Consider the Harmonic Oscillator

\[ L(x, \dot{x}) = \frac{1}{2} \dot{x}^2 - \frac{1}{2} x^2 \]

under the following discretization procedure of the Feynman Path integral (lecture):

\[ T_0 = 2\pi \text{ classical period of oscillator} \]

\[ \Delta t = \frac{T_0}{128} \]

\[ N_D = 600 \quad x_0 = -4, \quad x_D = +4 \]

\[ x_{\text{start}} = 0.75 \]

\[ \psi_0 = \left( \frac{\alpha}{\pi} \right)^{1/4} e^{-\frac{x^2}{2} (x-x_{\text{start}})^2} \quad \text{initial wavefunction} \]

\[ \alpha = 2 \]
1. Calculate the propagator $K$ from the elementary $K_E$ matrix $(N_0 + 1) \times (N_0 + 1)$ dimensional,

$$E = \frac{T_0}{128} = \Delta t$$

for time period $\frac{T_0}{16}$

$$K = (\Delta x)^{N-1} \cdot K_E^N (\Delta t)$$

2. Evolve the wavefunction in time with $\frac{T_0}{16}$ stepsize and measure $\langle x \rangle$ as a function of time. Make a plot.

3. Calculate $\langle E \rangle$, $\langle K \rangle$, $\langle V \rangle$ as a function of time. Make a plot.

4. Calculate the evolution of the wavefunction as a function of time. Make plot.

5. Compare your plots with pages from lecture plots of 3-4-5-6

6. Bonus (142) Animation of the wavefunction

required for 242