FIELD ELECTRON EMISSION

Electron emission in intense electric fields (1928), R.H. Fowler and L. Nordheim

ABSTRACT

Electron emission from metal surfaces under high electric fields at low temperatures intrigued the researchers of early twentieth century. This article aims at understanding the historical development of this topic by targeting the most significant work in the area, viz. the 1928 paper of Fowler and Nordheim[1]. The treatment prior to Fowler-Nordheim (FN) has been examined in brief and recent developments have also been included. The paper seeks to emphasize the ubiquity of FN theory beyond emission from metals into vacuum, which accounts for its significance and present day relevance.

INTRODUCTION

Electron emission takes place when emissive materials are subjected to high temperatures (thermionic emission), applied fields (field emission), or light (photoemission). For thermionic and photoemission, the field is smaller than field-emission, and a usual approximation is that all energy more than the barrier are emitted. For thermionic emission, the energy of the transmitted electrons is assumed to be Maxwell-Boltzmann distributed leading to the Richardson-Laue-Dushman equation. However, the underlying physical phenomena for field-emission was not understood until the work of Fowler-Nordheim.

The experiments of Earhart[2], in 1901, indicated for the first time that electrons may be pulled out of metals by high electric fields. Thereafter, many successive experiments by Kinsley[3], Hobbs[4], Hoffman[5], Lilienfeld[6] and, Millikan and Shackelford[7] demonstrated the same without a theoretical discussion. In 1923, Schottky[8] developed an approximate theory in which he assumed that the electrons constituting the field-currents are identical with thermions. This was later refuted by much improved experimental data by Millikan-Eyring[9], and Millikan-Lauritsen[10].

The development of quantum mechanics around that time played a vital role in the understanding of this phenomenon. The theory was considered anew by Richardson[11] and Houston[12], borrowing from the concepts developed by Sommerfeld. But the breakthrough came with the seminal paper of Fowler and Nordheim, in 1928, built on the works of Nordheim[13] and Oppenheimer[14]. Their paper has shaped much of the subsequent work. The name 'Fowler-Nordheim tunneling' is now used for any field-induced electron tunneling through a roughly triangular barrier. The main modern contexts are: (i) vacuum breakdown in high-voltage apparatus, (ii) cold-cathode electron sources-their many applications include bright point sources, X-ray generators, electronic displays and space vehicle neutralizers, and (iii) internal electron transfer in some electronic devices.

The original 1928 equation used an unrealistic barrier model which seriously underpredicts the current densities. Various modified equations have been introduced known as 'FN-type equations'. Most frequently used is the 'standard FN-type equation' derived from Murphy and Good's[15] 1956 work. In recent years, several developments have been made in this direction by Forbes[16], Jensen[17] and, Forbes and Deane[18]. Forbes and Deane[19] also claim to have obtained an exact analytical theory for the exact triangular barrier problem which can replace Fowler-Nordheim's 1928 theory.

PRIOR TO FOWLER-NORDHEIM (1928)

In 1926, Millikan and Eyring first developed experimentally the laws governing the emission of electrons from metals by fields alone. Their experimental setup is shown in Fig. 1. They first proved that the electrons constituting the field-currents are not identical with thermions as had theretofore been assumed. The key results obtained by them were: (i) the field-currents in general have their origin in a few minute surface spots, and (ii) the critical gradients and the field currents are completely independent of temperature between 300 K and 1000 K.

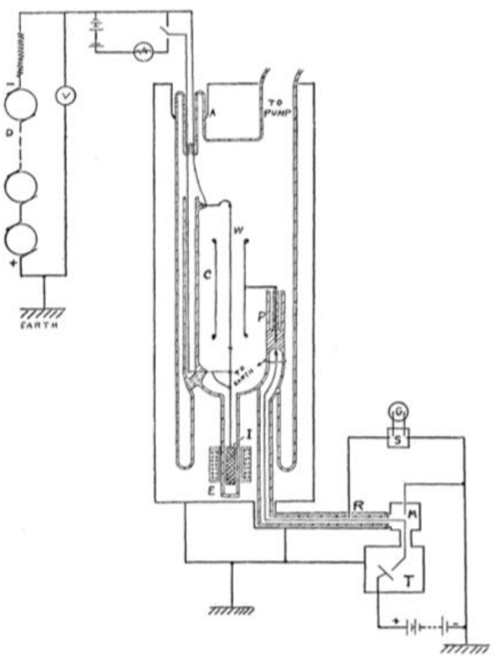

Fig. 1. Diagram of apparatus and electrical connections.

