Diamagnetism of Metals

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1 Biography

Lev Davidovich Landau was born on January 9th, 1908, in Baku, which was part of the Russian Empire at that time [1]. He received his bachelor's degree from Leningrad Polytechnical University (now St. Petersburg) from 1924-1927, only a few years after the Russian Revolution. Due to political turmoil, he did not receive a doctorate until 1934. In 1929, Landau was granted permission to go abroad to Germany, where he met with many prominent physicists of the time, such as Pauli and Peierls [2]. It was during that time, at the age of 22, that he wrote his paper on diamagnetism. His later work focused on developing a theoretical background for phase transitions, superfluidity of liquid helium, and superconductivity. Besides his contribution to theoretical physics, his most notable achievement was his famously difficult textbooks. Together with one of his students, Evegeny M. Lifshitz, he co-authored this series of physics texts, designed for graduate students. His life came to a tragic end after a car crash in 1962; he survived but never recovered completely and died while undergoing surgery in 1968.

2 Theory of magnetism up to 1930

2.1 Magnetism due to orbiting electrons

By 1929, the contributions to the magnetic properties of metals were believed to be due to several effects: the contribution from the core electrons, and the contribution from the conduction electrons of the atoms. In the following sections, the quantum theory will be applied to conduction electrons. By 1929, people had a good understanding of the contribution of the core electrons to the magnetic properties of metals. If an atom (ion) had filled shells, its core electrons exhibited a diamagnetism proportional to the sum of orbital radii. This result was obtained by Langevin in 1905, who treated the electron classically. He considered an electron as a point particle moving in a circular orbit. Electrons moving in a circular orbit can be thought of as a point current; using the definition $m = \oint IdA$ gives a result proportional to electronic charge and radius squared. His result was improved on in 1920 by Pauli [3], who obtained, using

his paper's notation, $\chi_c lassical = -\frac{\eta}{6c} L\Theta^2$. Pauli's values were $\eta = \frac{e}{mc}, L = N_A = 6.022 \times 10^{28}, \Theta = \overline{\sum eR^2}$, Z is the atomic number and R is the distance of electron orbitals from the center of mass. This result is now better known as $\chi = -Z_i N_A \frac{e^2}{6mc^2} \langle r^2 \rangle$, under the name of Langevin or Larmor diamagnetism. For ions with filled shells, such as the halide ions and the noble gases, this result gave fairly accurate predictions. On the other hand, if an atom had only a partially filled shell, then a paramagnetism was observed. A theory explaining this effect was developed in 1928 by van Vleck [4]. Using quantum mechanics and perturbation theory, he predicted a contribution to paramagnetism from the alignment of orbital and spin angular momenta with the external field. This contribution was on the order of $\chi = 10^-3$, meaning atoms with partially filled shells, where one of $\mathbf{L}, \mathbf{S}$ is nonzero. This paramagnetism followed Curie's law by an elementary argument from statistical mechanics. Finally, conduction electrons were believed to have no impact on the magnetic properties of a metal, since, from a classical point of view, the energy of (almost) free conduction electrons in metals depended only on their velocities squared, which meant that $\frac{\partial F}{\partial H} = 0$, and the contribution to magnetism vanished [5].

2.2 Pauli paramagnetism

Pauli's development of paramagnetism in 1927 and Heisenberg's model of ferromagnetism in 1928 were the first successful applications of quantum mechanics to the contribution of conduction electrons to magnetism in metals. Although crude approximations, they laid the basis for improved theories in the future [2]. Pauli found his famous paramagnetism by considering the contribution of spin to the energy of an electron in a magnetic field [6]. The crucial point was the application of Fermi-Dirac statistics to the electron gas; the classical approximation was inaccurate, since the gas was highly degenerate, and only electrons at the Fermi level contributed to the properties of the metal. The usage of FD statistics meant that the treatment of the problem was purely quantum mechanical, and the effect was also purely quantum mechanical. The magnetization of electrons in a magnetic field due to spin is given by equation 35, reproduced below

