Computer Simulation of Saturn's Ring Structure

New Mexico

Supercomputing Challenge

Final Report

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Team 58

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Summary

The main goal of this project is to develop a computer program to model the creation of structure in Saturn’s ring system. The computer program will be used to answer these questions: (1) How are gaps in Saturn’s Rings formed; (2) how accurately can I model gap formation with a 3D N-Body simulation; and (3) will my simulation compare to observed features, theoretical data, and professional simulations. Newton’s laws of motion and gravity as well as the velocity Verlet method are being used to orbit the particles around Saturn. Gaps in Saturn’s ring system are caused by three main methods: (1) Gravitational resonances; (2) moons that orbit inside the ring; and (3) an asteroid or comet impact. Gravitational resonances are a major part of formation in the ring system, and are caused by a special ratio of orbits between a moon and a ring particle. Many different simulations were run to determine which moons are responsible for which gaps in the ring system. A patch method was developed in order to get a high particle density which allows for more accurate results. Many professionals are using a rotating patch method which achieves the same thing, however my patch method is unique and very easy compared to the rotating method. A simulation that involved an asteroid impact was run to determine the effects on the ring system. This simulation showed that an asteroid will cause a ripple effect that may eventually form a large ring. There are no professional simulations for an asteroid impact to date, however my simulation matches a professional ‘cartoon’ of what is expected to happen. Many different computers were used for this project, and the code was parallelized using both MPI and OpenCL. This project requires the use of parallel computing in order to simulate the large number of ring particles needed to achieve an accurate simulation. My simulations proved certain moons cause certain gaps which matched the observed structure of Saturn’s rings.

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Introduction

There are three main ways gaps in Saturn's rings are formed: by resonances, moons within the rings, and an asteroid or comet impact. The first method for gap formation is a resonance which is a special interaction between a ring particle and a moon. There are many resonances in Saturn's ring system caused by many different moons. One of the more popular resonances is the 2:1 resonance with the moon Mimas. This resonance creates the small Huygen's gap in the Cassini Division, a large gap about 5,000 km wide visible from Earth. A 2:1 resonance means the ring particle orbits Saturn twice for each time the moon (in this case Mimas) orbits once. This particular resonance only occurs at a specific location in the ring, other locations do not feel this special interaction (the moon's effects average out to zero for them), and will orbit normally. The particles that are on the resonance location, will build up an acceleration due to that moon over time. Over time, the increasing acceleration the particle feels from the moon will cause it to shift its orbit which causes the gap to form. A resonance is entirely dependent on the location of the moon. If the moon's orbit changes then the resonance location will change. This is why it is important to keep moon orbits stable when modeling this effect.

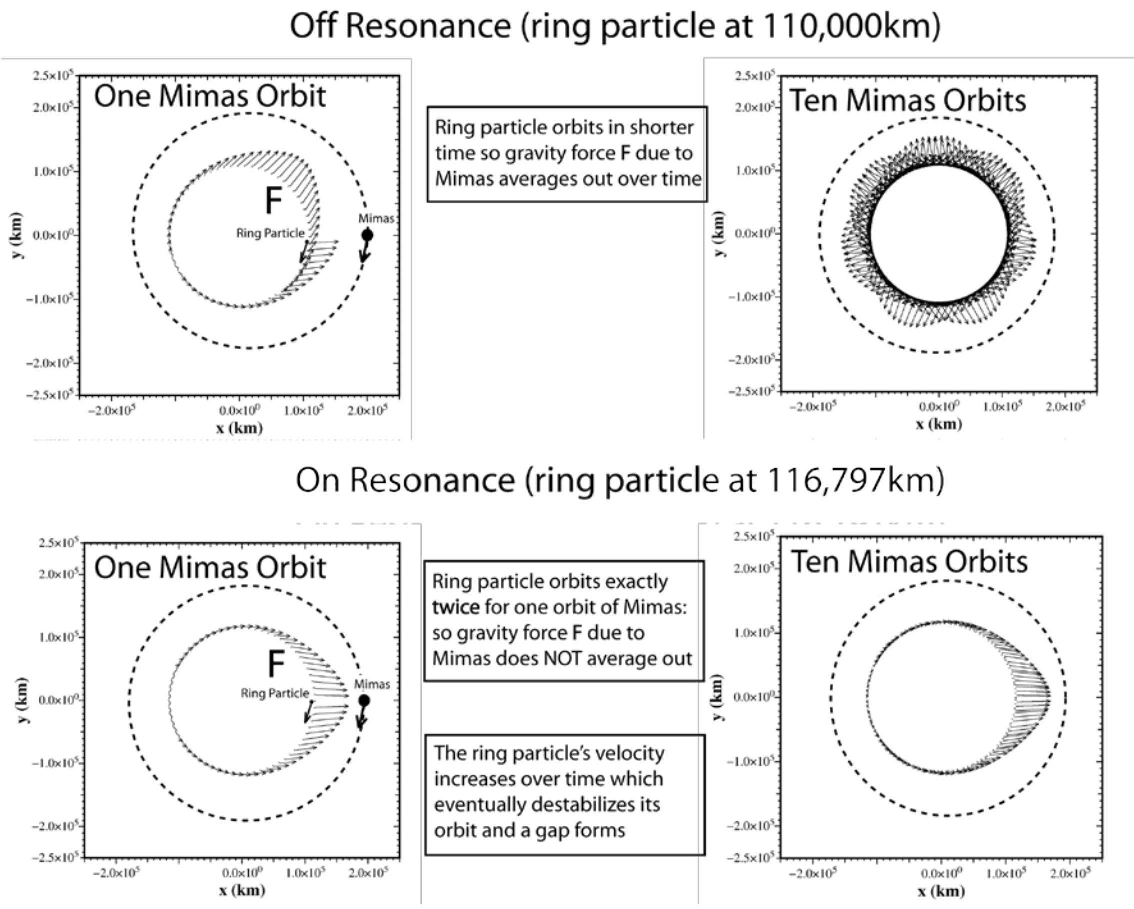

Figure 1: Shows the different interactions a ring particle feels when it is on a resonance location and when it is not on a resonance location.

Another way gaps are by moons that orbit within the ring. These cause the ring particles close to their orbit to get a large gravitational kick which pushes them out of the path, clearing a gap.

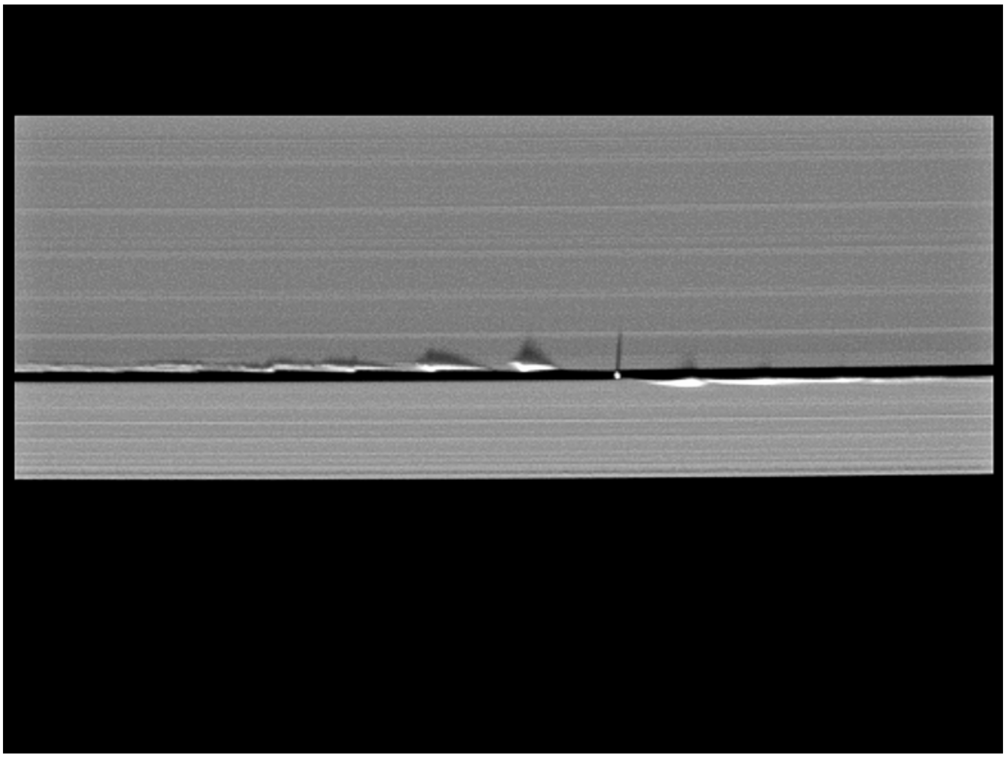

Figure 2: Saturn’s moon Daphnis orbiting within the ring (“plowing” a gap).

The third method for gap formation is an asteroid or comet impact in the ring structure. Many astronomers believe that a impact in 1983 caused a set of ripples in Saturn’s ring. My simulation will try to simulate all three gap formation mechanisms.

Problem

The main questions this project will answer are: how are the gaps in Saturn's rings formed, how accurately will I be able to model gap formation with a 3D N-Body simulation, will my simulation compare to observed and professional results. My simulation of Saturn's Ring structure consists of individual ring particles randomly distributed from Saturn's D Ring to the edge of Saturn's F ring which accounts for the visible sections of the ring system. These particles orbit around Saturn using Newton's Laws of gravity and the Velocity Verlet method. My simulation currently treats 15 of the most important moons. Due to the large force particles feel from Saturn, the moon interactions are captured in separate R_{moon}, V_{moon}, and A_{moon} arrays. If the moon interactions are included in the original R, V, and A arrays, their effects get essentially buried due to the large interaction from Saturn. This causes the moon effects to never add up significantly (they are smaller than the numerical errors). Including moon interactions in a separate array allows the simulation to pick up moon interactions much more efficiently. The program will add the R_{moon}, V_{moon}, and A_{moon} arrays to the original arrays every two hours. Ring particle interactions are not included in my simulation since the particle density is too low (they are treated as collisionless particles). My simulation is written in the C programming language also using MPI (Message Passing Interface) and OpenCL for parallelization.