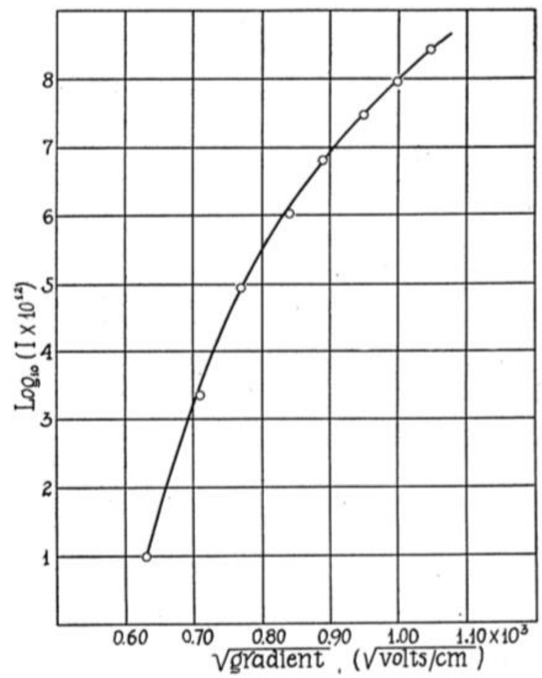

Fig. 2. Logarithm of field current vs square root of field

The conclusions that were made:

(i) Field currents are due to conduction electrons.

(ii) The field currents consist of electrons which escape only from isolated points on the surface where the work function b has been enormously reduced by microscopic geometrical roughness, or chemical impurities, or both.

(iii) Energy of conduction electrons is independent of temperature. Independence upon temperature of the field-currents over a range of 700 ˚C constitutes evidence that equipartition does not hold for the bulk of conduction electrons in tungsten at ordinary temperatures. For when these conduction electrons escape as thermions, the law governing their escape is

$$i = AT^n e^{-b/T} \qquad (1)$$

in which b is the work function of Richardson. The fact, then, that in the present experiments i is not at all dependent upon T over a 700 ˚C interval means that the electrons pulled out by the fields here are not thermions at all.

(iv) Relations of field-currents and thermionic currents- For at a sufficient distance x_1 from the surface of the wire (a distance large compared to the surface irregularities) it is the image-force alone which must be overcome to cause the electron to leave the surface. The value of this image-force per unit charge is $e/4x_1^2$. Every electron must escape which reaches the distance $x_1 = \sqrt{e/4F}$. Now, the total work necessary to bring an electron out to x_1 is the work b_0 necessary to bring it up to point x_0 at which the image law begins to be valid plus the integral $\int_{x_0}^{x_1} \left(\frac{e}{4x^2} - F\right) e\,dx$. That is, the work function is

$$b_0 + \int_{x_0}^{\sqrt{e/4F}} \left(\frac{e}{4x^2} - F\right) e\,dx = b_0 + \frac{e^2}{4x_0} - \sqrt{e^3F} + eFx_0 \qquad (2)$$

Since x_0 is very small (10^{-8} cm) the last term may be dropped while the first term is the total work necessary to bring the electron out where there is no field. Hence $b - \sqrt{e^3F}$ is the work function in the presence of an external field F and the thermionic equation becomes $i = AT^n e^{-(b-\sqrt{e^3F})/T}$. This equation shows that when T is constant and F alone varies $\log i \propto \sqrt{F}$. However, plot for log i against $\sqrt{F}$ is not a straight line (Fig. 2).

