Supplement: Gas Dynamic Shocks
lecture III

(a) Scale

Consider Burgers Eqn.

\[ \frac{\partial}{\partial t} V + V \frac{\partial}{\partial x} V - \gamma \frac{\partial^2}{\partial x^2} V = 0 \]

\[ \bar{P} \]
\[ \int \Delta V \]

For shock width

\[ \Delta V \frac{\Delta V}{W} \sim \gamma \frac{\Delta V}{W^2} \]

\[ W \sim \frac{\gamma}{\Delta V} \]

dissipation sets shock thickness

for \( \gamma \sim \text{lsmp} \epsilon_s \)

\[ \Delta V \sim \epsilon_s \]

\[ W \sim \text{lsmp} \]

characteristic thickness of shock layer.
6. Entropy Production

In gas dynamics
- initially ideal dynamics
- entropy constant

but... pulse steepens and shocks

- sharp gradients produced in shock
  ⇒ couple to diffusive dissipation
  ⇒ drive collisional transport
  ⇒ produce entropy.

N.B.: − Entropy production required in shock
  − sets arrow of time
- to calculate:

\[ \frac{dS}{dt} = \left( 1 - \frac{\Gamma^x}{A^x} \right) \rightarrow \]

\[ \Delta x \rightarrow \text{thermodynamic force} \]

\[ \Gamma^x \rightarrow \text{Flux} \]

\[ \therefore \Gamma^x = -A^x \Delta x \]

\[ \frac{dS}{dt} \sim \left( 1 - \frac{\Gamma^x}{A^x} \right) \Delta x^2 \]

\[ \Delta x^2 \rightarrow \text{entropy production rate density} \]

then, for Burgers shock:

\[ \frac{dS}{dt} \sim \Delta \left( \frac{\Delta v}{V} \right)^2 \]

\[ \sim \frac{1}{W^2} \frac{\Delta v^2}{V} \sim \frac{\Delta v^2}{V} \frac{\Delta v^2}{\Delta v^2} \]

\[ \sim \Delta v^4 / V \]
but total entropy production is integrated over shock thickness

\[ \int dx \frac{dS}{dt} \sim \frac{dS'}{dt} \implies \frac{dS}{dt} \frac{dS'}{dt} \]

\[ \sim R \frac{v^2}{A v^2} (A v)^2 \]

\[ \sim (A v)^3 \]

so total entropy production:

\[ \frac{dS}{dt} \sim (A v)^3 \]

- independent of \( v \)

- entropy / heating produced by collisions but total \( \frac{dS}{dt} \) independent of \( v \).