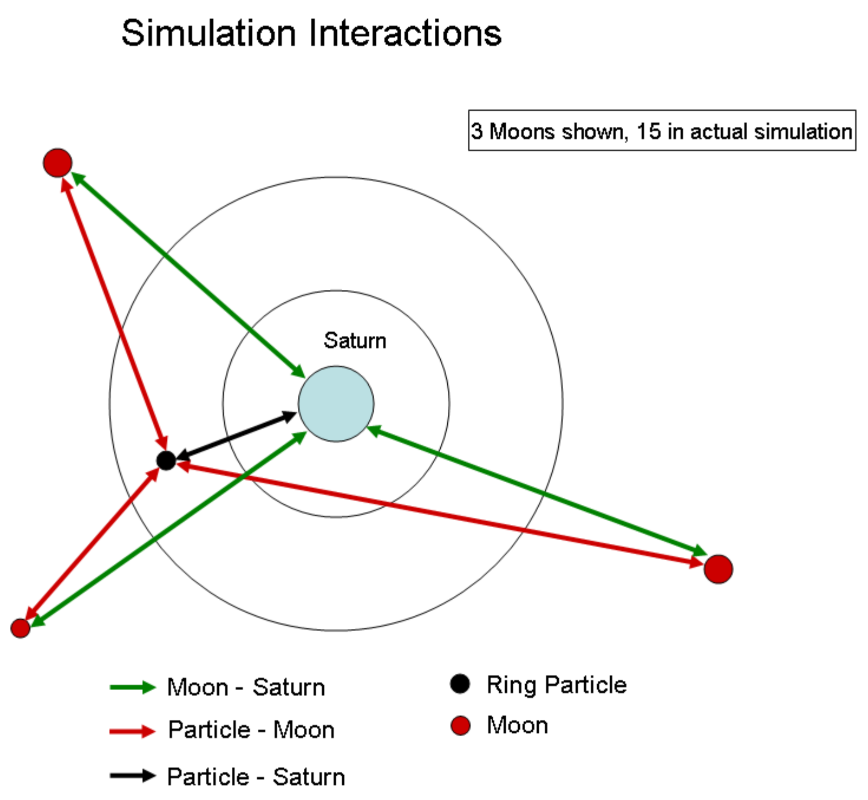

Figure 3: Model of gravity interactions included in the simulation.

Results

Many simulations were run with different moons included to determine which moons caused certain gaps, and how the ring system changed as different moons were added or subtracted. Most simulations were run with one million ring particles and a one second time step. Figure 4 (below) compares a detailed photo of Saturn's ring structure with my 3D simulation results. Velocity magnitude is plotted, showing spikes which are due to interactions from the moons (labeled with the numbers). The five spikes on the right edge are due to moons that orbit within the ring. The other three spikes toward the middle of the ring are caused by resonances. The 2:1 resonance with Mimas mentioned earlier is the largest resonance picked up by my simulation. These spikes from the moon interactions match up with the structure of Saturn's ring which proves that my simulation is able to pickup large features of the ring system accurately.

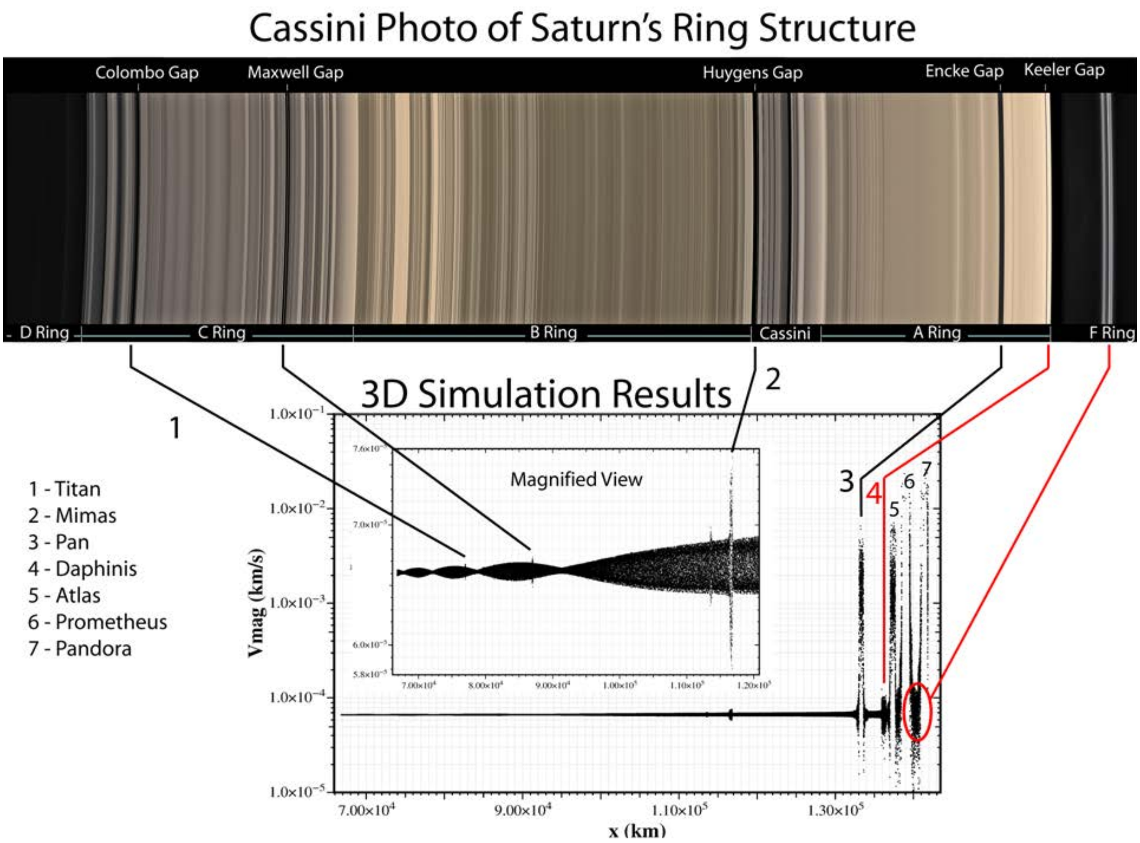

Figure 4: Detailed photo of Saturn's rings with major gaps labeled compared to my 3D simulation results.

Figure 5 (below) shows a 3D plot of my simulation results above. The velocity magnitude is colored, red is faster while blue is slower. The major spikes seen above correspond to the red bands forming around the ring system.

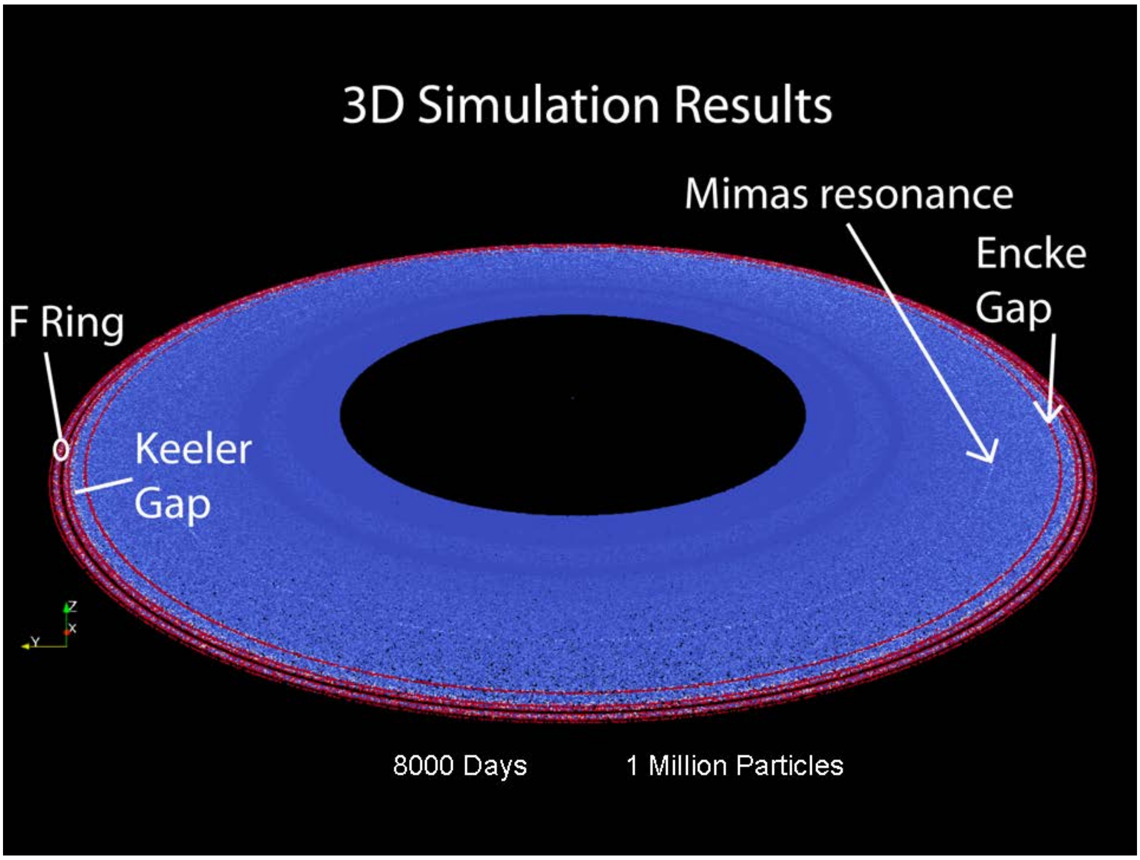

Figure 5: A 3D view of my simulation results plotted in figure 4 with major gaps labeled.

Patch Method

Spreading particles out over a full ring system has a very low particle density which caused only a few moon interactions to be picked up. Since Saturn's rings are symmetrical, splitting the rings into a small patch would allow for a higher particle density and pickup more moon interactions. The Patch method (shown below in Figure 6) is another way for computing Saturn's rings without modeling the whole entire system. Ring particles are now distributed within the patch area which is determined by the angle $\Delta\theta$. The particles orbit Saturn moving downward, when a particle exceeds the patch boundary it is rotated by the angle $\Delta\theta$ to the top of the patch. Each particle has a counter that gets incremented when that particle gets 'reset'. This counter allows the program to be able to compute the actual position of the particle as if it were normally orbiting Saturn. The moons orbit normally around Saturn.

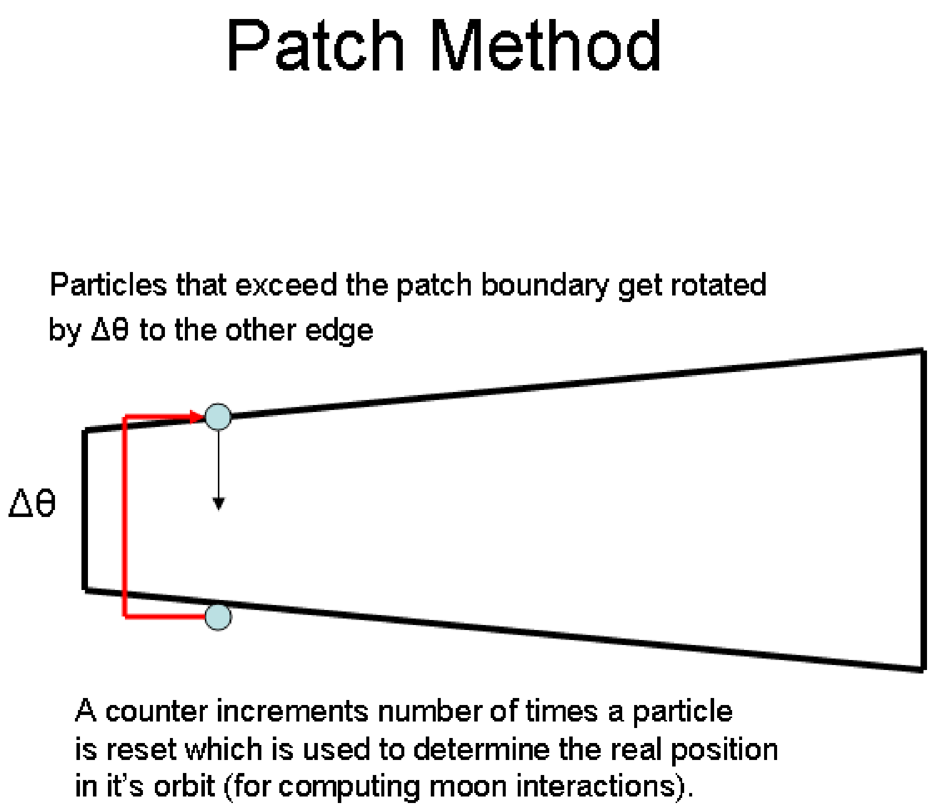

Figure 6: Diagram of the patch method, shows how the ring particle is rotated back to the top when it exceeds the patch boundary.