As a continuation of the last conclusion, Millikan and Lauritsen plotted log i against 1/F. The result shown in Fig. 3 reveals that log i plots as quite as good a straight line against 1/F as is obtained in the thermionic work when log i is plotted against 1/T.

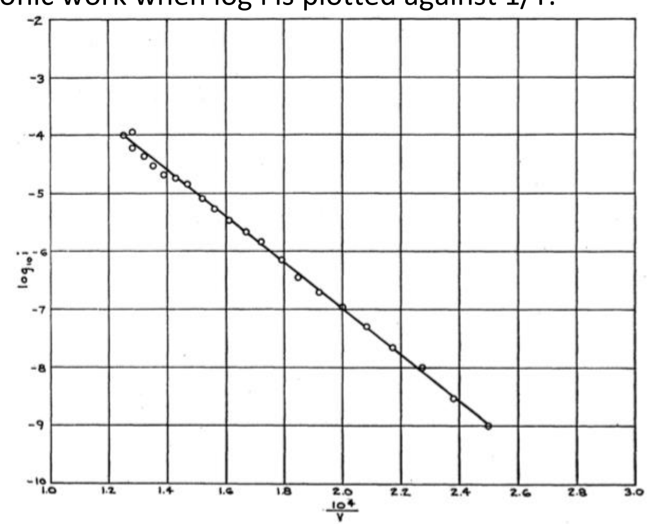

Fig. 3. Logarithm of field-current vs inverse of field

Further, in view of the fact that except at high temperatures the field currents have been shown to be independent of temperature, they justified that log i - 1/F curves would have precisely the form given above at absolute zero. They combined the empirical formula governing field currents ($i = Ce^{-b/F}$) with the usual formula governing thermionic currents ($i = Ae^{-b/T}$) and obtained the general formula for the extraction of electrons by fields or by temperature-or by both-in the form $i = Ae^{-\frac{b}{T+cF}}$. For the sake of more exact identification of form with the customary thermionic equation,

$$i = A(T + cF)^2 e^{-\frac{b}{T+cF}} \qquad (3)$$

They concluded that application of an external field is equivalent to increasing the temperature of the electrons within the metal.

FOWLER-NORDHEIM'S PAPER

Millikan and his associates asserted that a distinction should be drawn between the electrons which can function as thermions and the ordinary conduction electrons which yield the emission at great field strengths and are absolutely independent of the temperature. Fowler and Nordheim found this deduction liable to mislead. They showed that Sommerfeld's picture of a metal yields the formula both for strong fields and thermionic emission. A single set of free or conduction electrons distributed according to the Fermi-Dirac statistics suffices for both purposes.

The Reflection of Electrons at a Potential Jump when an Electric Field acts on one Side-
When a uniform external field acts, potential energy of the electrons is as shown in Fig. 4. The corner at the top will really be rounded off by the image effect. Fowler-Nordheim claimed that this modification is unimportant in calculating the strong field emission at ordinary temperatures. To study the emission through the potential energy step of Fig. 4, one needs to solve-

$$\frac{d^2\psi}{dx^2} + \kappa^2(W - C + Fx)\psi = 0 \ (x > 0) \qquad (4)$$

$$\frac{d^2\psi}{dx^2} + \kappa^2 W\psi = 0 \ (x < 0) \qquad (5)$$

subject to the conditions that ψ and $d\psi/dx$ are continuous at x=0 and that for x>0 ψ represents a stream of electrons progressing to the right only. Constant κ is defined by

$$\kappa^2 = 8\pi^2 m/h^2 \qquad (6)$$

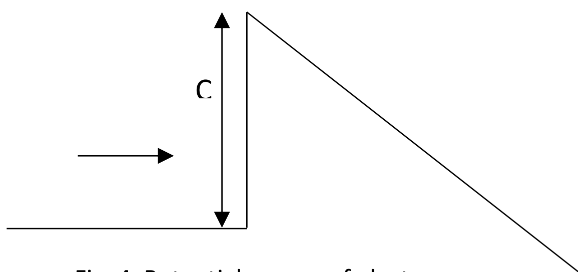

Fig. 4. Potential energy of electrons

Let

$$\left(-\frac{C-W}{F} + x\right)(\kappa^2 F)^{\frac{1}{3}} = y \qquad (7)$$

then Eq. 4 becomes

$$\frac{d^2\psi}{dy^2} + y\psi = 0 \qquad (8)$$

of which the solutions are expressible in terms of Bessel's functions of order 1/3,

$$\psi = \sqrt{y} J_{\pm\frac{1}{3}}\left(\frac{2}{3} y^{\frac{3}{2}}\right) \qquad (9)$$

