of \( A_{\text{src}} \) and \( A_{\text{sink}} \). In the initial condition, a weak co-rotation is given to study momentum transport, and the ion temperature profile is far above linear and nonlinear thresholds at \( R_0/L_{\text{ni}} \sim 4.5 \) and \( R_0/L_{\text{ni}} \sim 6 \), respectively. Here, this nonlinear threshold value is identified using two different gyrotropic codes, GTSD and GT3D, through collisionless and sourceless ITG benchmark simulations with similar cyclone like parameters [1] (see figure 7). Plasma parameters at \( r = r_s \) are \( n_e = n_i \sim 5 \times 10^{19} \text{ m}^{-3}, T_e \sim T_i \sim 2 \text{ keV}, R_0/L_{\text{ni}} = 2.22, R_0/L_{\text{te}} = 6.92, R_0/L_{\text{ni}} = 10.0 \) and \( v^* \sim 0.025 \). Near the magnetic axis, the heat source model (7) is given by \( f_{\text{ST1}}(n_i = \bar{n}_i, U_i = 0, T_i = T_f) \) and \( f_{\text{SK}}(n_i = \bar{n}_i, U_i = 0, T_i = T_f) \), where \( \bar{n}_i \) and \( T_f \) are the volume averaged density and temperature. Power scan simulations with \( \dot{P}_{\text{in}} = 2 \text{ MW} \) and \( \dot{P}_{\text{in}} = 4 \text{ MW} \) are performed, where \( \tau_{\text{src}} \) is determined by equation (21) and the source deposition profile \( A_{\text{src}} \) is chosen to be broad enough so that the heating process does not produce negative values of \( f \) in the heating region. It is noted that \( \dot{P}_{\text{in}} \) is defined as power input in a full torus configuration, and therefore, practical power input is \( \dot{P}_{\text{in}} \) in a 1/3 wedge torus configuration. In a boundary region, the time constant in the simulation model (8) is given as \( \tau_{\text{sink}}^{-1} = 0.1 v_i/\alpha \). This parameter is chosen to be large enough to fix \( U_{\text{in}} \) and \( T_{\text{in}} \) in the boundary region. Since the present simulation model satisfies a power balance in a steady state and the heat flux is imposed by input power \( \dot{P}_{\text{in}} \), the simulation is not so sensitive to \( \tau_{\text{sink}} \), provided that the sink is strong enough and the boundary conditions for \( U_i \) and \( T_i \) are unchanged.

Figure 8 shows the time histories of the kinetic energy \( E_{\text{kin}} \), the field energy \( E_{\text{fld}} \), the collisional power transfer \( E_{\text{col}} \), the input power \( E_{\text{src}} \) and the output power in the sink \( E_{\text{sink}} \) in the source driven ITG turbulence simulation with \( \dot{P}_{\text{in}} = 2 \text{ MW} \). The plot is normalized by the initial kinetic energy \( E_{\text{Ekin}} \) and \( E_{\text{Ekin}} = E_{\text{Ekin}} \) are scaled by \( 1/100 \). In the plot, the energy conservation is shown by a balance between \( E_{\text{fld}} = E_{\text{col}} \) and \( E_{\text{kin}} = E_{\text{Ekin}} + (E_{\text{coli}} + E_{\text{col}}) \). After \( \tau_{\text{in}}/R_0 \sim 300 \), a power balance condition, \( E_{\text{coli}} + E_{\text{sink}} = 0 \), is established.

The initial excitation of \( E_f \) and GAMs. After the damping of GAMs, the system relaxes towards a kinetic equilibrium condition, \( \{f, H\} \sim C(f) \). However, in the simulation with non-axisymmetric components, the ITG mode shows linear growth, provided that \( R_0/L_{\text{ni}} \) exceeds the nonlinear threshold. In the initial nonlinear phase, the saturation of linear ITG modes shows transient bursts, which produce an order of magnitude larger heat transport than the quasi-steady transport level. Although these initial bursts due to linear ITG modes are unphysical phenomena, their large heat transport quickly adjusts the \( T_i \) profile towards nonlinear marginal states in turbulent time scales. In collisionless and sourceless ITG turbulence simulations, the turbulent transport is almost quenched after these bursts [1] (see figure 7).