$$M = N \frac{1}{3} \frac{\mu^2 H}{kT} \frac{-F'(\alpha)}{F(\alpha)} \tag{1}$$

Pauli uses $\mu = \sqrt{3} \frac{e\hbar}{2m_e c}$, (for spin $\frac{1}{2}$) and defines $F'(\alpha) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{x} dx}{e^{x+\alpha}+1}$. Later on in the same paper, it is shown that $\alpha = \frac{\omega}{kT}$, where ω is the chemical potential. The susceptibility is then obtained by $\chi = \frac{\partial M}{\partial H}$. The values are on the order of 10^{-6}. This result, although completely ignoring lattice effects and electron-electron interaction, was nevertheless an important result for identifying a paramagnetic contribution from conduction electrons. As a result of the aforementioned assumptions, Pauli's results held fairly well for some the alkali metals; however, for Rb and Cs, a diamagnetism on the order of the Pauli paramagnetism was not completely ruled out. For Na and K, on the other hand, the measured paramagnetism was greater than what was predicted [2]. At that time, it was not immediately

obvious if this was a result of the oversimplified model or of a fundamentally different phenomenon, but the results were fairly consistent, at least for alkali metals. An interesting feature of this result was the temperature independence of the paramagnetism; Curie's law predicted the dependence of susceptibility on the inverse temperature, which was not observed in metals. However, this expression, although it explicitly depends on T, the ratio of integrals contains the Fermi function, which results in a characteristic temperature T_F, instead of T. Another well-known form for Pauli paramagnetism is $\chi = \mu_B^2 g(E_f)$, where this relationship is more obvious. The equivalence of the two will be shown to be equivalent later in this paper.

2.3 Open questions at that time

After the work of Pauli and Heisenberg, there were relatively few unanswered questions, and there was no published work on fluctuations of electric and magnetic susceptibility yet. However, one of them was the large observed diamagnetism of certain elements. Bismuth, for instance, was observed to have a diamagnetism on the order of 10^{-4}, and there was no good explanation as to why that might be. Experimentalists were not certain where the diamagnetism came from, because it could be coming from either the free or from the bound electrons (that is, the core), in the case of metals [7]. However, as mentioned previously, people believed that the contribution of conduction electrons (considered as a free electron gas) to the diamagnetism must vanish, and the core contribution did not completely explain the strong diamagnetism of bismuth, for instance. There were arguments trying to explain this anomaly using either the Langevin diamagnetism or effects of the crystal lattice, but there was no treatment of it using the formalism of quantum mechanics [8]. In summary, by 1929, the magnetic properties of metals were still poorly understood in the context of quantum mechanics, and experiments were only just beginning to reveal more about the electrical and magnetic properties of metals.

3 Landau's paper

Landau had arrived in Germany in 1929, at age 21, where Paul Ehrenfest suggested the problem of diamagnetism to him. There was almost nothing suggesting there was a diamagnetism missing besides the case of bismuth or antimony. However, the understanding of magnetism was, except in the case of Pauli and Heisenberg, was based on a classical treatment of the problem. As mentioned previously, the assumption that there was no contribution to susceptibility from conduction electrons was not even semi-classical, which led Ehrenfest, among others, to continue working on diamagnetism [8]. Landau published his paper before de Haas, Shubnikov, and van Alphen published the results of their experiments [2], which will be discussed after the presentation of Landau's paper. This means that, curiously enough, oscillations in susceptibility were predicted before they were observed, which is certainly uncommon. The

paper was eventually published in 1930, although Peierls later recalled that Landau had solved the problem nearly a year earlier. However, he first wanted to discuss his result with Petr Kapitsa, who was much more familiar with experiments and the limitations on, for instance, the magnetic fields realizable in the labs of the time [9].