We require that solution which for large x (i.e. y) represents a wave travelling to the right. Therefore,

$$\psi = \sqrt{y} H^{(2)}_{\frac{1}{3}}\left(\frac{2}{3} y^{\frac{3}{2}}\right) \qquad (10)$$

where $H^{(2)}$ denotes second function of Hankel. For large y,

$$\psi \sim \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{1}{y^{\frac{1}{4}}} \frac{1}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} e^{-i\left[\frac{2}{3}y^{\frac{3}{2}} - \frac{5\pi}{12}\right]} \qquad (11)$$

As $y \to \infty$,

$$|\psi|^2 \sim \frac{A}{(W-C+Fx)^{\frac{1}{2}}} = \frac{A\prime}{v} \qquad (12)$$

where A, A' are constants and v is the velocity of electrons. Hence the density of electron stream behaves as it should.

For x<0 we get,

$$\psi = \frac{1}{W^{\frac{1}{4}}}\left[a e^{i\kappa x\sqrt{W}} + a'^{e^{-i\kappa x\sqrt{W}}}\right] \qquad (13)$$

Let us write

$$\frac{2}{3}\kappa\sqrt{F}\left(\frac{C-W}{F}\right)^{\frac{3}{2}} = Q \qquad (14)$$

so that Q is real and in practice large.

The equations of continuity of ψ and $d\psi/dx$ can therefore be reduced to

$$a + a' = W^{\frac{1}{4}}\left(\frac{C-W}{F}\right)^{\frac{1}{2}} H^{(2)}_{\frac{1}{3}}\left(e^{-\frac{3}{2}\pi i}Q\right) \qquad (15)$$

$$-a + a' = \frac{i}{\kappa W^{\frac{1}{4}}}\left[\frac{1}{2}\left(\frac{C-W}{F}\right)^{-\frac{1}{2}} H^{(2)}_{\frac{1}{3}}\left(e^{-\frac{3}{2}\pi i}Q\right) + \frac{C-W}{F}\kappa\sqrt{F}\frac{dH^{(2)}_{\frac{1}{3}}}{dQ}\left(e^{-\frac{3}{2}\pi i}Q\right)\right] \qquad (16)$$

By the definition of functions of Hankel, we can express $H^{(2)}_{\frac{1}{3}}\left(e^{-\frac{3}{2}\pi i}Q\right)$ in terms of real functions $I_{\pm\frac{1}{3}}(Q)$ by the equation

$$H^{(2)}_{\frac{1}{3}}\left(e^{-\frac{3}{2}\pi i}Q\right) = \frac{-1}{\sin\frac{1}{3}\pi}\left\{I_{-\frac{1}{3}}(Q) + e^{\frac{1}{3}\pi i} I_{\frac{1}{3}}(Q)\right\} \qquad (17)$$

Let us now write

$$\alpha + i\beta = \frac{I'_{-\frac{1}{3}} + e^{\frac{1}{3}\pi i} I'_{\frac{1}{3}}}{I_{-\frac{1}{3}} + e^{\frac{1}{3}\pi i} I_{\frac{1}{3}}} \qquad (18)$$

where α and β are real. Let us also write D(W) for the fraction of W electrons penetrating the boundary peak and emerging under the influence of the external field F.

