Problem 1: \[ C_{\text{H}_2\text{O}} = 1 \text{cal/g}^\circ \text{C}, \quad C_{\text{Ice}} = 0.5 \text{cal/g}^\circ \text{C}, \quad L_f = 79.7 \text{cal/g} \]

Heat released in cooling 10 g of H_2O from 10^\circ \text{C} to 0^\circ \text{C}:
\[ Q_{\text{H}_2\text{O}} = \frac{1 \text{cal}}{g} \times 10 g \times 10^\circ \text{C} = 100 \text{ cal} \]

Heat released in freezing 10 g of H_2O:
\[ Q_{\text{freeze}} = m L_f = 79.7 \times 10 \text{ cal} = 797 \text{ cal} \]

Heat to raise T of ice from -10^\circ \text{C} to 0^\circ \text{C}:
\[ Q_{\text{Ice}} = \text{Mass Ice} \times \Delta T = 100 g \times \frac{0.5 \text{ cal}}{g^\circ \text{C}} \times 10^\circ \text{C} = 500 \text{ cal} \]

Conclusion: In the final state, water and ice will coexist

\[ \Rightarrow T_{\text{final}} = 0^\circ \text{C} \] \hspace{1cm} (a)

(b) x grams of water will freeze, so that

\[ x \times L_f + Q_{\text{H}_2\text{O}} = Q_{\text{Ice}} \Rightarrow x \times 79.7 + 100 = 500 \Rightarrow \]

\[ x = \frac{400}{79.7} g = 5 \text{ g} \]

\( \Rightarrow \) total ice in final state: 105 g
Problem 2

(a) The work is \( W = \int P dV \), \( PV = RT \ln \text{mol} \)

\[
W_{12} = P_1 (V_2 - V_1) = P_1 (3V_1 - V_1) = 2P_1 V_1 = 2RT_1
\]

\( W_{23} = 0 \)

\[
W_{31} = \int \frac{RT_1}{V_3} dV = RT_1 \ln \frac{V_1}{V_3} = -RT_1 \ln 3
\]

Total net work in cycle: \( W = RT_1 (2 - \ln 3) = 0.90RT_1 \)

(b) The system absorbs heat in 12 because it is performing positive work, its temperature is increasing, hence internal energy is increasing, from \( \Delta E_{\text{int}} = Q - W \) we conclude \( Q_{12} \) is positive. The system releases heat in 23 because it is doing no work, its temperature is decreasing, hence \( E_{\text{int}} < 0 \), hence \( Q_{23} < 0 \).

The system releases heat in 31 because its internal energy is not changing since \( T_1 \) is constant, so \( Q_3 = W_{31} \), and \( W_{31} < 0 \) so \( Q_{31} < 0 \).

(c) Heat absorbed in cycle 1: \( Q_{12} \), process at constant pressure

\[
Q_{12} = C_p (T_2 - T_1) \quad ; \quad T_2 = 3T_1 \quad ; \quad C_p = \frac{5}{2} R \text{ for monatomic gases}
\]

\( Q_{12} = 5RT_1 \)

(d) \( e = W/Q_{13} = 0.9/5 = 0.18 \)

(e) Lowest \( T \geq T_1 \), highest \( > 3T_1 \)

\( \theta \text{ cannot} = \left| \frac{T_1}{3T_1} \right| = 0.66 \times e = 0.18 \)

(f) Heat in 23: \( Q_{23} = -C_v (T_3 - T_2) = -2T_1 \quad C_v = -3RT_1 = Q_{23} \)

\( Q_{31} = W_{31} = -RT_1 \ln 3 \)

\( Q_{\text{tot}} = Q_{12} + Q_{23} + Q_{31} = (2 - \ln 3)RT_1 = W \)

Since \( \Delta E_{\text{int}} = 0 \) in cycle
Problem 3

(a) Process 1: Free expansion
T doesn't change. So in gas, \( dQ = dW \),
\[
\Delta S_1 = \int \frac{dQ}{T} = \int \frac{RT}{V} \frac{dV}{V} = R \ln \frac{V_f}{V_i} = \Delta S_1 = R \ln 2
\]
No heat is exchanged with environment:
\[
\Delta S_{env} = 0
\]

(b) Process 2: Adiabatic expansion
\[
\Delta S_2 = 0 \quad \text{in gas, since no heat is exchanged}
\]
\[
\Delta S_{env} = 0 \quad \text{in environment}
\]

(c) At constant volume we have in the gas, \( W = 0 \implies \)
\[
d E_{int} = C_v \, dT = dQ \implies \]
\[
\Delta S_3 = \sum C_v \frac{dT}{T} = C_v \ln \frac{T_f}{T_i}, \quad T_f = T_i. \quad To \ find \ the \ T_i,
\]
Use \( TV^{x-1} = \text{constant} \) adiabatic \( T_i \cdot (2V_1)^{x-1} = T_i V_1^{x-1} = \)
\[
\Rightarrow T_i = 2^{\frac{1}{x-1}} T_1 \Rightarrow \Delta S_3 = C_v \ln 2^{\frac{1}{x-1}}; \quad C_v = \frac{3}{2} R, \ x-1 = \frac{2}{3}
\]
\[
\Rightarrow \Delta S_3 = -\frac{3}{2} R \cdot \frac{2}{3} \ln 2 = -R \ln 2 = -\Delta S_3
\]

Or, we could have found this from \( \Delta S_1 + \Delta S_2 + \Delta S_3 = 0 \) in cycle.
For environment, \( \Delta S_{env} = \frac{Q}{T_1}, \quad Q = C_v (T_i - T_1) = \frac{3}{2} R \left( 2^{\frac{2}{x-1}} - 1 \right) T_1
\]
\[
\Delta S_{env} = \frac{3}{2} \left( 2^{\frac{2}{3}} - 1 \right) R = 0.88 R
\]

(d) \( \Delta S_{env} (1, 2, 3) = \Delta S_3^{env} = 0.88 R \)
\[\text{since } \Delta S_{gas} = 0 \text{ in cycle, whether reversible or not}\]