3.1 Short summary of the paper

The paper begins with a remark on the assumption of zero contribution from conduction electrons; the author notes that the classical argument neglects the quantum aspect of the problem, since there will be some discreteness in the eigenvalues of an electron constrained to move in a plane [10]. He then defines P, Q from the velocities of the initial Hamiltonian for an electron in a magnetic field along the z axis.

$$v_1 = \frac{P}{\sqrt{m}}$$
$$v_2 = \frac{eH}{cm\sqrt{m}}Q$$

It can be shown that $[P, Q] = \frac{\hbar}{i}$, which means that $\frac{eH}{mc}$ can be identified with the frequency of a harmonic oscillator ω, meaning the problem reduces to that of a harmonic oscillator in the xy plane. Note that due to symmetry, the wave function separates into the xy component, which was shown to be mathematically identical to the oscillator, and into the z component, of which the last is identical to the wave function of a free particle moving in the z direction. Making substitution in the wave function, $\psi = e^{\frac{-ieHxy}{2\hbar mc}}\chi$, it can be verified that the Schrödinger equation will be independent of x as well, giving an equation for χ as the product of $e^{\frac{i\sigma}{\hbar}}$, and another function $\phi(y)$, which is only dependent on y. This last satisfies equation 13 in the text, which is given below along with the full wave function ψ for clarity.

$$\frac{d^2\phi}{dy^2} + \frac{2m}{\hbar^2}[E - \frac{m}{2}\left(\frac{eH}{mc}\right)^2 (y - \frac{c}{eH}\sigma)^2]\phi = 0$$
$$\psi = e^{\frac{i}{\hbar}(p_z z + \sigma x - \frac{eH}{2c}xy)}\phi_n\left[\sqrt{\frac{eH}{mc}}\left(y - \frac{c}{eH}\sigma\right)\right]$$

The first equation above makes it clear that ϕ_n is the nth Hermite polynomial, satisfying the equation for an oscillator with $\omega = \frac{eH}{mc}$, with the origin shifted by $\frac{c}{eH}\sigma$. The next step is to find the density of states; this is trivial in the x and z directions. Then, the crucial step is noting that the density of states in x is proportional to $d\sigma$. Next, the author observes that since ψ is rapidly damped in the y direction, we must have the wave function decay to 0 outside once we are a distance $\frac{c}{eH}\sigma$ away from the wall in the y direction, provided that we are dealing with a very large container of volume V, as is the case with density of states calculations. In this case, "far"means the argument of ϕ_n is far from

0. This adds a constraint relating σ and the volume, and gives the number of states per level n between $p, p+dp$:

$$R_{dp,n} = \frac{eH}{4\pi^2\hbar^2 c} V dp$$

Once the density of state is determined, Landau proceeds to use the machinery of statistical mechanics to obtain an expression for the free energy. As with Pauli's treatment, the usage of Fermi-Dirac statistics accounts for the degeneracy of the electron gas. This results in an integral over the continuous p_z variable and a sum over the discrete n, resulting from the energies of the "oscillator". To make progress, the summation is rid of using the Euler-Maclaurin formula for slowly converging sums, which imposes the condition $\mu H \ll kT$, introducing $\mu = 2\mu_B = \frac{e\hbar}{mc}$. The author also notes that even in high fields and low temperatures, it is still possible for this approximation to be valid. It would require an inhomogeneous field that varies more than the change between successive arguments near the point where $\omega = n\mu H$, since this is an argument of a decaying exponential in the integral defining the free energy. In that case, even though the field is large, the variation in field is larger and successive terms remain small. If these conditions are not fulfilled, however, the problem would not be reducible to the case of a linear relation between M and H, meaning that the magnetic moment would be some complicated function of the field. Specifically, if $\mu H \gg kT$, to find the susceptibility, it would be necessary to take two derivatives of the free energy with respect to the magnetic field; without the approximation, the relation between the field and the magnetization would not in general be linear in the field. To find the susceptibility in the strong field limit, one goes back to equation 27 in Landau's paper, which contains the full expression for the free energy:

$$\Omega = -kT \sum_{n=0}^{\infty} \int \ln \left[1 + e^{\frac{\omega - (n+\frac{1}{2})\mu H}{kT} - \frac{p^2}{2mkT}} \right] \frac{eH}{2\pi^2\hbar^2 c} V dp \tag{2}$$