$$D(W) = \frac{|a|^2 - |a'|^2}{|a|^2} = \frac{4\beta\left(\frac{C-W}{F}\right)^{\frac{3}{2}}\sqrt{F}}{\left\{W^{\frac{1}{4}}\left(\frac{C-W}{F}\right)^{\frac{1}{2}} + \frac{C-W}{\sqrt{F}}\frac{\beta}{W^{\frac{1}{4}}}\right\}^2 + \frac{1}{\kappa^2 W^{\frac{1}{2}}}\left\{\frac{1}{2}\left(\frac{C-W}{F}\right)^{-\frac{1}{2}} + \frac{C-W}{\sqrt{F}}\kappa\alpha\right\}^2} \qquad (19)$$

Calculation of α and β remains. It can be verified that

$$\beta = \frac{\sqrt{3}}{2}\frac{\left(I'_{\frac{1}{3}} I_{-\frac{1}{3}} - I'_{-\frac{1}{3}} I_{\frac{1}{3}}\right)}{\left(I_{-\frac{1}{3}} + I_{\frac{1}{3}}\right)^2 + \frac{3}{4} I_{\frac{1}{3}}{}^2} \qquad (20)$$

The numerator is the Wronskian of Bessel's equation of purely imaginary argument and we have exactly-

$$I'_{\frac{1}{3}} I_{-\frac{1}{3}} - I'_{-\frac{1}{3}} I_{\frac{1}{3}} = \frac{2\sin\frac{1}{3}\pi}{\pi Q} \qquad (21)$$

For the denominator, we can use the asymptotic expansion, Q being large. The denominator is

$$\left|\sin\frac{1}{3}\pi\, H_{\frac{1}{3}}^{(2)}\left(e^{-\frac{3}{2}\pi i}Q\right)\right|^2 \sim \frac{3}{4}\frac{2}{\pi Q}e^{2Q} \qquad (22)$$

so that

$$\beta \sim e^{-2Q} \qquad (23)$$

In evaluating α we can use asymptotic values throughout and find $\alpha = 1$.
Thus, with sufficient accuracy

$$D(W) = \frac{4\left(\frac{C-W}{F}\right)^{\frac{3}{2}}\sqrt{F}e^{-2Q}}{W^{\frac{1}{2}}\left(\frac{C-W}{F}\right)+\frac{1}{\kappa^2 W^{\frac{1}{2}}}\left\{\frac{1}{2}\left(\frac{C-W}{F}\right)^{-\frac{1}{2}}+\frac{C-W}{\sqrt{F}}\kappa\right\}^2} \qquad (24)$$

By considering the relative order of the terms in the denominator it is found that those independent of κ are dominant. Therefore, on inserting the value of Q

$$D(W) = \frac{4\{W(C-W)\}^{\frac{1}{2}}}{C}e^{-\frac{4\kappa(C-W)^{\frac{3}{2}}}{3F}} \qquad (25)$$

The Complete Electron Emission from a Cold Metal-

The number of electrons N(W) incident on a surface of unit area per unit time with a kinetic energy W normal to the surface has been evaluated by Nordheim according to Sommerfeld's theory. He finds

$$N(W) = \frac{4\pi mkT}{h^3}L\left(\frac{W-\mu}{kT}\right) \qquad (26)$$

where

$$L(\beta) = \int_0^\infty \frac{dy}{e^{\beta+y}+1} \qquad (27)$$

and μ is the usual parameter of the electron distribution in the Fermi-Dirac statistics equivalent to the thermodynamic partial potential of an electron. Hence the current I is given quite generally by-

$$I = \frac{4\pi m\varepsilon kT}{h^3}\int_0^\infty D(W)L\left(\frac{W-\mu}{kT}\right)dW \qquad (28)$$

where ε is the electronic charge. At ordinary and low temperatures, a sufficient approximation to $kTL\left(\frac{W-\mu}{kT}\right)$ is $\mu - W$ when $W < \mu$ and otherwise zero. Since μ is considerably less than C, we may then use Eq. 25 for D(W) and find

$$I = \frac{16\pi m\varepsilon}{Ch^3}\int_0^\mu W^{\frac{1}{2}}(C-W)^{\frac{1}{2}}(\mu - W)\,e^{-\frac{4\kappa(C-W)^{\frac{3}{2}}}{3F}}dW \qquad (29)$$

