PHYSICS 220 : GROUP THEORY
PROBLEM SET #1

[1] Show that the following is true for 2-cycles: \((jk)(kl)(jk) = (jl)\).

[2] Show that \(S_n\) is isomorphic to a subgroup of \(A_{n+2}\), and describe how to elicit this isomorphism.

[3] Show that \(A_4\) is not simple.

[4] Find \(\langle D_n, D_n \rangle\).

[5] Find the center of the quaternion group.

[6] Describe the Lie algebras \(e(n)\) and \(p(n, 1)\) for the Euclidean and Poincaré groups.

[7] Show that the set of all \(n \times n\) real upper triangular matrices is a matrix Lie group. Describe how to go about constructing the inverse of any element.

[8] Find a basis for the Lie algebra \(so(3, \mathbb{R})\). Show that you can choose a normalization \(\text{Tr}(X^a X^b) = -2 \delta^{ab}\). Find the structure constants.

[9] Show that for any traceless \(2 \times 2\) matrix \(X\), its exponential is given by

\[
\exp(X) = \cos \sqrt{\Delta} I + \frac{\sin \sqrt{\Delta}}{\sqrt{\Delta}} X,
\]

where \(\Delta = \det X\).