However, in the present source driven ITG simulation, the turbulent transport is sustained by constant power input, and a power balance condition, \( E_{\text{coli}} + E_{\text{sink}} = 0 \), is established in physically meaningful quasi-steady states. In figure 8, the energy conservation (9), which is shown by a balance between
Electric field shear \( \partial H / \partial r \) is tied to globally constant correlation time in the quasi-steady state. This is the stringent verification by active avalanches (see figure 9). Remarkable features found in the source-driven ITG turbulence simulations are that the turbulent heat transport is produced by avalanches, and their amplitudes are almost doubled with increasing \( P_m \) from 2 to 4 MW. These avalanches propagate with almost the same velocity, but the propagation width becomes shorter \( l_A \sim 10 \rho_i \) because of stronger \( T_i \) corrugation and local \( E_r \) shear, which suppresses the ballistic propagation of avalanches. Although \( \tau_A \) is not changed so much, a quasi-periodic feature becomes weak in the autocorrelation function of \( Q \), and an intermittent feature becomes strong. In figure 12, the power spectrum of \( Q \) shows a small peak at \( \omega_A \), where the power law changes from \( 1/f \) to stronger decay. This kind of \( 1/f \) type spectrum is a typical feature of SOC-like phenomena.

In figure 9, not only \( \chi_i \) but also \( L_\text{ui} \) and \( \partial E_i / \partial r \) show similar avalanches. According to cross correlation analyses, both \( L_\text{ui} \) and \( E_i \) show a delay \( \Delta t \sim 1.5 R_0 / v_i \) from \( \chi_i \), but there is no delay between \( L_\text{ui} \) and \( E_i \). This suggests that avalanche components of \( E_i \) are determined by some local force balance or equilibrium conditions. It is noted that the radial electric field \( E_r \) in the present source-driven ITG turbulence simulations is qualitatively different from that observed in the collisionless and sourceless ITG turbulence simulations, which are dominated by quasi-steady zonal flows [1]. In figure 13, it is found that quasi-steady zonal flows are observed only in a core region \( r/a < 0.4 \), and a source-free region \( r/a > 0.4 \) is dominated by mean \( E_i \) and avalanches. Although turbulence-driven zonal flows have been considered as an important saturation mechanism in the ITG turbulence, the source-free region, which is close to nonlinear marginal states, is not subject to this picture, and

\[
E_{\text{ad}} - E_{\text{col}} \text{ and } E_{\text{ui}} - E_{i=0} = (E_{\text{ad}} + E_{\text{ui}}),
\]

is satisfied for a long time in the quasi-steady state. This is the stringent verification of long time source-driven ITG turbulence simulations.

Figure 9 shows the spatio-temporal evolutions of the ion heat diffusivity \( \chi_i \), the normalized ion temperature gradient \( R_0 / L_{\text{ui}} \), the radial electric field shear \( \partial E_i / \partial r \), and the parallel flow \( U_i \). Here, the ion heat diffusivity \( \chi_i = -Q / (n_i \nabla T_i) \) is defined using the turbulent heat flux

\[
Q = \left( \frac{1}{2} m v_i^2 + \mu B \right) \int (\mathbf{R_i} \cdot \nabla \mathbf{r}) m_i^2 B_i^2 \, d\mathbf{r} \, d\mu \, da,
\]