Since the exponent in the integrand is very large for the largest values of W, it is easy to evaluate this integral to a sufficient approximation. We find using Eq. 6 and putting $C - \mu = \chi$, that

$$I = \frac{\varepsilon}{2\pi h}\frac{\mu^{\frac{1}{2}}}{(\chi+\mu)\chi^{\frac{1}{2}}}F^2 e^{-\frac{4\kappa\chi^{\frac{3}{2}}}{3F}} \qquad (30)$$

The χ of this equation is necessarily and exactly the thermionic work function.
If we express I in amperes per square centimeter of emitting surface, μ and χ in volts and F in volts per centimeter, and insert numerical values for the other constants, we find

$$I = 6.2 \times 10^{-6} \frac{\mu^{\frac{1}{2}}}{(\chi+\mu)\chi^{\frac{1}{2}}} F^2 e^{-2.1\times10^8 \frac{\chi^{\frac{3}{2}}}{F}} \qquad (31)$$

According to this formula emission begins to be sensible for fields of rather more than 10^7 V/cm. These values are higher than those commonly derived from experiments, which indicate measureable emission for values of F about 10^6 V/cm. They also attributed this to surface irregularities or peaks near which values of F will be larger.

To summarize, Fowler-Nordheim theory uses the following physical assumptions. That the metal: (i) has a free-electron band structure; (ii) has electrons obeying Fermi-Dirac statistics; (iii) is at zero temperature; (iv) has a smooth flat surface; and (v) has a work function that is uniform across the emitting surface and is independent of external field. It is also assumed that: (vi) there is uniform electric field outside the metal surface; (vii) the exchange-and-correlation effects may be neglected in a first approximation; and (viii) barrier penetration coefficients may be evaluated using the JWKB approximation.

<u>Murphy and Good's improvement</u>

Elementary FN theory neglects correlation-and-exchange effects. These can be included by taking the barrier as sum of a uniform electric field and an image potential, with the electrical surface and the image plane in the same place. Forbes refers to this as Schottky-Nordheim (SN) tunneling barrier. Murphy and Good treatment used this and also considers finite temperatures.

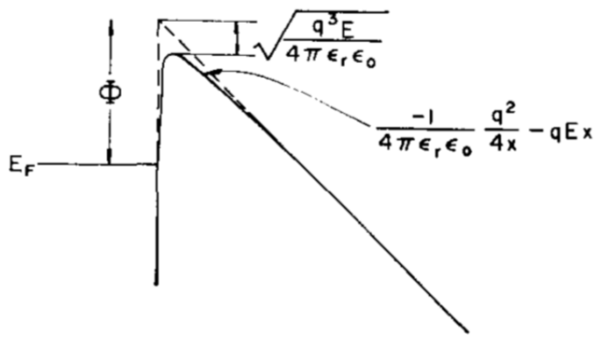

Fig. 5. Image-force barrier lowering

The final equation for field emission turns out to be-

$$j = \frac{F^2}{16\pi^2\phi t^2}\left(\frac{\pi ckT}{\sin \pi ckT}\right)\exp\left(-\frac{4\sqrt{2}\phi^{\frac{3}{2}}v}{3F}\right) \qquad (32)$$

where ϕ is the work function, arguments of t and v are $\frac{F^{\frac{1}{2}}}{\phi}$

$$c = 2\sqrt{2}F^{-1}\phi^{\frac{1}{2}}t\left(\frac{F^{\frac{1}{2}}}{\phi}\right) \qquad (33)$$

Numerical values of t(y) can be easily found from Burgess, Kroemer and Houston's tables; in terms of the functions v(y) and s(y) which they tabulate, t(y) is given by

$$3t(y) = 4s(y) - v(y) \qquad (34)$$

When ckT is small that $\frac{\pi ckT}{\sin \pi ckT}$ can be replaced by one, Eq. 32 becomes the Fowler-Nordheim formula.

Experimental Verification

Lenzlinger and Snow[20] studied Fowler-Nordheim tunneling in metal-silicon dioxide-silicon (MOS) structure and observed all the essential functional dependencies of the FN emission theory.