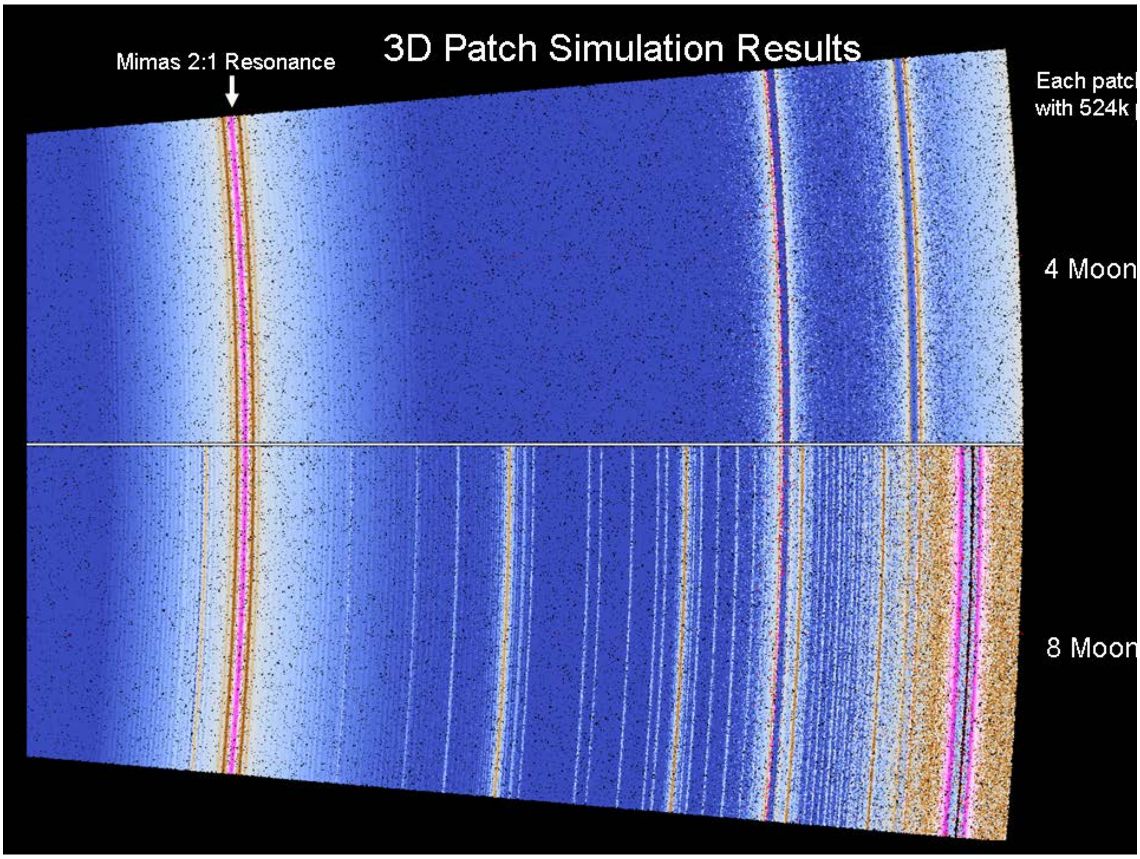

Figure 7: 4 moon (Mimas, Daphnis, Pan, and patch simulation (top) vs. 8 moon (previous four moons with Epimetheus, Prometheus, Janus, and Pandora) patch simulation (bottom).

Figure 7 (above) compares two different patch simulations. The top patch is with 4 moons and the bottom is with 8 moons included. This simulation shows that the patch method significantly increases the moon effects and my simulation is able to pick up finer and smaller resonances than before. With the patch method, the number of particles was able to decreased from one million to half a million but still keep a high particle density. The patch simulations above are zoomed in to the outer half of the ring. Figure 8 (below) shows resonance spikes for the moons Epimetheus (red) and Janus (blue). Theoretical predicted resonance locations are the bars plotted below. Both the Janus and Epimetheus resonances show up practically on top of each other since the two moons are only about 40 km apart from each other. Being close together allows the resonances to basically double up and increase in magnitude. Figure 9 shows resonance spikes for the moons Prometheus (blue) and Pandora (red). Prometheus and Pandora are also close together, however they are farther apart then Epimetheus and Janus which causes the resonances to stack close to each other instead of doubling in magnitude.

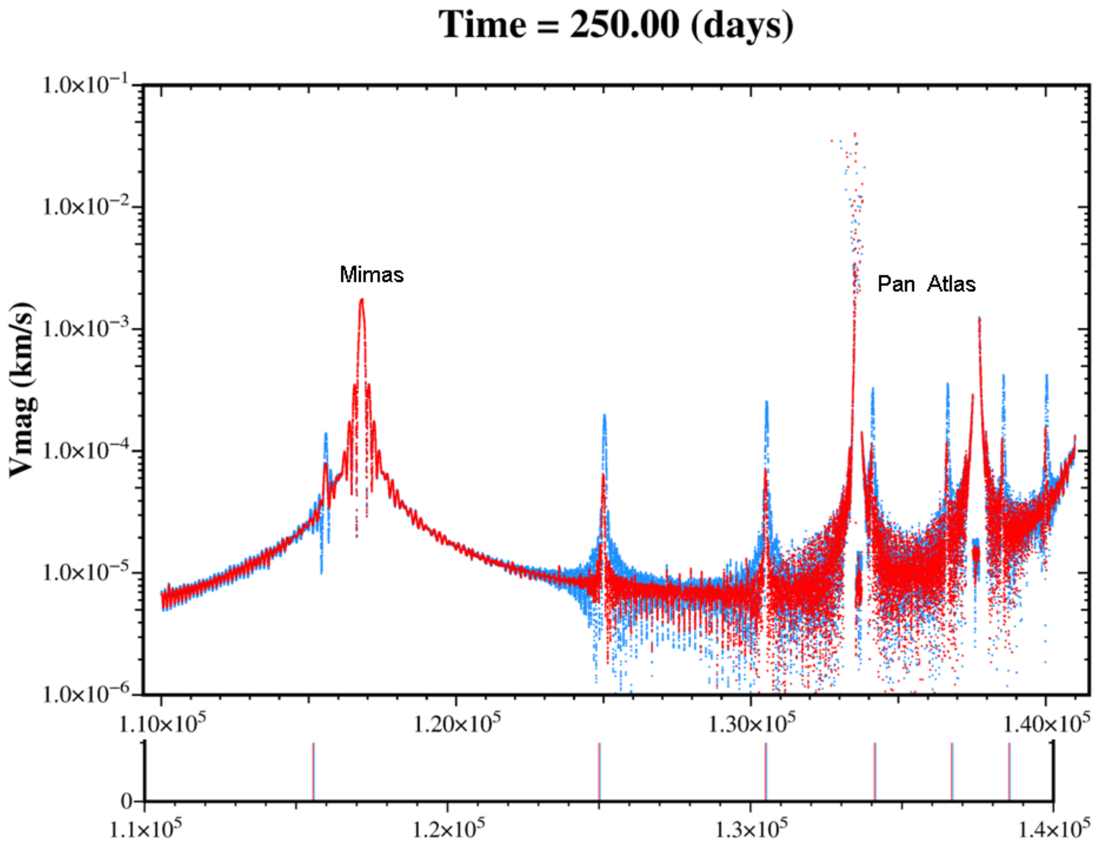

Figure 8: Resonance spikes for Janus (blue) and Epimetheus (red). Theoretical resonance locations plotted as bars underneath the simulation data.

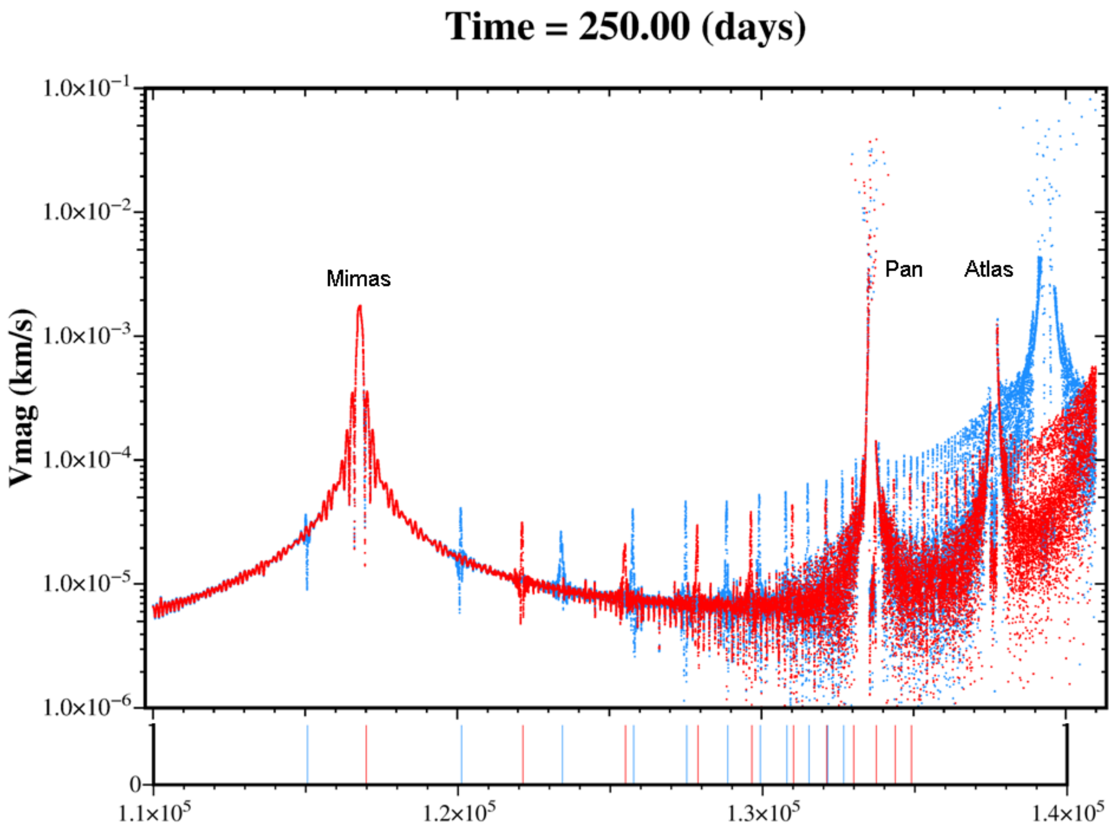

Figure 9: Resonance spikes for Prometheus (blue) and Pandora (red). Theoretical resonance locations plotted as bars underneath the simulation data.