where \( \mathbf{R_i} = (\mathbf{R}, \mathbf{H}) \) is given by a non-axisymmetric part of the Hamiltonian, \( \mathbf{H} = H - \langle H \rangle \), and \( n_i \) and \( T_i \) are given by evolving equilibrium profiles. Although the time history of \( E_{\text{ad}} \) suggests a quasi-steady turbulent state, figure 9 indicates active turbulent dynamics. Remarkable features found in the source-driven ITG turbulence simulation are that the turbulent transport in a source-free region \( r/a = 0.5-0.9 \) is dominated by active avalanches (see figure 9(a)), and the \( T_i \) profile in this region is tied to globally constant \( L_{\text{ui}} \) at \( R_0 / L_{\text{ui}} \sim 6.5 \), which is slightly above the nonlinear critical value at \( R_0 / L_{\text{ui}} \sim 6 \) (see figures 9(b) and 10(b)). It is noted that in figure 10(b), \( R_0 / L_{\text{ui}} \) at \( r/a = 0.5 \) is very close to the cyclone parameter \( R_0 / L_{\text{ui}} \sim 6.92 \). While the radial correlation length \( \Delta r_c \sim 5 \rho_i \) and the correlation time \( \tau_c \sim 0.7 R_0 / v_i \sim 2a / v_i \) of the turbulent fields suggest a gyro-Bohm-like picture, the propagation width and the fastest time scale of avalanches observed in the simulation with \( P_m = 2 \text{ MW} \) show an order of magnitude larger scales \( l_A \sim 20 \rho_i \) and \( \tau_A \sim 9 R_0 / v_i \), respectively. Their propagation velocity estimated from space–time autocorrelation analyses shows a ballistic feature with \( V_A \sim \rho_i v_i / R_0 \). It is noted that in this simulation, GAM activities are not observed except for the initial relaxation phase, and the time scale of avalanches \( \omega_A \sim 2 \pi / \tau_A \sim 0.7 v_i / R_0 \) is slower than that of GAMS \( \omega_{\text{GAM}} \sim 2 v_i / R_0 \). The avalanche propagation of heat flux with similar spatio-temporal scales was reported also in other gyrokinetic simulations [7, 36]. In the power scan, it is found that with increasing \( P_m \) from 2 to 4 MW, \( Q \) is doubled with almost the same \( L_{\text{ui}} \), showing strong profile stiffness (see figure 10). Here, there is no significant change in \( \Delta r_c \) and \( \tau_c \), and the increase of \( \chi_i \) is mainly due to enhanced amplitudes of avalanches. In figure 11, it is shown that a significant part of the turbulent heat transport is produced by avalanches, and their amplitudes are almost doubled with increasing \( P_m \) from 2 to 4 MW. These avalanches propagate with almost the same velocity, but the propagation width becomes shorter \( l_A \sim 10 \rho_i \) because of stronger \( T_i \) corrugation and local \( E_r \) shear, which suppresses the ballistic propagation of avalanches. Although \( \tau_A \) is not changed so much, a quasi-periodic feature becomes weak in the autocorrelation function of \( Q \), and an intermittent feature becomes strong. In figure 12, the power spectrum of \( Q \) shows a small peak at \( \omega_A \), where the power law changes from \( 1/f \) to stronger decay. This kind of \( 1/f \) type spectrum is a typical feature of SOC-like phenomena [12].

In figure 9, not only \( \chi_i \) but also \( L_{\text{ui}} \) and \( \partial E_i / \partial r \) show similar avalanches. According to cross correlation analyses, both \( L_{\text{ui}} \) and \( E_i \) show a delay \( \Delta t \sim 1.5 R_0 / v_i \) from \( \chi_i \), but there is no delay between \( L_{\text{ui}} \) and \( E_i \). This suggests that avalanche components of \( E_i \) are determined by some local force balance or equilibrium conditions. It is noted that the radial electric field \( E_r \) in the present source-driven ITG turbulence simulations is qualitatively different from that observed in the collisionless and sourceless ITG turbulence simulations, which are dominated by quasi-steady zonal flows [1]. In figure 13, it is found that quasi-steady zonal flows are observed only in a core region \( r/a < 0.4 \), and a source-free region \( r/a > 0.4 \) is dominated by mean \( E_i \) and avalanches. Although turbulence-driven zonal flows have been considered as an important saturation mechanism in the ITG turbulence, the source-free region, which is close to nonlinear marginal states, is not subject to this picture, and

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Figure 10. The quasi-steady profiles (time average over $t_{\text{Ti}}/R_0 = 400–650$) of (a) $T_\text{e}$ (log scale) and (b) $L_\text{Ti}$ observed in power scan simulations with $P_{\text{in}} = 2$ MW and $P_{\text{in}} = 4$ MW. Similar quasi-steady profiles are observed in two simulations with different input power, showing strong profile stiffness. Both profiles are far from the initial condition (dotted curve), and source free regions ($r/a = 0.5–0.9$) are tied to globally constant $L_\text{Ti}$ ($R_0/L_\text{Ti} \sim 6.5$) near the nonlinear critical value ($R_0/L_\text{Ti} \sim 6$).