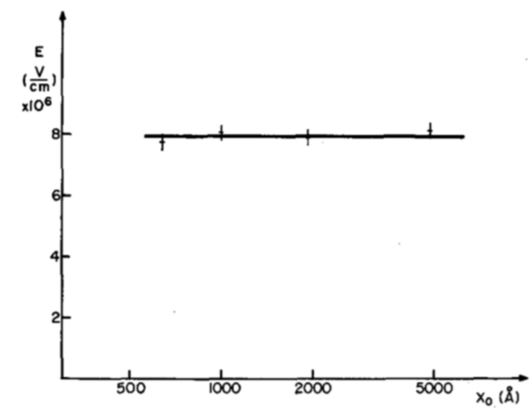

Fig. 6. Field at a given current density ($J=0.2\mu A/cm^2$) vs oxide thickness

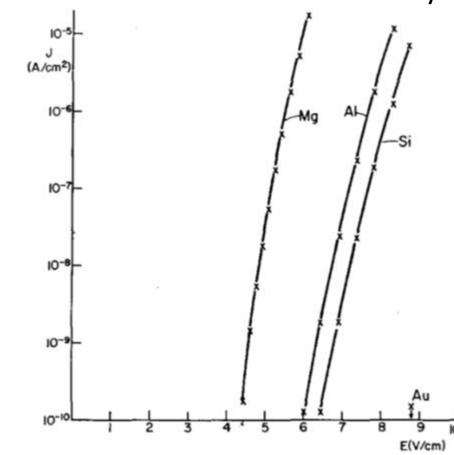

Fig. 7. Current density vs field for Mg, Al and Si

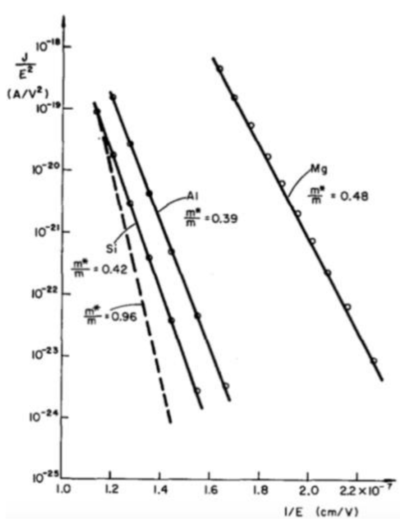

Fig. 8. Fowler-Nordheim characteristics ($\log J/E^2$ vs $1/E$)

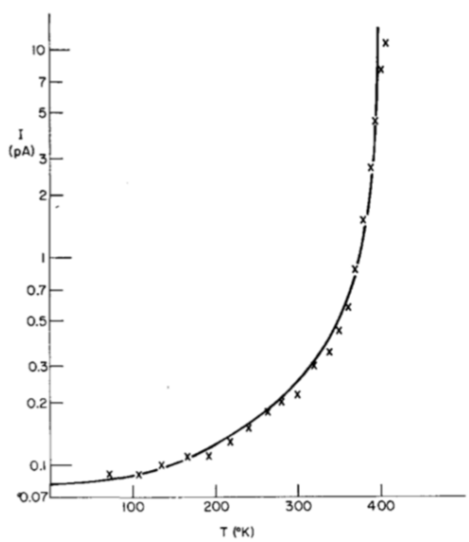

Fig. 9. Current vs temperature for Al for $E=6.1\times10^6$ V/cm

RECENT DEVELOPMENTS

Forbes, in 1999, suggested that relaxing one or more conditions used in deriving the FN equation leads to equations in various forms and approximations. All can be regarded as specialized versions of the generalized Fowler-Nordheim equation-

$$I = \lambda R^{el} F^2 \exp\left\{\mu \frac{S^{el}}{F}\right\} \qquad (35)$$

where $I = R^{el}F^2 \exp\left\{\frac{S^{el}}{F}\right\}$ is the elementary FN equation; λ and μ are generalized correction factors, whose form in a given case depends on the particular assumptions and approximations made.