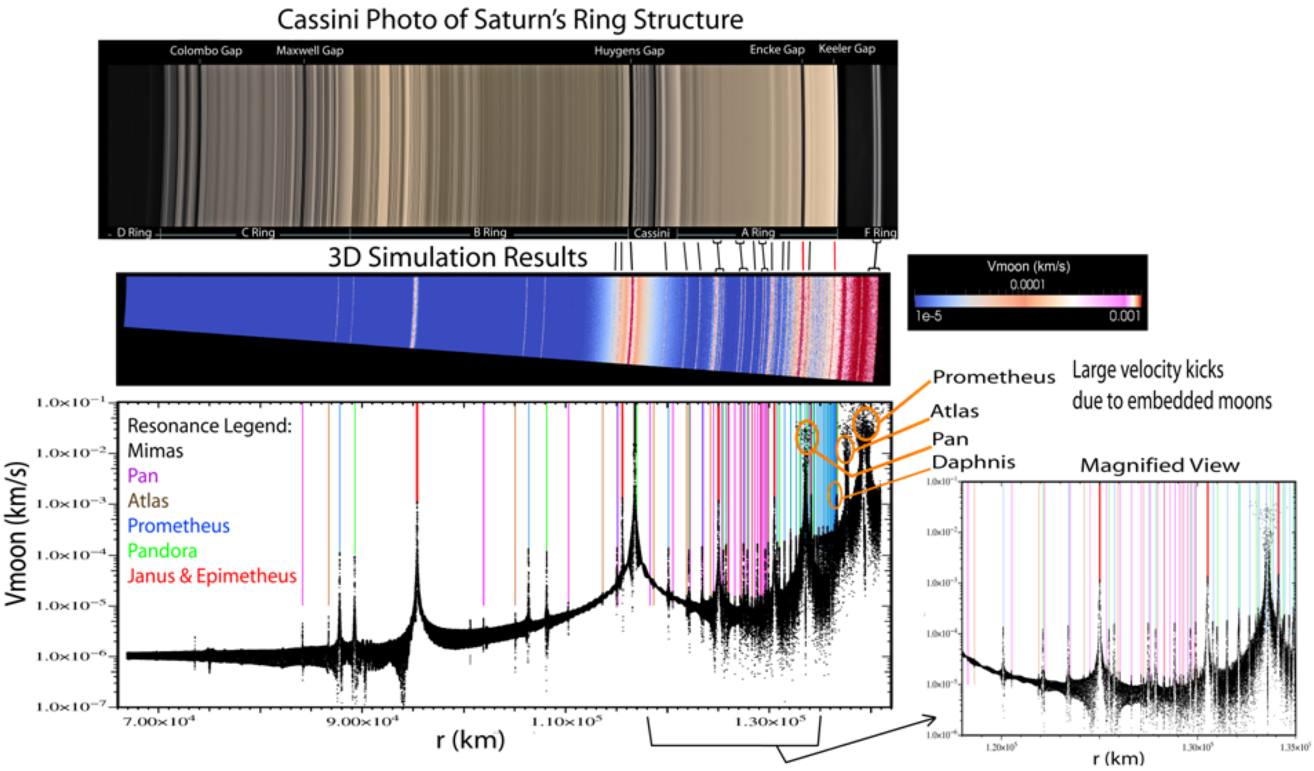

Figure 10: My higher resolution 3D patch simulation comparison to Cassini photo of Saturn's Rings. Velocity due to the moons are plotted. Resonance spikes are also compared while the theoretical bars are plotted with color. Two million particles were used in this small patch region versus one million for the full ring in Figs. 4 and 5. The simulation took 8 full days on two Nvidia GTX-580s.

Figure 10 (above) compares the new patch method simulation with the detailed photo of Saturn's rings. Four small patches with 524 thousand particles each were simulated separately then combined to get a patch with 2 million particles total. Below the patch, the particle velocities due to the moon interactions are plotted in a 1D profile. The resonance spikes are due to the moon resonances. Theoretical resonance locations are plotted as colored bars which run vertically through the data. The theoretical resonance locations are determined by setting the time required for a ring particle to orbit n times equal to (n-1) moon orbits to get $R_{res} = [(n-1)/n]^{2/3} R_{moon}$. The location of the resonance is R_{res} and the moon's orbit is R_{moon}. The resonance spikes picked up from my simulation match the theoretical resonance locations and the observed patterns in the Cassini image. Compared to the previous plot (Figure 4) the patch method significantly increased the particle density and allowed my simulation to see more moon effects and realistic features. Figure 11 (below) shows the full ring simulation with the small section of the patch overlaid. This figure shows how small the patch is compared to the full ring. The magnified views on the right show how much the moons push the particles outward in the Z direction.

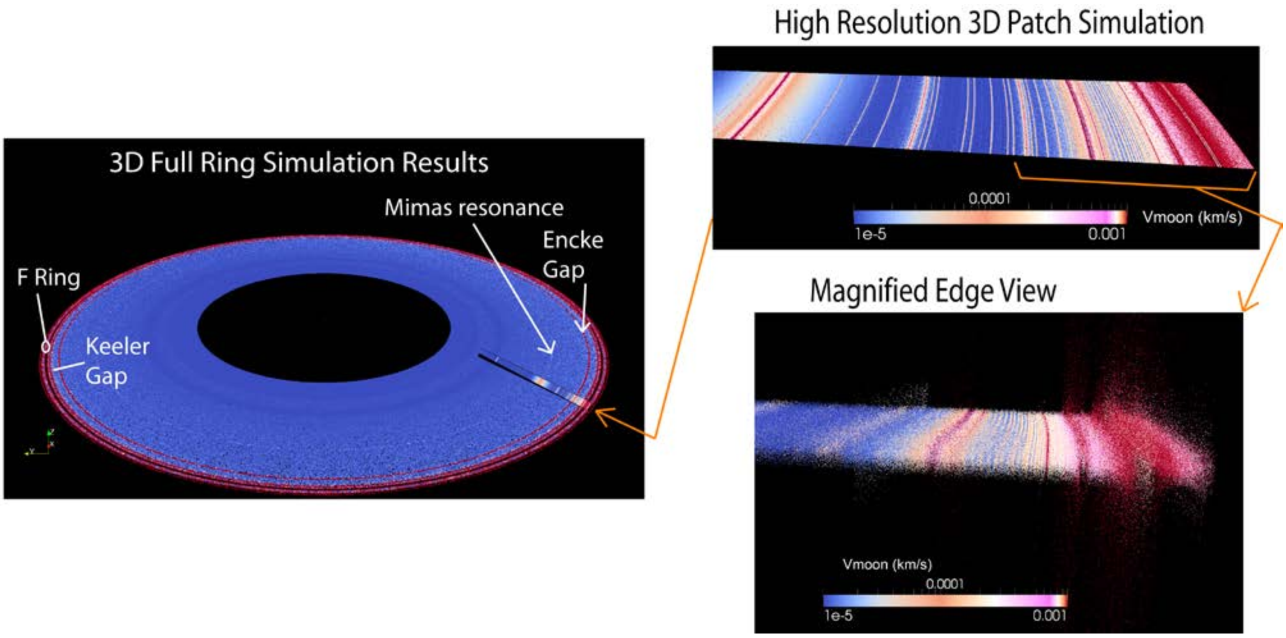

Figure 11: Full ring simulation with patch simulation overlaid. Magnified on right to show the moons kicking particles out in the Z direction.

Comparison to Professional Results

Figure 12 below plots the results from the first professional numerical N-body simulation of Saturn's ring. In order to do the calculation in 1981, the simulationo was 2D, only one moon was considered, and 2,000 ring particles were used. Only a small section of the ring was simulated for just 15 days. Also, in order to see an effect, the mass of Saturn's moon Mimas was increased by 100,000 times. For comparison, my 3D simulations used the real Mimas mass and included up to 15 moons and 2 million particles and simulated thousands of days. This is now possible due to the large number of cores on GPUs. Figure 13 plots the results from another professional simulation for a 3D model ring system. Again the mass of Mimas was increased this time by about 1,000 times, only one moon was considered, and the simulation was run to 400 days. Resonance spikes similar to the ones seen in my 3D simulations (Figs. 4, 8-10 above) were observed. Most modern professional 3D ring simulations use around 300,000 particles and consider a very tiny region of the ring. They also use a rotating "patch method" which moves with the ring and is different than mine. A rotating patch must include rotational forces which complicates the calculation. My patch is stationary and so it doesn't have to include rotational forces which is easier to code and runs faster.

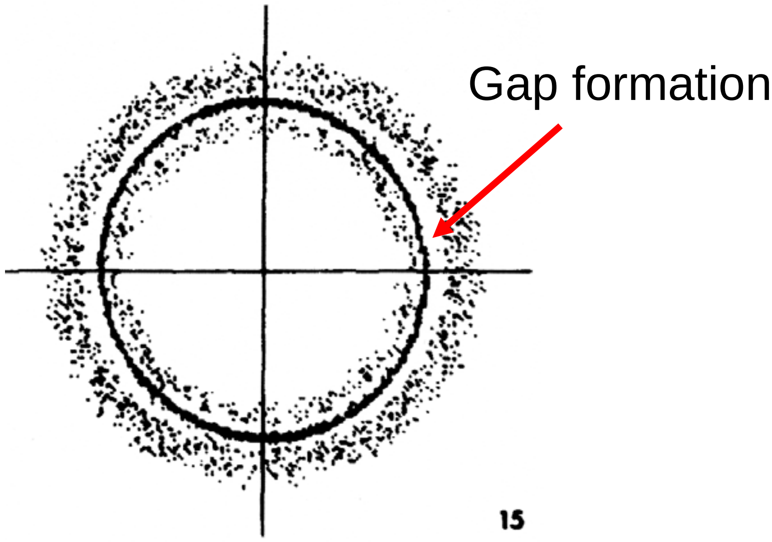

Figure 12: Professional two-dimensional N-body simulation of the ring region near the Mimas 2:1 resonance (2,000 particles, 15 days, used Mmimas/Msaturn = 0.002, actual value is 0.67×10^{-7}) Schwarz, P. M. "Clearing the Cassini Division." *Icarus* ***48*** (1981): 339-342.