Figure 11. The time histories of $L_\text{Ti}$ and $Q$ at $r/a = 0.7$. The time average of $\chi_\text{i}/(v_\text{Ti} \rho_\text{Ti}^2/t_\text{Ti}/L_\text{Ti})$ over $t_{\text{Ti}}/R_0 = 400–650$ is $\sim 0.4$ and $\sim 0.8$ for $P_{\text{in}} = 2$ MW and $P_{\text{in}} = 4$ MW, respectively. The neoclassical heat transport is less than $\chi_\text{i}/(v_\text{Ti} \rho_\text{Ti}^2/t_\text{Ti}/L_\text{Ti}) \sim 0.1$.

Figure 12. The power spectrum of the turbulent heat flux $Q$ averaged over source free regions ($r/a = 0.5–0.9$). The spectra in low frequency region show $1/f$ type spectra.

Figure 13. The radial profiles of $E_\text{r}$ and $L_\text{Ti}$ observed in a simulation with $P_{\text{in}} = 2$ MW ($t_{\text{Ti}}/R_0 \sim 550$). ‘H–H’ shows $E_\text{r}$ estimated by a force balance relation (30) with the H–H formula. The shear of equilibrium $E_\text{r}$ affects propagation of avalanches. In figure 9, the propagation direction of avalanches is changed depending on the sign of $dE_\text{r}/dr$. At $r/a < 0.6$ ($r/a > 0.6$), the $E_\text{r}$ shear is negative (positive) on average.
and avalanches propagate inwards (outwards). This can be understood from a relation between $E_\|$ and $L_\|$. In the avalanche front, flattening of $T_\|$, occurs, and local maxima of $E_\|$, and $-R_0/L_\||$ are produced following a local force balance relation (see figure 13). As a result, the avalanche front is bounded by positive and negative local $E_\|$ shear regions where local $E_\|$ shear is shifted on average by mean $E_\|$ shear (see figure 15). In the positive (negative) mean $E_\|$ shear region, local $E_\|$ shear outside (inside) is always weaker than the other side, and ITG modes in the avalanche front tend to couple with modes outside (inside), leading to one-sided propagation of avalanches. This mechanism may explain a change in the direction of the avalanche propagation depending on toroidal rotation reported in [7].

Another important effect of the $E_\|$ shear is its influence on the momentum transport. Before discussing the momentum transport, we discuss a difference between the parallel flow and the toroidal rotation. Figure 16 shows the toroidal rotation profile and its parallel, perpendicular $E \times B$ and perpendicular $\nabla P$ components observed at $t = \tau_\|$. Although the parallel and perpendicular flows observed are the same order in the simulation with zero momentum input, toroidal rotation of the latter becomes an order of magnitude smaller than the former. Therefore, we focus only on the parallel momentum and its transport. In the present simulation, initial parallel flows with $U_{\|}/v_\| \sim 0.1$ are given in the co-current direction, and the momentum diffusion is observed during initial transient bursts leading to a relaxation of the $U_{\|}$ profile (see figures 9(d) and 17(a)). However, in the quasi-steady turbulent state, co-rotation in the core region and counter-rotation in the source free region build up without momentum input, which suggests the existence of non-diffusive momentum transport. In order to see the uniqueness of this rotation profile, we perform the same simulation starting from $U_{\|} \sim 0$. In the simulation, properties of turbulent ion heat transport and $T_\|$ profiles are not changed. Figure 17(b) shows evolutions of the $U_{\|}$ profile observed in the simulation. Here, small initial parallel flows with $U_{\|}/v_\| \sim 0.02v_\|$ come from the $v_\|$ dependence of the Jacobian $m^2 B_i^2$. The result shows that even with different initial conditions, similar co-rotation in the core region and counter-rotation in the source free region build up without momentum input. This means that the rotation profiles are intrinsic. It should be noted that in figure 9, the heat transport and the $T_i$ profile are in the quasi-steady state after transient bursts. However, $U_{\|}$ is slowly evolving even in this stage, while its evolution rate becomes slower in the later phase. Since this level of $U_{\|}$ does not work as a turbulence drive, the $U_{\|}$ profile is not stiff and its evolution is slow.