The full mathematical analysis of this expression is rather involved; in the end, it reveals [11, pp. 207-210] that the free energy is periodic in H for strong fields. However, in the weak field limit, the free energy becomes:

$$\Omega = \Omega_0 - \frac{\mu^2 H^2}{24} \frac{\partial^2 \Omega_0}{\partial \omega^2}$$

From this equation, recalling that $M = -\frac{\partial \Omega}{\partial H}$, and also that $N = -\frac{\partial \Omega}{\partial \omega}$, we finally have the result for the diamagnetism of conduction electrons, given below. It must be noted that Landau uses $\mu = 2\mu_B$.

$$M_{Landau} = -\frac{\mu^2 H}{12 \frac{\partial \omega}{\partial N}} = -\frac{\mu_B^2 H}{3 \frac{\partial \omega}{\partial N}} \tag{3}$$

Pauli, in his paper, defines $\alpha = \frac{\omega}{kT}$[6], and equation (1) in section 2.2 contains the term $\frac{-NF'(\alpha)}{kTF(\alpha)}$. Noting that $F(\alpha) \propto N$ and adding in a factor of kT from

the derivative with respect to α gives

$$M_{Pauli} = \frac{\mu_B^2 H}{\frac{\partial \omega}{\partial N}} \tag{4}$$

To recast this result in the familiar form $\chi_{Pauli} = \mu_B^2 g(E_F)$, it is only necessary to recall the definition of the Fermi energy as the chemical potential at low temperature. Then, it follows immediately that $\frac{dN(E)}{dE}|_{E=E_F} = g(E_F)$, which is the desired result. So it is seen that the contribution to the susceptibility of electrons moving in a plane perpendicular to an applied magnetic field is $-\frac{1}{3}$ of the result that comes from the contribution of spins. This value is, in general, different from the zero that a classical treatment of the problem gives. Landau then goes on to mention that his analysis applies to free electrons and this same analysis will change when considering electrons in a periodic potential, which may change the ratio of diamagnetism and paramagnetism in some materials, following Ehrenfest [8]. The paper concludes with a remark that these results imply that, for strong fields, there should be periodic oscillations in the magnetic moment as a function of the field, as noted above. The ratio of diamagnetism to paramagnetism was also predicted to go as $\left(\frac{m}{m^*}\right)^2$, which turned out to be the case for Bi, as shown in a later paper by Jones [2]. He also attempts to relate his results to Kapitsa's experiment as a final remark.

3.2 Follow-up experiments

It must first be noted that Landau's result was much more important for a theoretical understanding of quantum mechanics, rather than for experimental physics. Overall, conduction electrons were still paramagnetic, and experiments at that time did not separate the contributions from the core and conduction electrons. Landau's work also dealt with a free electron gas, whereas lattice effects were known to have significant impact on the properties of materials. Landau's predictions were therefore considered to be imperfect, since it was unknown what effect the lattice structure would have on his model. It was clear, however, that lattice effects implied that conduction electrons were involved in the diamagnetism. The predicted periodicity of the susceptibility in a strong field, was supposed to be unobservable. The best known experiment that showed precisely that periodicity was one by de Haas and van Alphen in 1930. They were conducting a series of experiments that showed that the susceptibility of bismuth was in fact a periodic function of the field, which is exactly what Landau's paper predicted. An inquiry into the variation of susceptibility with the field was a consequence of earlier work done by Shubnikov and de Haas, which showed that there were oscillations in the resistance of bismuth due to an applied magnetic field. De Haas then investigated the susceptibility of bismuth under a strong field, and obtained the result mentioned above. Their results were published only a few months later [7]. It is remarkable that even though they acknowledged and cited Landau's work, they did not apply his theory to their result, since their field rather inhomogeneous. Landau claimed that to observe

oscillations in magnetic moment, the field must have been homogeneous to a degree not achieved in their experiment; if that were not satisfied, then the oscillations should average out. In any case, this effect, along with the similar Shubnikov-de Haas effect, did not fit into the classical model of magnetism, since the periodicity in the field could not be explained using the current model at all. The unusually high diamagnetism also did not fit into the framework of classical theory. The authors themselves suggest the possibility of the contribution of valence electrons in their paper. A more complete explanation of this effect was given in 1933 by Rudolf Peierls.