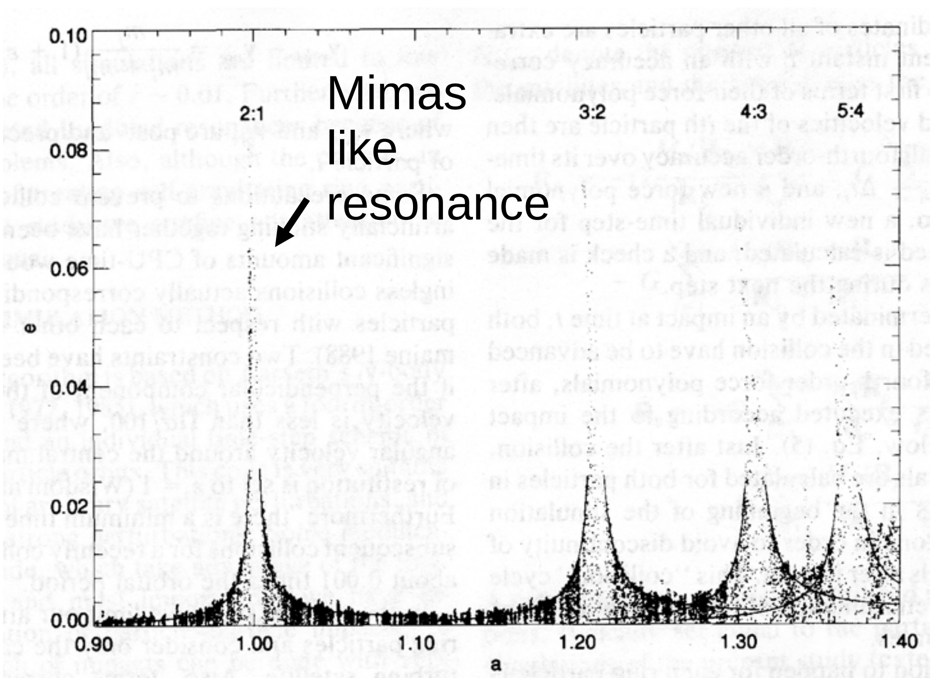

Figure 13: Three-dimensional N-body simulation of a model ring system (1,000 particles, 400 days, used Mmimas/Msaturn = 1×10^{-4}, actual value is 0.67×10^{-7}). Hanninen, J., Salo, H. "Collisional Simulations of Satellite Lindblad Resonances." *Icarus* **97**(1992): 228-247.

Asteroid Impact

With the asteroid impact simulation, an asteroid the size of the moon Titan (the second largest moon in our solar system) moved through the ring system. The main gaps were added in by hand to determine how the asteroid affected already existing gaps and if it would change their behavior. This asteroid impact simulation was with one million particles and ran for one day of simulation time. The calculations were done on an ATI HD5850 and required about 5 hours. Figure 14 (below) shows the first hour of simulation time. The asteroid just collided with the rings causing large spikes in the Z direction. This impact can be seen almost like dropping a rock into water. As the asteroid neared the surface of the ring system, it pulled particles up creating the spike at the top, then went through the ring system, pulling particles downward creating the lower spike. After seven hours of simulation time (Figure 15) large waves have formed from the initial two spikes and have spread throughout the ring. Since the particles closer towards Saturn move faster then those that are farther away, it causes a sheering effect which causes the waves to spread out rapidly in a spiral pattern. At 16 hours of simulation time (Figure 16) more large spiral waves have formed have spread father out through the ring system. Towards the end of the simulation, at 22 hours (Figure 17) a large tear to form which consumes almost allof the top half of the ring system. This tear is due to the sheering effect of the initial hole punctured by the asteroid. In summary, an asteroid impact on Saturn's ring structure greatly disturbs the structure by causing large waves to spread throughout the ring and only after a day a large gap forms which could create a gap eventually.

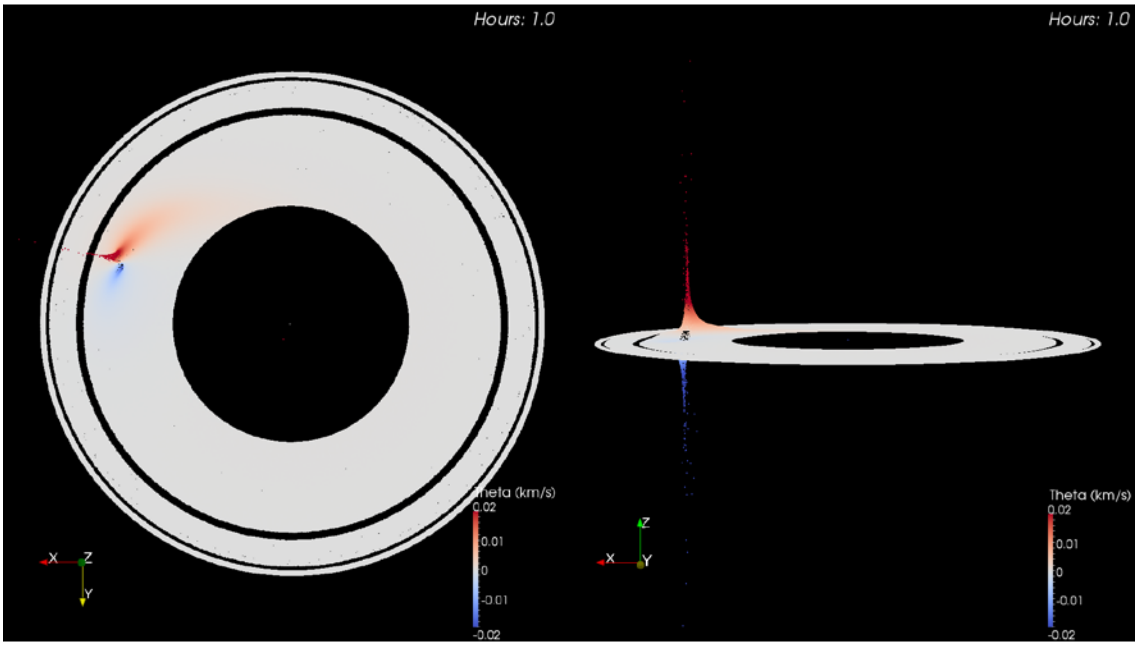

Figure 14: Asteroid impact simulation after one hour of simulation time. Initial spikes form. The ring particles are moving clockwise. The red and blue colors correspond to the particle's velocity in the V_θ direction.

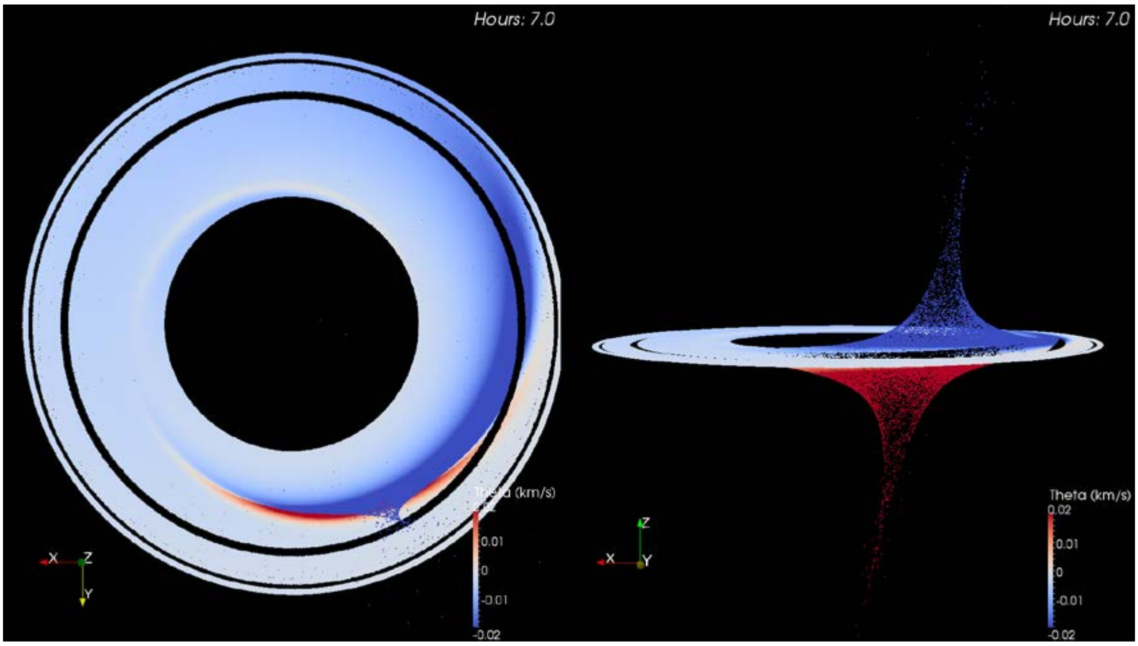

Figure 15: Asteroid impact simulation after 7 hours of simulation time. Large spiral waves form and spread out through the rings.

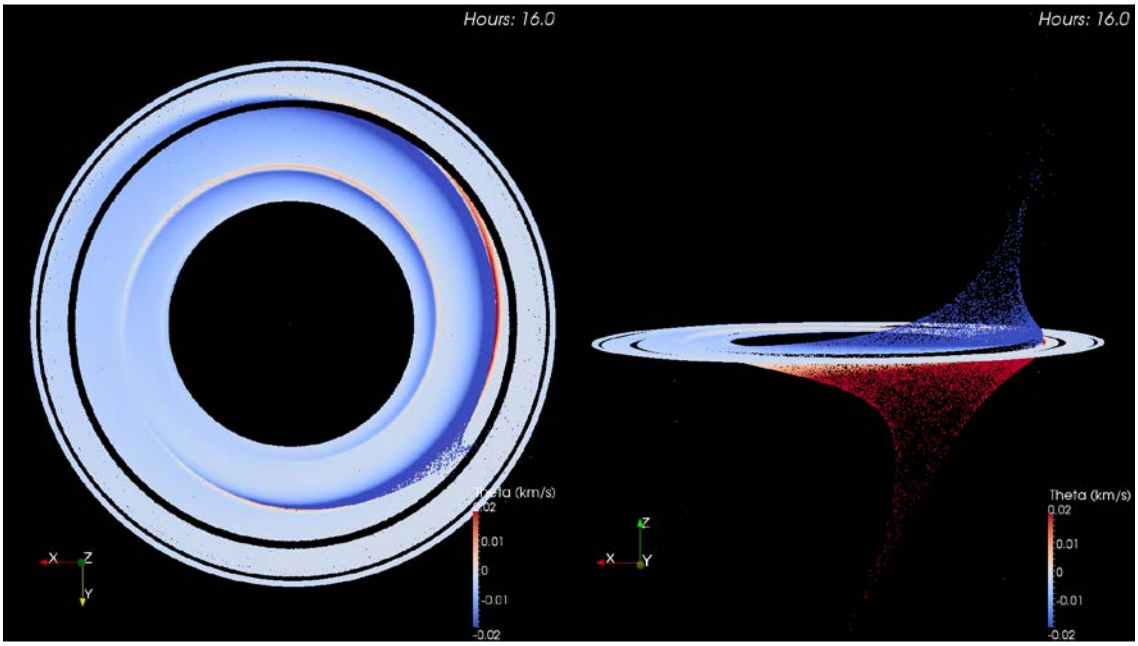

Figure 16: Asteroid impact simulation after 16 hours of simulation time. More large spiral waves continue to spread out through the rings.