The turbulent momentum transport consists of the momentum diffusion, the momentum pinch [38, 39] and the $E_\|$ shear stress [40, 41]. Since the rotation profile builds up even from the initial condition with $U_{\|} \sim 0$, the $E_\|$ shear stress is considered to play a role in the momentum pinch observed. In figure 18, the time averaged momentum flux $\Pi$ is in the opposite direction to the momentum gradient or $-dU_{\|}/dr$, showing a non-diffusive feature. Here, the turbulent parallel momentum flux $\Pi$ is defined as

$$\Pi \equiv \left( \int v_{\|} f (R_1 \cdot \nabla r) m_i^2 B_i^2 \, dv_{\|} \, d\mu \, d\sigma \right) \, .$$  

(32)
Simulation parameters are the same. Simulations starting from different initial momentum input. The result shows a signature of the turbulent momentum transport. Although full-tube and turbulence suppression due to \( \rho/a > 0 \) region at investigated by relative importance of these mechanisms was quantitatively especially after the rotation profile builds up. In [42], the balance among the above three mechanisms is complicated. Figure 18 also shows a correlation between momentum and the radial electric field shear \( \partial E_r/\partial r \) observed in a simulation with \( P_{in} \) = 2 MW (time average over \( t_{vB}/\tau_0 = 400-650 \)).

Figure 18 also shows a correlation between \( \Pi \) and \( \partial E_r/\partial r \), and \( \Pi \) is outwards (inwards) in a positive (negative) \( \partial E_r/\partial r \) region at \( r/a > 0.6 \) (\( r/a = 0.4-0.6 \)). Although this result shows a signature of the \( E_r \) shear stress, a quantitative balance among the above three mechanisms is complicated especially after the rotation profile builds up. In [42], the relative importance of these mechanisms was quantitatively investigated by \( \delta 
abla \) flux-tube simulations, where \( U_i \), \( dU_i/\partial r \) and \( dE_r/\partial r \) are imposed. In contrast, in the present full-\( f \) global simulations, all these parameters are self-consistently determined through complicated nonlinear processes. On the one hand, \( U_i \) is determined by the turbulent momentum transport, and \( U_i \) and evolving equilibrium profiles dictate \( E_r \) through a force balance relation. On the other hand, the non-diffusive momentum transport depends on \( U_i \) and \( dE_r/\partial r \), and turbulence suppression due to \( dE_r/\partial r \) also affects the turbulent momentum transport. Although full-\( f \) simulations are useful to dictate the intrinsic toroidal rotation through the above complicated nonlinear processes, further investigations are needed to understand each mechanism separately.

5. Summary

In this work, long time source driven ITG turbulence simulations are developed by extending sources and collisions in a global gyrokinetic toroidal full-\( f \) 5D Vlasov code GT5D. The key features of GT5D are summarized as follows.

1. The NDCFD enables robust and accurate long time full-\( f \) simulations, where the turbulent transport and equilibrium profiles are evolved self-consistently based on the same first principles.
2. Ion–ion collisions are implemented using the linear Fokker–Planck collision operator, which is important not only as a physically relevant dissipation mechanism of fine scale velocity space structures but also as the neoclassical physics which dictates the equilibrium \( E_r \) and a baseline of transport levels.
3. Choices (and extensions) of source and sink models are flexible. In this work, two source models are developed reflecting conditions of on-axis heating and H-mode like edge plasmas.