3.3 Follow-up theory

The model of the free electron gas, treated quantum mechanically gave a new result not only accounted for quantum effects, but also gave a result on the order of Pauli's, although originating from a completely different physical mechanism. Whereas Pauli's result arose from considering electron spins in a magnetic field, Landau's result came about from the motion of electrons in a plane, only considering spin as adding a factor of two to the degeneracy. The quantization of cyclotron orbits turned out to have very important consequences later on, but, for the moment, the result was interesting as revealing more about the properties of metals as a result of the fully quantum mechanical treatment. The discovery that movement of electrons in a plane contributed to diamagnetic response also provided a way for crystal effects to affect the magnetic properties of a metal, since lattice effects would change the motion of electrons and therefore its magnetic properties. If the diamagnetism only came from the core electrons, crystal effects would not be important in determining magnetic response, since they affected the conduction electrons. However, the treatment of the problem was not satisfactory in several aspects; first of all, Landau's remark that the oscillatory effect would not be observable was discouraging, and it offered no insight into why the dHvA or SdH effects were still observed. His prediction of a weak diamagnetism was also hard to measure at the time, since it would have to be somehow separated from the van Vleck and Langevin contribution. The effects of the lattice were also not included, and only a qualitative analysis of the oscillations and the lattice effects were included [10]. Perhaps, due to these issues, de Haas and van Alphen did not use Landau's theory to explain their results. Many theorists of the time, notably Peierls, initially glossed over the effect as impossible to realize in the laboratory and promptly forgot about the predictions, but that did not prevent them from recognizing the superiority of the argument compared to the classical result. Some, such as Pauli, tried to extend Landau's argument to strong fields and classical gases [2]. The next important step in this question of the diamagnetism of bismuth came in 1933, when Peierls published his papers on the diamagnetism of conduction electrons. He published three papers, each of which dealt with diamagnetism of free electrons. The second paper dealt with the oscillations in susceptibility, and it basically repeated Landau's argument to derive the density free energy, with more focus on the strong-field regime and oscillations. Its argument revolved

around the filling of bands at different field strengths. For instance, in two dimensions, the jumps would happen at $H = \frac{const}{m}$, where m is an integer and the constant depends on the material [12]. This can be seen from the following argument: suppose H is so strong that all electrons are in the n=0 level. Then, if H is decreased, some electrons move into the n=1 level, which increases the energy. Decreasing H decreases the energy, until n=1 is full, etc. Clearly, that would mean jumps happen at values of H where bands were full. This argument is more complicated in 3D, where the energy depends not only on n, but also on p_z, but the results are similar. The important conclusion was that the period of susceptibility oscillations were proportional to the inverse of the field strength. Peierls also argued that the lattice effects would prevent the averaging, which Landau feared would wash out the oscillations. These results were consistent with the observations of de Haas and van Alphen, and also included a description of diamagnetism from conduction electrons without ignoring the band structure. In the end, pieces of Landau's model, such as the expression for the free energy, remained correct, as noted by Peierls in his paper. The periodicity also remained. However, the lattice effects made the calculation of the exact eigenvalues and eigenstates impractical, but added more interesting physics to the problem and helped explain many anomalies that could not be explained before. The remaining step was to identify the Fermi surface as a physically relevant concept, which was done in the 1940's.