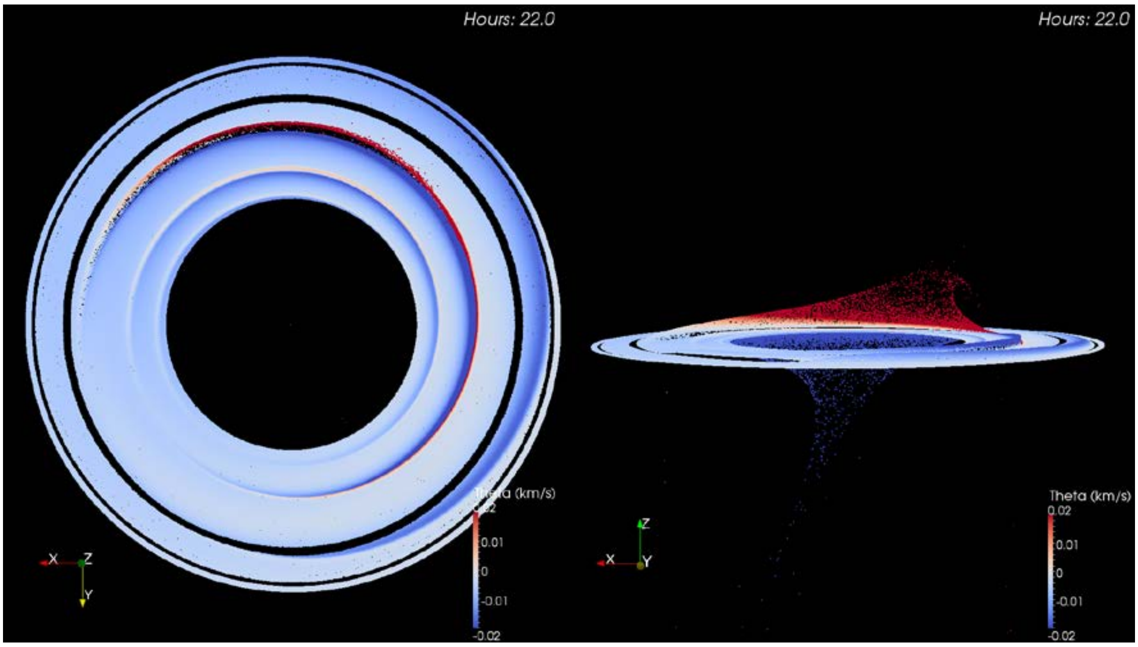

Figure 17: Asteroid impact simulation after 22 hours of simulation time. Large tear in ring can be seen on the top half edge.

To date, no professional N-body simulations have been done for a comet/asteroid impacting with Saturn's rings. Professional theoretical studies suggest that spiral wave structures will form (see Fig. 18 below) and observed Cassini images indicate that ripples in the rings may be the result of a 1983 impact. The date of the impact was estimated based on the rate of decrease in the ripples amplitude over time. In future work, my N-body ring impact simulation could be used to confirm this date.

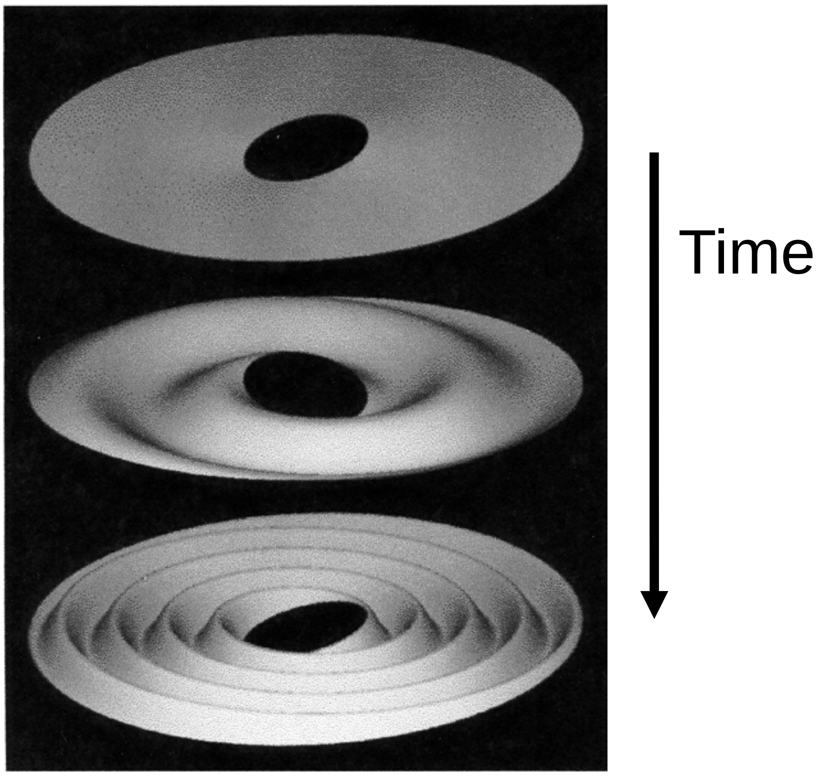

Figure 18: Cartoon representation of how a comet impact leads to a spiral wave due to the shearing effect of the ring particles' rotation (Note: this is <u>not</u> a simulation). Hedman, M., Burns, J., Evans, M., Tiscareno, M., Porco, C. "Saturn's Curiously Corrugated C Ring." *Science* **332** (2011): 708.

Program

My code is written in the C programming language and uses MPI (Message Passing Interface) and OpenCL to parallelize the simulation. My code development history is summarized below:

My Code Development History

- Created original single CPU C code
- Added 26 moons to the calculation
- Made a MPI full N-Body version
- Tested and ran many MPI simulations (on LANL's Mapache)
- Made a OpenCL N-Body version to run on GPUs
- Created a post processing program
- Added **R**moon **V**moon and **A**moon arrays to OpenCL code
- Removed particle-particle interactions and increased particle count significantly
- Ran many tests with different moons to determine 15 most optimal
- Collected data from many simulations for results
- Implemented the patch idea on the OpenCL code
- Currently writing a fast tree searching nearest neighbor method for the OpenCL code

Figures 19, 20, and 21 are flowcharts of how the code works. Figure 19 shows the host program which is run on the computer's CPU. Figure 20 shows the kernel which is run on the computer's GPU (Graphical Processing Unit) and handles all the particle computation and updating. OpenCL is used over MPI for most of this project since the graphics cards contain hundreds of small processors which when compared to a CPU's several cores out perform it significantly. Figure 21 shows the post processing code which is used to read in the data files and generate 3D vtk plot files for ParaView. The detailed steps and command line options for my processing code are outlined in Figure 22. Figure 23 plots the OpenCL and MPI computational time of my code for four different graphics cards (ATI HD7970, Nvidia GTX 580, Nvidia Tesla 2050, and an ATI HD5850) and Mapache. Figure 24 plots the performance in GFLOPS of my OpenCL and MPI code for each of the four graphics cards and Mapache. Except for the ATI HD 7970, the peak performance is reached by 512,000 particles. The new architecture of the 7970 is harder to keep busy and optimal performance is not reached until a very large number of particles (2 million) is used. Figure 25 plots the energy conservation of 512,000 ring particles over a 200 day simulation for different time steps. The energy is well conserved and remains stable. A time step of 1 to 2 sec was used in most of the simulations.

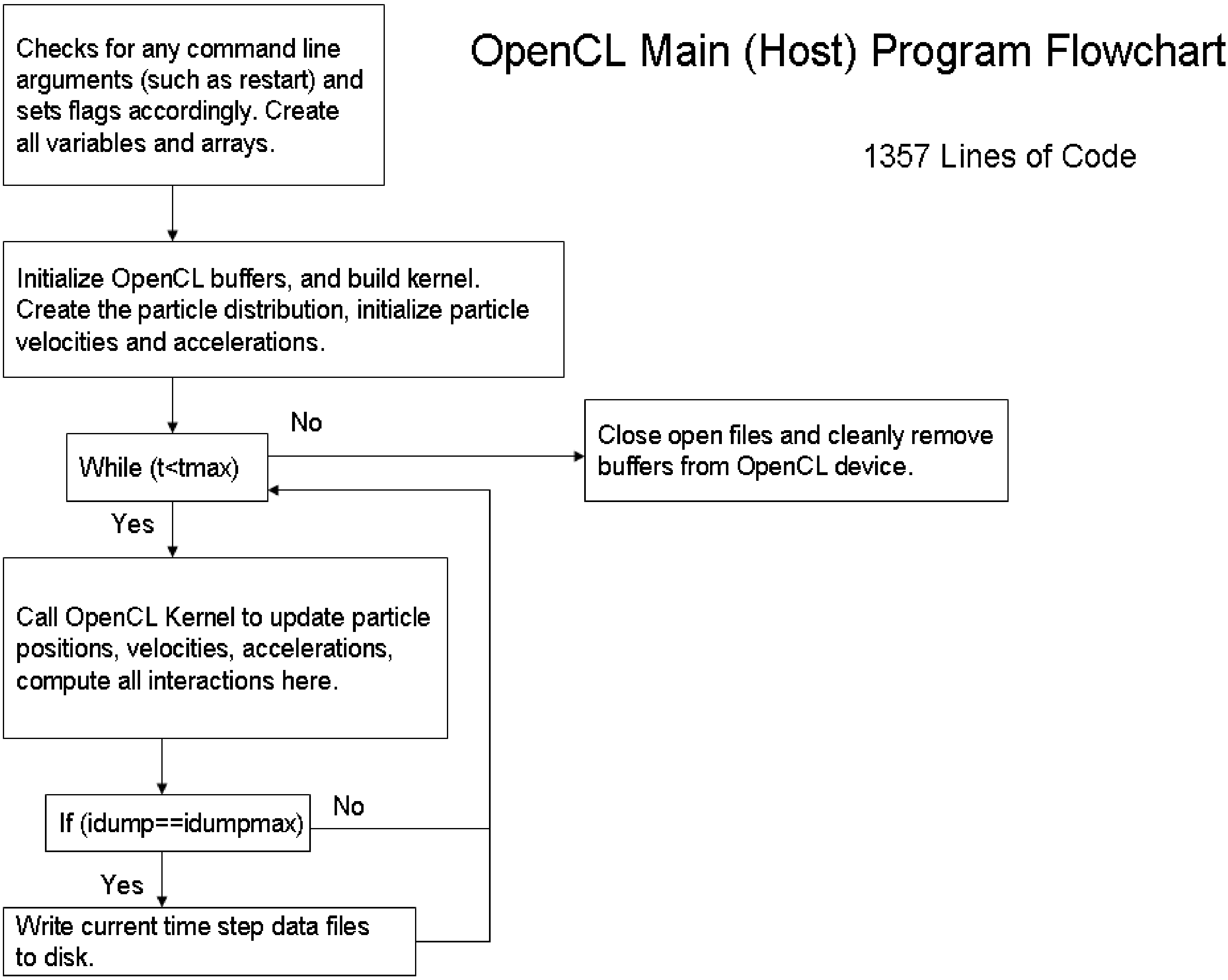

Figure 19: Flowchart of my OpenCL Host program.