The collision operator is verified through benchmark calculations of the neoclassical transport, in which standard local neoclassical theories are recovered in the quasi-steady phase. However, in the transient phase, it is found that significant heat transport is driven by GAMs, and that the heat diffusivity and the poloidal rotation approach the neoclassical levels slowly in a collision time, which gives the minimum time duration to simulate the neoclassical physics.

Source driven ITG turbulence simulations in a normal shear tokamak with \( \rho^{-1} \sim 150 \) and \( \nu^* = 0.025-0.1 \) are performed using the source models to fix power input (zero momentum input) near the axis and \( T_i \) and \( U_i \) /\( \tau_0 \) at the edge. In the simulation, long time behaviour of the turbulent transport and profile formations is traced over a collision time, and the following key features of the ion turbulent transport are clarified:

1. The \( T_i \) profile in a source free region is tied to globally constant \( L_q \) near the nonlinear threshold, and strong stiffness is observed in the power scan. This kind of stiff \( T_i \) profile with globally constant \( L_q \) was typically observed in H-mode plasmas in JT60U [43].
2. In the source free region, a significant part of the heat flux is carried by avalanche like phenomena, which have an order of magnitude larger spatio-temporal scales than the radial correlation length \( \Delta r_c \sim \delta \) and the correlation time \( \tau_c \sim 2a/\nu_i \) of the turbulent fields. This suggests a possibility of non-local or Bohm like features of the turbulent transport produced by gyro-Bohm like turbulent fields.
3. The intermittent heat flux of avalanches shows 1/f type spectra, which are typically observed in SOC-like phenomena. Similar 1/f type spectra were observed also in the experiment [44]. The criticality of $L_0$ and 1/f type spectra suggests that stiff $T_0$ profiles produced in the ITG turbulence may be explained by a SOC type picture.

4. Roles of the mean $E_i$ shear are found. The $E_i$ shear dictates the direction of the avalanche propagation. The mean $E_i$ shear profile and the non-diffusive momentum flux show a clear correlation. This suggests a signature of the $E_i$ shear stress.

5. Without momentum input near the axis, non-diffusive momentum transport keeps non-zero toroidal rotation in the co- (counter-) current direction in the core (outer) region, which may be related to the intrinsic toroidal rotation in the experiment [45, 46].

These features show, at least, qualitative agreement with the experiment, and suggest the validity of the source driven ITG turbulence simulation using GT5D. Since GT5D has a capability of using shaped MHD equilibria and equilibrium profiles in the JT60U database, further quantitative validation will be addressed.

The SOC-like phenomena of the ion heat transport is a unique feature observed in the full-f gyrokinetic simulation, and may be related to non-local or Bohm-like features of the turbulent transport. To study their impact on the turbulent transport in future large devices, we need $\rho^*$ scan with full-f gyrokinetic simulations. In this study, non-diffusive momentum fluxes are observed and their correspondence with the $E_i$ shear stress is discussed. However, a detailed balance among the momentum diffusion, the momentum pinch and the $E_i$ shear stress as well as the Prandtl number have not been identified yet. To study the momentum transport further in detail, we need simulations with different momentum input. These simulations will be addressed in future works.

Acknowledgments

One of the authors (YI) would like to thank Drs H. Sugama, S. Satake, T.-H. Watanabe, M. Honda, N. Hayashi and M. Kikuchi for useful comments on the neoclassical phenomena. Similar 1/f type spectra were observed also in the experiment [44]. The criticality of $L_0$ and 1/f type spectra suggests that stiff $T_0$ profiles produced in the ITG turbulence may be explained by a SOC type picture.