3.4 Further development in the field

Before making a jump of several decades, it is important to understand why that jump exists in the first place. In the late 1920s and the early 1930s, when quantum mechanics was just being developed and applied to solve problems having to do with properties of materials, solid state physics was flourishing. However, by 1935, there was nothing "exciting" left to solve in the new theory, at least according to Peierls and Pauli [9]. Many moved on to other fields, most notably Heisenberg and Pauli. It was also becoming clear that Europe was very politically unstable at the time, given the situations in both the USSR and Germany, causing many notable physicists to emigrate to the USA and to leave solid state physics for, say, atomic physics. That being said, it is not at all the case that work on solid state physics came to a halt. One notable achievement was the discovery of the dHvA and the SdH effect in many metals other than Bi. Another achievement that is relevant to the 1930 paper was Landau's calculation, in 1939, of the magnetization of the free electron gas in the high field limit [11]. By 1950, the concept and the importance of the Fermi surface were generally understood.

4 Relevance up to today

Landau's discovery of the quantization of cyclotron orbitals in an external magnetic field in his paper led to the nomenclature "Landau levels"and "Landau

quantization". The consequences of this quantization of orbitals are still an active field of research; the two papers mentioned below are both investigating unusual properties of various materials in magnetic fields.

4.1 Quantum Hall Effect

When Landau levels are brought up, many people think of the Quantum Hall effect and the quantization of resistance in an magnetic field. This phenomenon occurs in two-dimensional materials in a strong magnetic field and the resistance takes values of $R_K = \frac{h}{\nu e^2}$, the von Klitzing constant divided by a dimensionless filling factorν. On a basic level, this result can be understood by thinking about the consequences of the quantization of kinetic energy on the conductivity of a material. This quantization only allows certain values of current, leading to certain allowed resistance values, as written above. Returning to the equation, the filling factor is given as a function of the field by $\nu = \frac{hc\rho}{eH}$, and it is found by repeating Landau's analysis to find the density of states, but this time in two dimensions. The resulting expression involves area instead of volume, which in turn gives the result above by multiplying this ratio of degeneracy per area by the density of electrons in two dimensions, giving a number of electrons per state or filled Landau levels, hence the name "filling factor"[13]. The filling factor, at weaker fields, is restricted to integers (IQHE); however, as the field is increased, electron-electron interactions become significant, and certain fractional filling factors also become possible, giving rise to the fractional quantum Hall effect (FQHE). People are still carrying out further research into these and other related effects, usually with some adjective followed by "Hall effect". They all deal with a phenomenon which, at its core, deals with the quantization of electron orbitals in a plane due to a transverse magnetic field. One somewhat recent paper on the quantum Hall effect exploits the exact quantization of the kinetic energy of electrons to define a standard for resistance. A graphene sample was used over other materials due to the large spacing between Landau levels, which allowed for higher precision. The use of graphene confirmed the quantization of resistance to three parts per billion[14], suggesting a standard for resistance in terms of fundamental constants, given by $R_K = \frac{h}{e^2}$.

4.2 Oscillations in a strong field

After a very brief introduction to the quantum Hall effect and Landau levels, one can also continue an inquiry into the effect of magnetic properties of materials and, specifically, their behavior in strong fields. Although interest in the de Haas-van Alphen effect, and, more generally, magnetic oscillations, stemmed from an inquiry into unusual properties of bismuth, it turned out to have a very useful application for collecting information about the Fermi surface of a sample. Both Peierls and Landau looked to connect this effect to the Fermi surface of the metal[9], but it was not until 1952 that Lars Onsager arrived at the well-known

result: [2]:

