Problem 1

\[ y(x,t) = A \cos(kx - \omega t) = 0.45 \cos(2.4x - 18t) \]

(a) Wavelength \( \lambda = \frac{2\pi}{k} = \frac{2\pi}{2.4} \) m = 2.62 m = \lambda

(b) Frequency \( \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{18}{2\pi} \) s\(^{-1}\) = 2.86 Hz = f

(c) \( v = \frac{\omega}{k} = \frac{18}{2.4} \) m/s = 7.5 m/s = \( v \)

(d) Speed of particles in the cnd \( \omega \) magnitude of

\[ \dot{y}(x,t) = \omega A \sin(kx - \omega t) \]

Maximum speed: \( U_{\text{max}} = \omega A = 18 \times 0.45 \frac{m}{s} = 8.1 \frac{m}{s} = U_{\text{max}} \)

Minimum speed: \( U_{\text{min}} = 0 \)

(e) Acceleration of particle in the cnd \( \ddot{y} \)

\[ \ddot{y}(x,t) = -\omega^2 A \cos(kx - \omega t) \]

Maximum acceleration: \( a_{\text{max}} = \omega^2 A = 18^2 \times 0.45 \frac{m}{s^2} = 146 \frac{m}{s^2} = a_{\text{max}} \)
Problem 2

\[ \omega = \lambda \phi = \sqrt{\frac{T}{\mu}} \quad ; \quad T = 40 \text{N}, \quad \mu = \frac{100 \text{g}}{1 \text{m}} = 100 \text{g/m} \]

\[ \omega = \sqrt{\frac{40 \text{N.m}}{0.1 \text{kg}}} = 20 \text{ m/s} \]

The lowest frequency \( f \) in the lowest \( \lambda \), largest \( \lambda \) is \( \lambda = 2L \)

\[ \lambda = 2L = 2m \Rightarrow f = \frac{\omega}{\lambda} = \frac{20 \text{ m/s}}{2 \text{ m}} = 10 \text{ HZ} = f_1 \] \( (a) \)

(b) \[ \lambda = \frac{\omega}{f} = \frac{20 \text{ m/s}}{1000 \text{ s}^{-1}} = 2 \text{ cm} = \lambda (f = 1000 \text{ HZ}) \]

(c) 10 masses instead of continuous string:

approximate lowest frequency:

almost same as continuum \( f \approx 10 \text{ HZ} \) \( f_{\text{min}} \)

highest frequency:

so 5 wavelengths in 1 m \( \lambda = 20 \text{ cm} \Rightarrow \)

\[ f \approx \frac{\omega}{\lambda} = \frac{20 \text{ m/s}}{20 \text{ cm}} = 100 \text{ HZ} \approx f_{\text{max}} \]

Exactly:

\[ \omega_n = 2\omega_0 \sin \left( \frac{n\pi}{2(N+1)} \right) = 2\pi f_n \]

\[ \omega_0 = \sqrt{\frac{T}{mL}} \quad ; \quad m = 10 \text{g} = 0.01 \text{kg}, \quad l = 10 \text{cm} = 0.1 \text{m} \Rightarrow \omega_0 = \sqrt{\frac{40}{0.01 \times 0.1}} \text{ rad/s} \Rightarrow \]

\[ \omega_0 = 200 \text{ rad/s} \quad ; \quad N = 10 \Rightarrow f_n = \frac{1}{\pi} \cdot \omega_0 \cdot \sin \left( \frac{n\pi}{2 \cdot 11} \right) \]

lowest frequency:

\[ f_1 = \frac{1}{\pi} \cdot 200 \cdot \sin \left( \frac{\pi}{22} \right) \text{ HZ} = \frac{9.1 \text{ HZ}}{f_1} \] \( f_{\text{min}} \)

highest frequency:

\[ f_{10} = \frac{1}{\pi} \cdot 200 \cdot \sin \left( \frac{10\pi}{22} \right) \text{ HZ} = \frac{6.3 \text{ HZ}}{f_{10}} \] \( f_{\text{max}} \)
Problem 3

\( y(x, t) = A \sin(bx) \cos(wt) \)

\( A = 2 \text{ cm}, \ b = 0.2 \pi \text{ cm}^{-1}, \ w = 80 \pi \text{ s}^{-1}. \ L = 20 \text{ cm}, \ m = 40 \text{ g} \)

(a) The velocity \( v \) of the string at point \( x, t \) is

\[ v(x, t) = -wA \sin(bx) \sin(wt) \]

