19. Table 19-1 gives \( M = 28.0 \) g/mol for nitrogen. This value can be used in Eq. 19-22 with \( T \) in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using \( v \) for \( v_{\text{rms}} \):

\[
\frac{v_2}{v_1} = \frac{\sqrt{\frac{3RT_2}{M}}}{\sqrt{\frac{3RT_1}{M}}} = \sqrt{\frac{T_2}{T_1}}.
\]

(a) With \( T_2 = (20.0 + 273.15) \) K \( \approx 293 \) K, we obtain \( v_2 = (517 \text{ m/s}) \sqrt{\frac{293 \text{ K}}{300 \text{ K}}} = 511 \text{ m/s} \).

(b) In this case, we set \( v_3 = \frac{1}{2}v_2 \) and solve \( \frac{v_3}{v_2} = \sqrt{\frac{T_3}{T_2}} \) for \( T_3 \):

\[
T_3 = T_2 \left( \frac{v_3}{v_2} \right)^2 = (293 \text{ K}) \left( \frac{1}{2} \right)^2 = 73.0 \text{ K}.
\]

which we write as \( 73.0 - 273 = -200^\circ \text{C} \).

(c) Now we have \( v_4 = 2v_2 \) and obtain

\[
T_4 = T_2 \left( \frac{v_4}{v_2} \right)^2 = (293 \text{ K})(4) = 1.17 \times 10^3 \text{ K}
\]

which is equivalent to \( 899^\circ \text{C} \).
21. **THINK** According to kinetic theory, the rms speed is (see Eq. 19-34)
\[ v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \], where \( T \) is the temperature and \( M \) is the molar mass.

**EXPRESS** The rms speed is defined as
\[ v_{\text{rms}} = \sqrt{\langle v^2 \rangle_{\text{avg}}} \], where
\[ \langle v^2 \rangle_{\text{avg}} = \int_0^\infty v^2 P(v)dv \], with the Maxwell’s speed distribution function given by
\[
P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}.
\]

According to Table 19-1, the molar mass of molecular hydrogen is \( 2.02 \text{ g/mol} = 2.02 \times 10^{-3} \text{ kg/mol} \).

**ANALYZE** At \( T = 2.7 \text{ K} \), we find the rms speed to be
\[
v_{\text{rms}} = \sqrt{\frac{3 \left( 8.31 \text{J/mol} \cdot \text{K} \right) (2.7 \text{K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.8 \times 10^2 \text{ m/s}.
\]

**LEARN** The corresponding average speed and most probable speed are
\[
v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \left( 8.31 \text{J/mol} \cdot \text{K} \right) (2.7 \text{K})}{\pi (2.02 \times 10^{-3} \text{ kg/mol})}} = 1.7 \times 10^2 \text{ m/s}
\]
and
\[
v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2 \left( 8.31 \text{J/mol} \cdot \text{K} \right) (2.7 \text{K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.5 \times 10^2 \text{ m/s},
\]
respectively.
23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is \( 2mv \cos \theta \), where \( m \) is the mass of the molecule, \( v \) is its speed, and \( \theta \) is the angle between its velocity and the normal to the wall. If \( N \) molecules collide with the wall, then the change in their total momentum is \( 2Nmv \cos \theta \), and if the total time taken for the collisions is \( \Delta t \), then the average rate of change of the total momentum is \( 2(N/\Delta t)mv \cos \theta \). This is the average force exerted by the \( N \) molecules on the wall, and the pressure is the average force per unit area:

\[
p = \frac{2}{A(\Delta t)} \left( \frac{N}{\Delta t} \right) mv \cos \theta = \left( \frac{2}{2.0 \times 10^{-4} \text{ m}^2} \right) \left( 1.0 \times 10^{23} \text{ s}^{-1} \right) \left( 3.3 \times 10^{-27} \text{ kg} \right) \left( 1.0 \times 10^3 \text{ m/s} \right) \cos 55^\circ
\]

\[
= 1.9 \times 10^3 \text{ Pa}.
\]

We note that the value given for the mass was converted to kg and the value given for the area was converted to m\(^2\).
26. The average translational kinetic energy is given by \( K_{\text{avg}} = \frac{3}{2} kT \), where \( k \) is the Boltzmann constant \( (1.38 \times 10^{-23} \text{ J/K}) \) and \( T \) is the temperature on the Kelvin scale. Thus

\[
K_{\text{avg}} = \frac{3}{2} \left( 1.38 \times 10^{-23} \text{ J/K} \right) (1600 \text{ K}) = 3.31 \times 10^{-20} \text{ J}.
\]
29. **THINK** Mean free path is the average distance traveled by a molecule between successive collisions.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

\[ \lambda = \frac{1}{\sqrt{2\pi d^2 N/V}}, \]

where \( d \) is the diameter of a molecule and \( N \) is the number of molecules in volume \( V \).