Appendix A. Implementation of collision operator

By neglecting FLR corrections, the linear Fokker–Planck collision operator [25] is given as

$$C(f) = C_r(f) + C_v,$$

(A1)

$$C_r(f) = \frac{\partial}{\partial s}(v_{\perp 1} v_{\perp 1} f) + \frac{\partial}{\partial u}(v_{\parallel 1} uf) + \frac{1}{2} \frac{\partial^2}{\partial s^2}(v_{\perp 2} v_{\perp 2} f)$$

$$+ \frac{1}{2} \frac{\partial^2}{\partial u^2}(v_{\parallel 2} v_{\parallel 2} f) + \frac{\partial^2}{\partial s \partial u}(v_{\perp 1} v_{\perp 1} f),$$

(A2)

$$C_v = P_{FM},$$

(A3)

where $s = 2\mu B/m_i$, and $u = v_{\parallel 1} - U_1$ consist of a moving frame with respect to the parallel flow velocity $U_1$, $v^s = u^s + s$ and $v^f = v_{\perp 1} + \sqrt{2\mu B/m_i}$. $v_{\parallel 1}$ can be determined by solving the parallel momentum equation. $U_1$ can be calculated from the parallel flow velocity $U_1$, i.e., $U_1 = \sqrt{2\mu B/m_i}$.

The SOC-like phenomena of the ion heat transport is a unique feature observed in the full-f gyrokinetic simulation, and may be related to non-local or Bohm-like features of the turbulent transport. To study their impact on the turbulent transport in future large devices, we need $\rho^*$ scan with full-f gyrokinetic simulations. In this study, non-diffusive momentum fluxes are observed and their correspondence with the $E_i$ shear stress is discussed. However, a detailed balance among the momentum diffusion, the momentum pinch and the $E_i$ shear stress as well as the Prandtl number have not been identified yet. To study the momentum transport further in detail, we need simulations with different momentum input. These simulations will be addressed in future works.
The conservation of momentum and energy is satisfied with $C_T$. All the conservation properties are further improved up to the machine precision compared with $C_T$ by adding $C_{corr}$.

<table>
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<th>$C_T + C_F + C_{corr}$</th>
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<td>$\frac{1}{m_| n_| v_|} \int m_| v_| C , d^3v (N_{|} = 128)$</td>
<td>$0.1782E-09$</td>
<td>$-0.5210E-12$</td>
<td>$-0.1326E-24$</td>
</tr>
<tr>
<td>$\frac{1}{2 \sqrt{\pi} n_| v_|} \int m_| v_|^2 C , d^3v (N_{|} = 128)$</td>
<td>$0.1290E-09$</td>
<td>$-0.1967E-11$</td>
<td>$-0.1702E-22$</td>
</tr>
</tbody>
</table>

In the field particle operator (A3), a function $P$, which is determined from the momentum and energy conservation, is written as [26]

\[
P = -3 \sqrt{\frac{3}{2}} \Psi^{-1} \hat{\rho} - 3 \sqrt{\frac{3}{2}} \left( \Psi - \Psi^* \right) \eta^{-1} \hat{E}, \quad (A15)
\]

\[
\hat{\rho} = \frac{1}{n_\| v_\|^3} \int uC_T(f) \, d^3v, \quad (A16)
\]

\[
\hat{E} = \frac{1}{3n_\| v_\|^2} \int u^2C_T(f) \, d^3v. \quad (A17)
\]

With this field particle operator, $C(f)$ conserves the particle number, the momentum and the energy. However, the conservation property is not exactly satisfied with a finite velocity grid number and a limited velocity space. Although the remaining numerical error is small compared with $f$, its accumulation (in particular, erroneous particle accumulation) in a long time simulation may not be negligible compared with $\delta f$. In order to compensate the remaining error, we add a correction term given by

\[
C(f) = C_T(f) + C_F + C_{corr}, \quad (A18)
\]

\[
C_{corr} = v_{corr}(f_{M1} - f_{M2}), \quad (A19)
\]

where $v_{corr}$ is a constant and $f_{M1}$ and $f_{M2}$ are determined in an iterative manner to satisfy the conservation properties, equations (11)–(13). Table 1 shows the conservation properties observed when the collision operator is operated on a test function given by a gyrokinetic Vlasov equilibrium $f_c(P_c, \nu, \mu)$ defined at $r/a = 0.5$ and $\theta = 0$ in a cyclone like configuration used in section 4. The results show that the errors of momentum and energy are cancelled by $C_T$. However, by adding $C_{corr}$, all the conservation properties are improved up to the machine precision compared with the original errors in $C_T$.

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