For a small element of mass, \( dm \), at point \( x \) and time \( t \), kinetic energy is

\[ dK = \frac{1}{2} dm \dot{y}^2 = \frac{1}{2} dm \cdot w^2 A^2 \sin^2(bx) \sin^2(wt) \]

Minimum kinetic energy occurs when \( \sin(wt) = 0 \) then \( K = 0 \)

Minimum kinetic energy occurs when \( \sin^2(wt) = 1 \). Integrating over the string,

\[ K = \frac{1}{2} m w^2. \ A^2 \cdot \frac{1}{2} = \frac{1}{4} m w^2 A^2 \]

\[ = K = \frac{1}{4} \cdot 40 \text{ g} \cdot 80^2 \pi^2 \cdot 2^2 \text{ cm}^2 = 2.5 \times 10^6 \text{ erg} = K_{\max} \]

(b) The wavelength is \( \lambda = \frac{2\pi}{b} = \frac{2\pi}{0.2\pi} \text{ cm} = 10 \text{ cm} = L / 2 \)

The possible wavelengths are \( \lambda_n = 2L/n \Rightarrow n = 4 \)

\[ \lambda_n = n \lambda_1 = 80 \pi \text{ s}^{-1} \Rightarrow \lambda_1 = 20 \pi \text{ s}^{-1} = 2 \pi f \Rightarrow f_1 = 10 \text{ Hz} \]

So the three lowest frequencies are \( 10 \text{ Hz}, 20 \text{ Hz}, 30 \text{ Hz} \)

(c) \[ A \sin(bx) \cos(wt) = \frac{A}{2} \sin(bx - wt) + \frac{A}{2} \sin(bx + wt) \]

Speed of wave: \( v = \lambda_1 / \lambda = 80 / 0.2 \text{ cm/s} = 400 \text{ cm/s} \)

(d) Using the formula:

\[ \bar{p} = 2\pi^2 \mu \nu f^2 A^2 \text{ but } A \text{ should be } \frac{A}{2} \]

\[ \mu = \frac{40 \text{ g}}{20 \text{ cm}}, \ \nu = 40 \text{ Hz}, \ A = 1 \text{ cm} \Rightarrow \]

\[ \bar{p} = 2\pi^2 \times \frac{2g}{\text{cm}} \times 400 \text{ cm} \times 40^2 \cdot 1 \text{ cm}^2 = \frac{2.5 \times 10^7 \text{ ergs}}{5} = \bar{p} \]
Derivation of the formula for power \( P \) in a steady wave:

\[ y = A \sin(\lambda x - \omega t) \]
\[ \dot{y} = A \omega \cos(\lambda x - \omega t) \]

Consider 1 wavelength: kinetic energy is

\[ K = \frac{1}{2} m \dot{y}^2 , \quad m = \mu \lambda = \frac{\lambda}{\text{mass/unit-length}} \]

\[ K = \frac{1}{2} \mu \lambda A^2 \omega^2 \langle \cos(\lambda x - \omega t) \rangle^2 . \quad \text{On average, } \langle \cos^2 \rangle = \frac{1}{2}. \]

There is also potential energy which \( \propto \) magnitude; the average energy in one wavelength is then \( \bar{E} = \frac{1}{2} \mu \lambda A^2 \omega^2 \). If it flows in 1 period \( T = \frac{1}{f} = \frac{\lambda}{\omega} \)

\[ \bar{P} = \frac{\bar{E}}{T} = \frac{1}{2} \mu \lambda A^2 \omega^2 = \boxed{2 \pi^2 \mu \lambda f^2 \frac{\omega^2 A^2}{2}} \]

Using that \( \omega = 2\pi f \) and \( \nu = \lambda f \)

(c) Relation between results in (a) and (d):

In (a), the maximum kinetic energy = average kinetic potential energy.

There are 2 wavelengths in the string, and the strands wave is superposition of 2 steady waves, so energy per wavelength of steady wave is on average \( \frac{K_{\text{max}}}{4} \) in one wavelength. The period is

\[ T = \frac{1}{f_4} , \quad f_4 = f_1 \cdot f_4 = \frac{f}{4} = 40 \text{ Hz} \], so energy per unit time is

\[ \bar{P} = \frac{K_{\text{max}}}{4} \cdot f_4 = K_{\text{max}} \cdot \frac{10}{5} \]

\[ = K_{\text{max}} \cdot \frac{10}{5} \]