**ANALYZE** (a) Substituting \( d = 2.0 \times 10^{-10} \text{ m} \) and \( N/V = 1 \times 10^6 \text{ molecules/m}^3 \), we obtain

\[ \lambda = \frac{1}{\sqrt{2\pi (2.0 \times 10^{-10} \text{ m})^2 (1 \times 10^6 \text{ m}^{-3})}} = 6 \times 10^{12} \text{ m}. \]

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

**LEARN** Mean free path is inversely proportional to the number density, \( N/V \). The typical value of \( N/V \) at room temperature and atmospheric pressure for ideal gas is

\[ \frac{N}{V} = \frac{p}{kT} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(298 \text{ K})} = 2.46 \times 10^{25} \text{ molecules/m}^3 = 2.46 \times 10^{19} \text{ molecules/cm}^3. \]

This is much higher than that in the outer space.

30. We solve Eq. 19-25 for \( d \):

\[ d = \sqrt{\frac{1}{\lambda \pi \sqrt{2} (N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5 \text{ cm}) \pi \sqrt{2} (2.7 \times 10^{19} / \text{cm}^3)}} \]

which yields \( d = 3.2 \times 10^{-8} \text{ cm}, \) or 0.32 nm.
35. (a) The average speed is

\[ v_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} v_i = \frac{1}{10} [4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s}. \]

(b) The rms speed is

\[ v_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10} [4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s} \]

(c) Yes, \( v_{\text{rms}} > v_{\text{avg}}. \)
37. **THINK** From the distribution function \( P(v) \), we can calculate the average and rms speeds.

**EXPRESS** The distribution function gives the fraction of particles with speeds between \( v \) and \( v + dv \), so its integral over all speeds is unity: \( \int P(v) \, dv = 1 \). The average speed is defined as \( v_{\text{avg}} = \int_0^\infty v P(v) \, dv \). Similarly, the rms speed is given by \( v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}} \), where \( (v^2)_{\text{avg}} = \int_0^\infty v^2 P(v) \, dv \).

**ANALYZE** (a) Evaluate the integral by calculating the area under the curve in Fig. 19-23. The area of the triangular portion is half the product of the base and altitude, or \( \frac{1}{2} av_0 \). The area of the rectangular portion is the product of the sides, or \( av_0 \). Thus,

\[
\int P(v) \, dv = \frac{1}{2} av_0 + av_0 = \frac{3}{2} av_0 ,
\]

so \( \frac{3}{2} av_0 = 1 \) and \( av_0 = 2/3 = 0.67 \).

(b) For the triangular portion of the distribution \( P(v) = av/v_0 \), and the contribution of this portion is

\[
\frac{a}{v_0} \int_0^{v_0} v^2 \, dv = \frac{a}{3v_0} v_0^3 = \frac{av_0^2}{3} = \frac{2}{9} v_0 ,
\]

where \( 2/3v_0 \) was substituted for \( a \). \( P(v) = a \) in the rectangular portion, and the contribution of this portion is

\[
a \int_{v_0}^{2v_0} v \, dv = \frac{a}{2} \left( 4v_0^2 - v_0^2 \right) = \frac{3a}{2} v_0^2 = v_0 .
\]

Therefore, we have

\[
v_{\text{avg}} = \frac{2}{9} v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{\text{avg}}}{v_0} = 1.22 .
\]
(c) In calculating $v_{\text{avg}}^2 = \int v^2 P(v) \, dv$, we note that the contribution of the triangular section is

$$\frac{a}{v_0} \int_0^{v_0} v^3 \, dv = \frac{a}{4v_0} v_0^4 = \frac{1}{6} v_0^2.$$

The contribution of the rectangular portion is

$$a \int_{v_0}^{2v_0} v^2 \, dv = \frac{a}{3} \left( 8v_0^3 - v_0^3 \right) = \frac{7a}{3} v_0^3 = \frac{14}{9} v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6} v_0^2 + \frac{14}{9} v_0^2} = 1.31v_0 \quad \Rightarrow \quad \frac{v_{\text{rms}}}{v_0} = 1.31.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N \int_{1.5v_0}^{2v_0} P(v) \, dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N \, a v_0 = N/3,$$

where $2/3v_0$ was substituted for $a$. In other words, the fraction of particles in this range is $1/3$ or 0.33.
40. We divide Eq. 19-31 by Eq. 19-22:

\[
\frac{v_{\text{avg}2}}{v_{\text{rms}1}} = \sqrt{\frac{8RT/\pi M_2}{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}
\]

which, for \( v_{\text{avg}2} = 2v_{\text{rms}1} \), leads to

\[
\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8} \left( \frac{v_{\text{avg}2}}{v_{\text{rms}1}} \right)^2 = \frac{3\pi}{2} = 4.7
\]