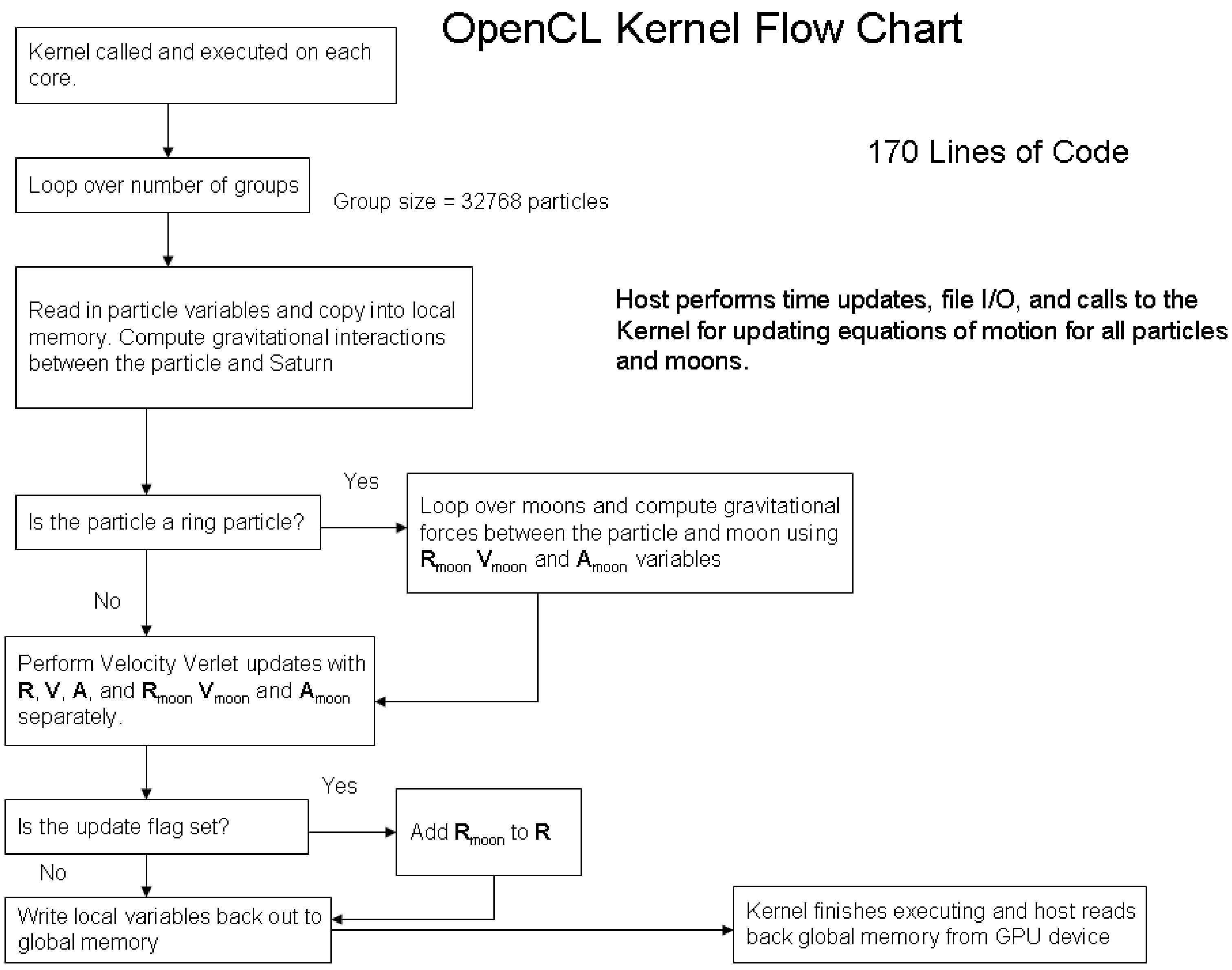

Figure 20: Flowchart of my OpenCL Kernel.

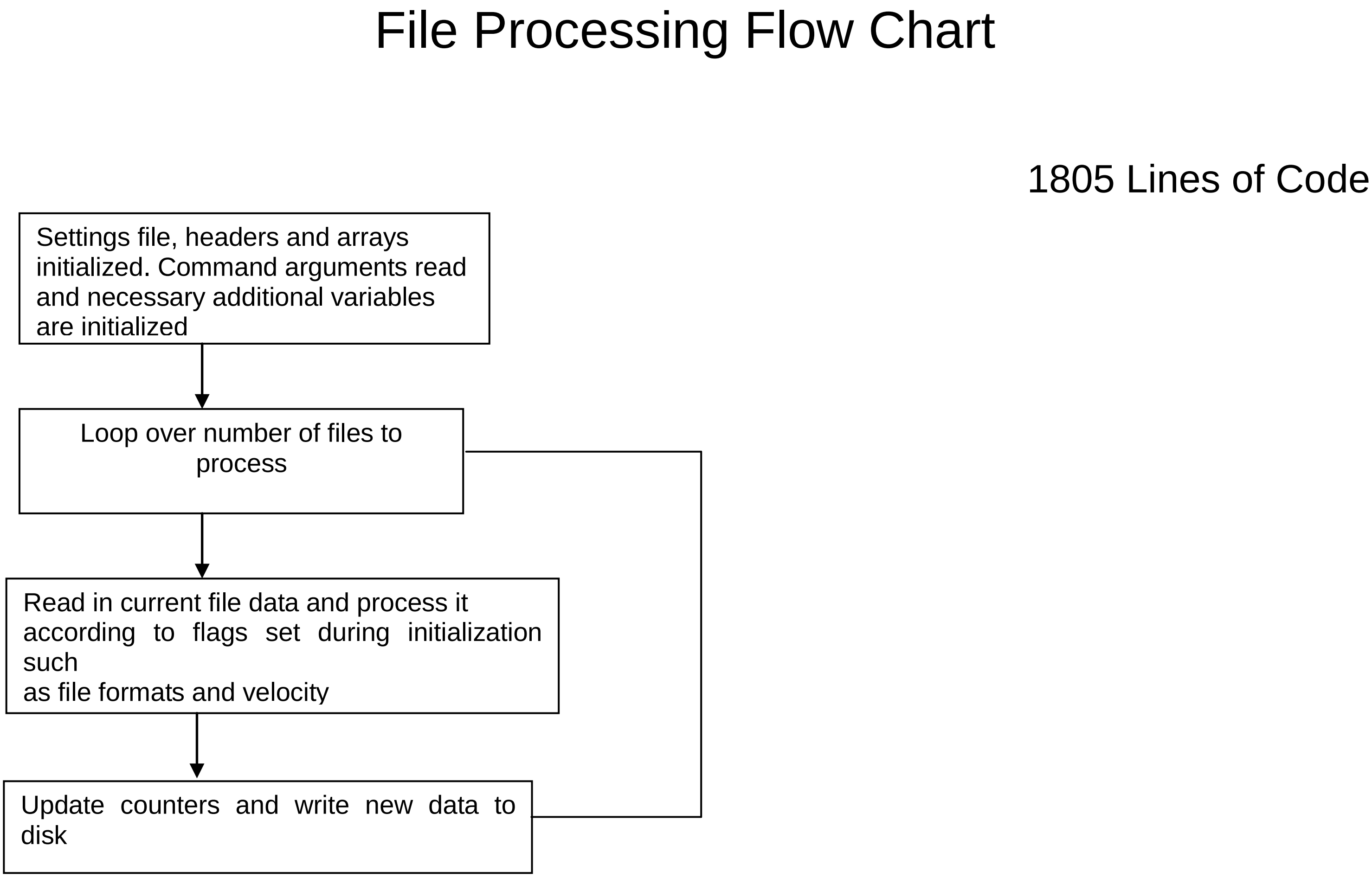

Figure 21: Flow chart of my post-processing code.