In 2003, Jensen developed a generalized thermal-field emission methodology developed to account for low work function, high fields, photoexcitation, and other conditions in which the incident electron energy is near the barrier maximum. He also examined specialized topics like application to multidimensional structures and statistical nature of emission site variation.

In 2007, Forbes and Deane reformulated the standard FN theory by a different treatment of the SN barrier which can serve as a paradigm for other barrier shapes. The same authors, in 2011, found an exact solution to the exact triangular (ET) barrier problem, which was investigated in the original 1928 paper of Fowler-Nordheim. They found equations for different regimes as shown in Fig. 10. Their analysis reproduces the FN formula for deep tunneling regime.

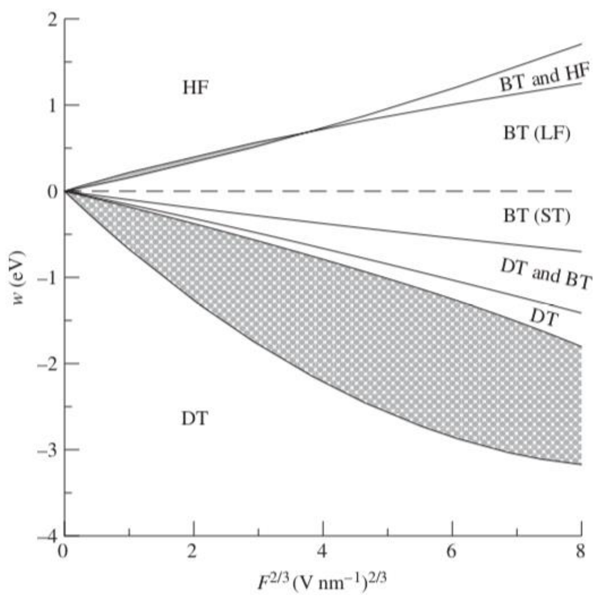

Fig. 10. Transmission regime diagram. The chosen regimes are: deep tunneling (DT); barrier-top regime (BT), which includes shallow tunneling (ST) and low flyover (LF); and high flyover (HF). For shaded regions, no good working formula has been found.

EPILOGUE

The ET barrier is not a physically realistic model for the actual surface barrier experienced by escaping electrons. The SN barrier, which contains an image-PE term, is a better physical model, certainly for metals. However, the Schrödinger equation for the SN barrier cannot be solved exactly. The ET barrier has the marked advantage that an exact analytical solution to the related Schrödinger equation exists. It has the disadvantage that-although trends can be found-quantitative predictions of experimental quantities such as current densities are not accurate.

The standard FN equation has been invoked even when operational conditions violate one or more of the approximations upon which it is based: Semiconductor band bending effects change the supply function; tunneling barriers may be so reduced or narrowed that the classical image charge modification to the potential is compromised; the emitters run relatively hot; the geometry of a typical field emitter is hardly one dimensional and results in potential barriers that are not linear for emission into a vacuum; oxides, coatings, and semiconducting layers exist or are purposely applied; the electric fields are often so large or the work functions so low that the barrier maximum is not well above Fermi level; the surface is in a state of flux due to adsorbates, ion bombardment, and/or evolving nanoprotrusions; adsorbate migration and other surface effects introduce noise; and other mechanisms (from resonant tunneling to photoexcitation to thermal emission) supplement, complicate, or dominate the emission process.

ABOUT THE SCIENTISTS

R.H. Fowler: Sir Ralph Howard Fowler was a British physicist and astronomer. Fifteen fellows of the Royal Society and three Nobel laureates (Chandrasekhar, Dirac, and Mott) were supervised by him. He is best known for Fowler-Nordheim tunneling and Darwin-Fowler method. He was the first to formulate and label the zeroth law of thermodynamics.

L. Nordheim: Lothar Wolfgang Nordheim was a German born Jewish American theoretical physicist. Upon his immigration to the United States Nordheim served as a visiting professor at Purdue University, moving on to a permanent faculty position at Duke University. He is best known for Fowler-Nordheim tunneling.

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