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A_c}$$

A_c is the cross sectional area of the Fermi surface normal to the applied field and $\Delta\left(\frac{1}{H}\right)$ is the change in $\frac{1}{H}$ over one period of the oscillation of susceptibility. This result was also obtained by Ilya M. Lifshitz (brother of Evgeny M. Lifshitz) and Arnold M. Kosevich in 1955 [15], who pick up Landau's expression for the free energy given by equation (2), and, adding in spin, obtain an expression for the magnetization. The crucial difference from Landau's work is the incorporation of a more general dispersion relation as opposed to the parabolic relation assumed for free electrons; however, there is an oscillatory term regardless in the high-field limit. The authors obtain the same result as Onsager in their equation (3.11) and also proceed to show that one can obtain the shape of the Fermi surface by using the de Haas-van Alphen effect. They derive the free energy in terms of the maximal and minimal values of the Fermi surface and its derivatives, and then proceed to show that the largest non-negligible term is a periodic function in $\frac{1}{H}$ with period $\frac{2\pi e}{\hbar A_c}$, with A_c the area of the Fermi surface perpendicular to the applied magnetic field. The complete argument is given in their paper. Their result is only one application of quantum oscillations; the study of quantum oscillations is far from over. In 1930, Landau's paper focused on a non-interacting gas of free electrons in three dimensions; in 1933, Peierls considered electrons in a lattice, and in 1955, Lifshitz extended the analysis to deduce the magnetization of electrons with a more general dispersion relation $\mathscr{E} = \mathscr{E}(p_x, p_y, p_z)$. The study of quantum oscillations in two-dimensional electron gases is much more recent and contemporary with studies of the quantum Hall effect; even more recent is the treatment of magnetic oscillations in topological insulators [16]. Topological insulators require a somewhat different treatment since, by definition, they rely heavily on spin orbit coupling, an effect ignored in previous models, the model must be further modified. The results for the surface states of the TI Bi_2Se_3 are very different from the usual model of the two-dimensional electron gas (2DEG). In general, the Hamiltonian can be written as follows:

$$H(\mathbf{k}) = \frac{\hbar^2}{2m^*}k^2 + \hbar v_F(k_x\sigma_y - k_y\sigma x)$$

Here, the first term is the standard parabolic dispersion relation, and the second term, containing the Fermi velocity and the Pauli spin matrices, contains the spin-orbit coupling terms. Considering this Hamiltonian in the presence of a magnetic field in the z-direction, after some algebra, shows that the second term dominates. The energy of the nth state(recall that the energies are still indexed by the quantum number n from the oscillator) is also found, and from it, the free energy and the magnetization follow. A remarkable feature of this energy is that instead of going as $(n + \frac{1}{2})B$, as for the 2DEG, it goes as $\sqrt{nB}$, since the spin-orbit coupling term is now dominant, as opposed to the parabolic term. Further calculations reveal several more differences between the two cases. The chemical potential of the electrons in the TI exhibits the dHvA oscillations, but

if the zeroth level is filled, the mean value decays with applied field; if it is empty, it grows with applied field, and if half-filled, it stays constant. In the case of the 2DEG, the chemical potential stays constant. In the case of magnetization, the difference is even more pronounced; the magnetization is positive and constant if the 0th level is not empty, and it is the same constant, but negative, if the level is empty. In the 2DEG case, the magnetization oscillates about zero in all three cases. It is also shown that the Hall conductance and the susceptibility also split into three cases based on the filling of the 0th Landau level. In summary, the filling of the 0th level determines some of the electrical and magnetic properties of the material. The paper concludes with a remark that there may be significant contribution from the mixing of bulk bands and surface bands which has not been accounted for.

4.3 Conclusion

The study of magnetic properties of materials is therefore seen to be far from over; although the mystery of paramagnetism and diamagnetism has been more or less solved by Pauli, Landau and Peierls, the study of magnetic properties of materials is not over. At the outset of quantum mechanics, people were trying to understand the properties of simpler materials such as metals and insulators from a quantum-mechanical standpoint. Now, people have moved onto more exotic materials, such as two-dimensional, one-dimensional, or even zero-dimensional materials. Although the quantization of kinetic energy still takes place, there are now more effects that must be accounted for. Landau's model must now incorporate the periodic crystal lattice, a more or less arbitrary dispersion, electron-electron interaction, and spin-orbit coupling, to name a few. However, the application of quantum theory to properties of materials is still at the heart of these inquiries.

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