Post Processing Steps

- Run process on dataset (Asteroid impact dataset consumed about 850GB) and export .vtk files
- Import .vtk files into ParaView
- Add labels, coloring, and export each frame as a .png file
- Run ffmpeg to stack all .png files into a movie

```
Process Options:
    -v                              Output the radial velocity and color code particles in masC files.
    -v3d                            Output masC files that are written with x, y, vmag.
    -vv                             Output with vector option. Colors are set due to vmag
    -ps [1080p|720p|res XxY]        Export .ps files. Use -ps for standard use or see options below.
                                        1080p - Export .ps with 1980x1020 resolution
                                        720p - Export .ps with 1280x720 resolution
                                         res XxY - Enter a manual resolution in XRESxYRES format Ex. -ps res 420 x 380
    -ps3d [1080p|720p|res]          Export .ps files in 3d format
    -ppm                            Export .ppm files
    -pv [csv|vtk|xyz]               Export for ParaView
                                        csv - Comma Seperated Value
                                        vtk - ParaView Data Format
                                        xyz - X,Y,Z File Format
    -c|--clean [int]                Put all particles at 0,0,0 that exede a certain distance from the center. (Helpful for ParaView plotting)
    -mass                            Export mass files with the loaded header file
    -all                            Export all supported file types including .ps and .ppm files.
    -mf|--moon-force [mode]         Compute individual particles forces on a particular moon and dump results to file
                                        total - Output files with Fmag & r
                                        sep - Output files with Fx, Fy, Fz, & r
                                        vec - Output filews with FxyMag & r
    -neighbors [search] [star] [opts]   Write star & its neighbors inside search radius to file If star=all then do it for n.
                                        overlay - Display neighborhood ontop of ring
                                            track - Track the entered neighborhood throughout the fileset
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Figure 22: Post processing steps and command line options for my processing code.

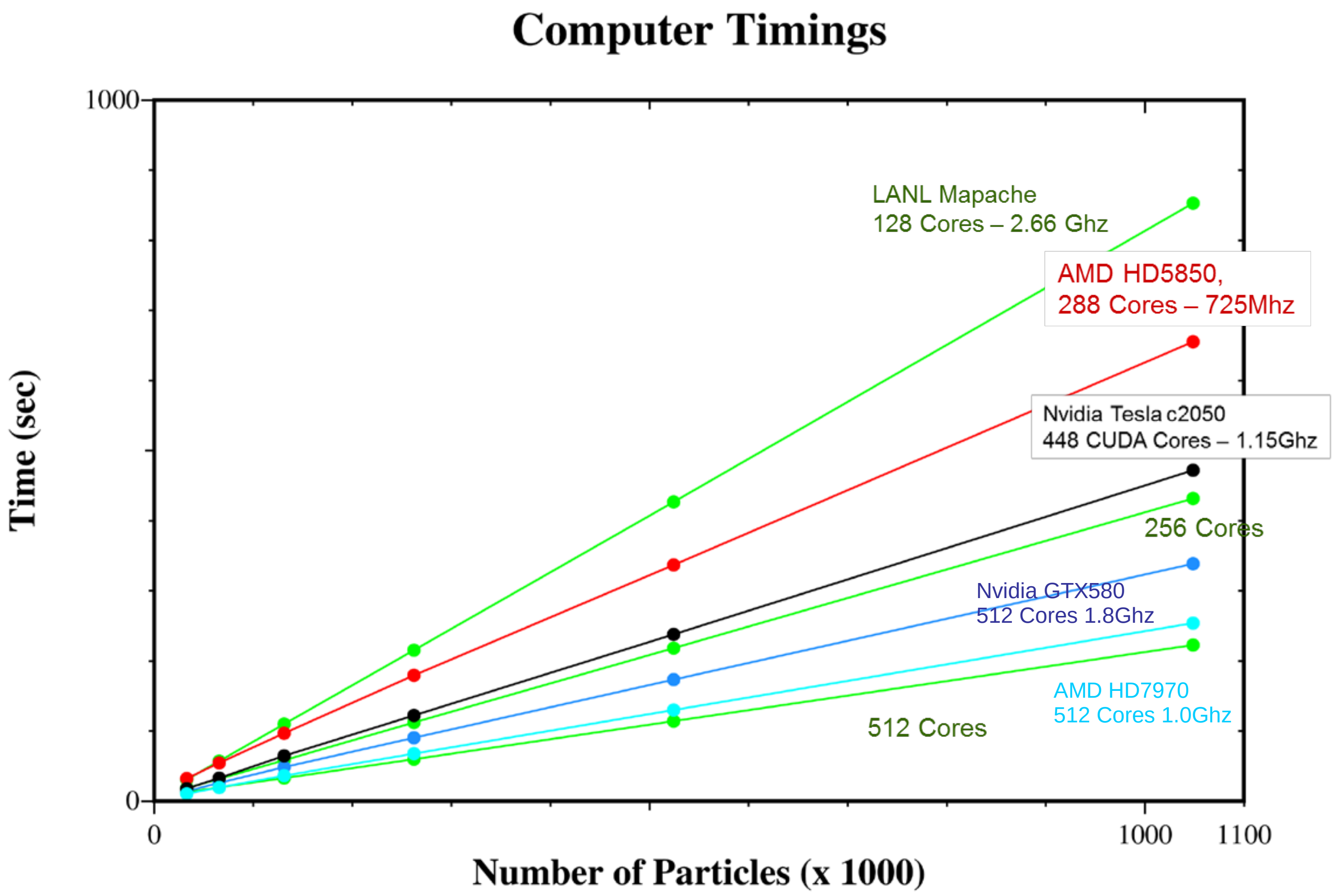

Figure 23: OpenCL and MPI performance of my Saturn ring simulation code on four different graphics cards and Mapache.

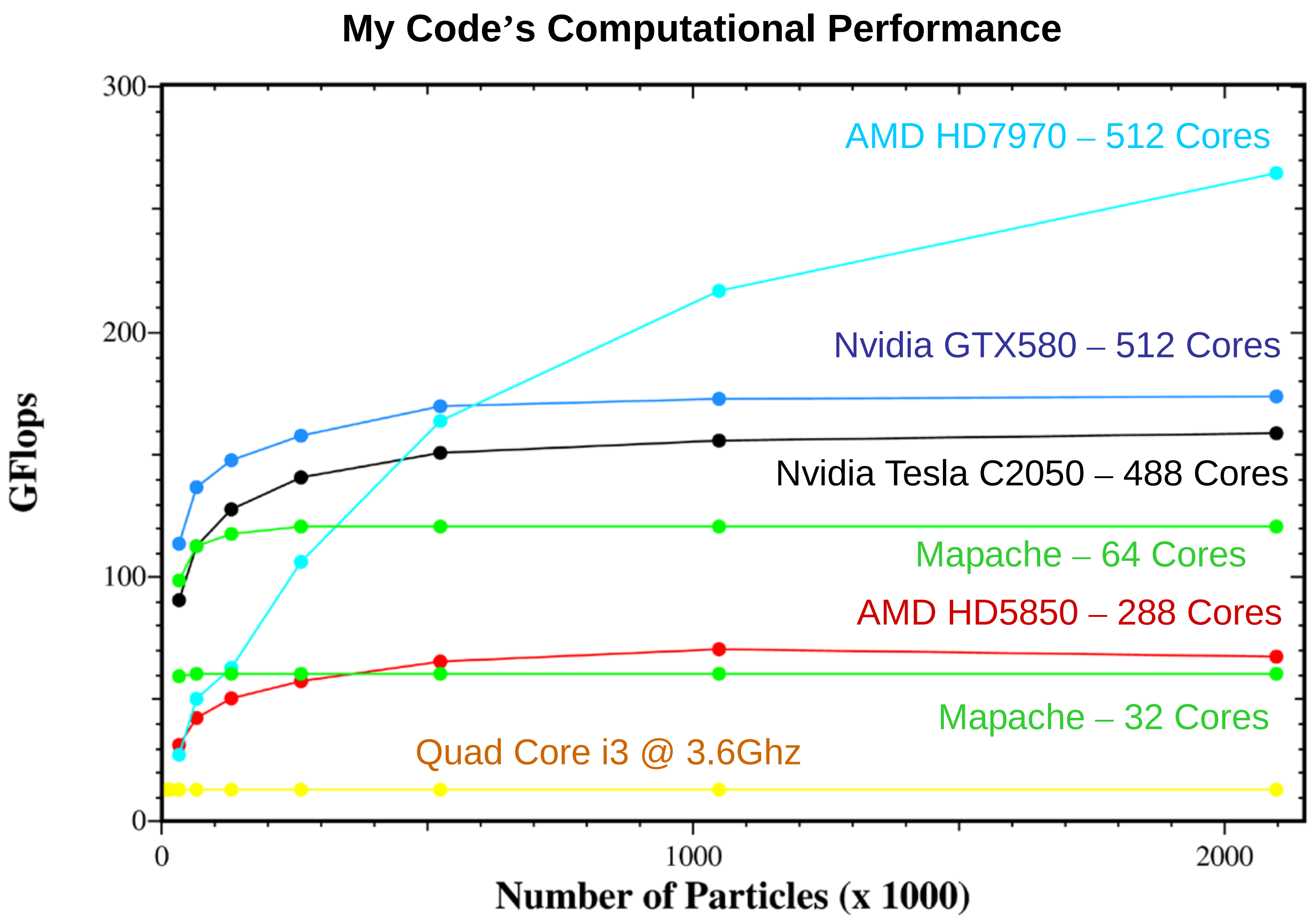

Figure 24: Computational performance in GFLOPS of my OpenCL and MPI Saturn ring simulation code for four different GPUs, Mapache, and a quad core Intel desktop.

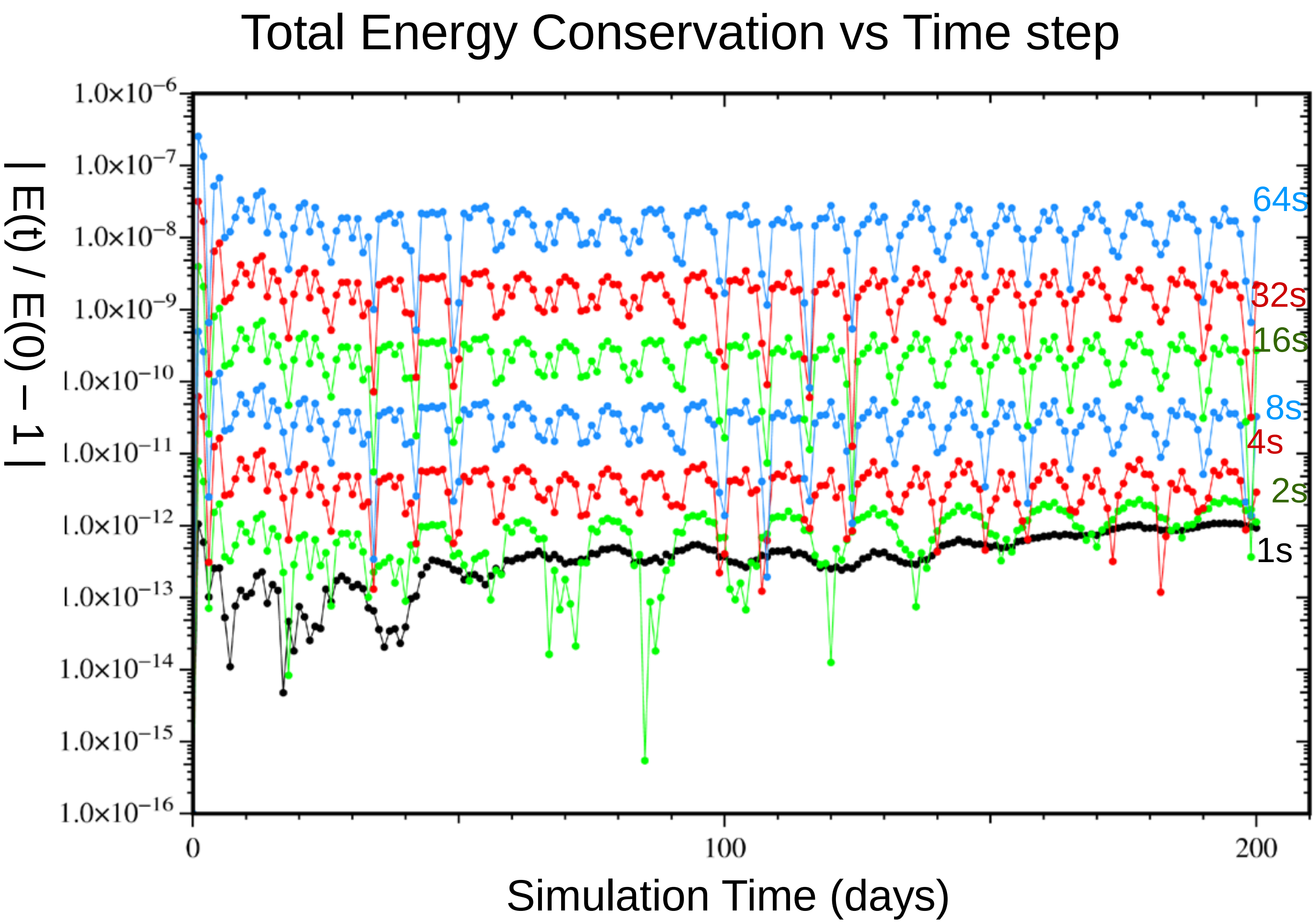

Figure 25: Energy conservation versus time step: 1s, 2s, 4s, 8s, 16s, 32s and 64s.

Conclusions

A 3D N-body based computer simulation of Saturn's rings and moon system was developed for both the entire ring and a small patch of the ring. The patch simulation was needed to increase the ring particle density and increase the ability of the simulation to capture more moon effects on the ring. Twenty-six of Saturn's moons were investigated and fifteen were included in the final simulations. By plotting the ring particles velocity magnitude due to each moon's gravity interaction over time, the tiny gravity resonances on the ring particles could be detected. The location of the resonances due to each moon agreed with the theoretical predictions and aligned well with the experimental data (Cassini images). The 3D comet or asteroid impact simulation showed what would happen if a large comet or asteroid were to hit the rings. An asteroid impact on the ring system disturbs the rings and causes large spiral waves to quickly spread out throughout the rings. A gap might also form over time and the ripples may help explain many of the unexplained features seen in Saturn's rings. This simulation is consistent with the small ripples and spiral wave patterns that professionals have predicted but not yet simulated. Ripple effects have been seen in the observations from the Shoemaker-Levy 9 comet collision with Jupiter's rings. Several different GPUs and LANL's Mapache were used to run the simulations. The computational performance of the different computers was measured and compared. For this kind of calculation the GPU performance is similar to a large supercomputer. In conclusion, this model can be used to help determine which moons form certain gaps, simulate impacts, and provide an accurate model for understanding Saturn's ring structure.

Future Work

Future work includes including particle to particle interactions and collisions within the patch method. Collision would allow for finer scale structure formation which would help cause larger scale gap formation. Particle to particle interactions will be done with a nearest neighbor tree method which only uses particles and their neighbors to interact instead of a particle interacting with everyone else like a full N-Body method.

Acknowledgements

I would like to thank David Kratzer-LANL for helping me get access to Mapache, and also the HPC (High Performance Computing) Advisory Council and Kenneth Lloyd for access to 4 Nvidia Tesla c2050's. I would also like to thank my Dad for purchasing two Nvidia GTX580s.

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www.nvidia.com see developers page for CUDA and OpenCL examples.

www.amd.com see developers page for OpenCL examples/documentation.

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