# Thermal Ratchets

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## Introduction

There exist a number of systems which can extract work out of unbiased random fluctuations. A windmill, for example, harnesses air currents, and a self-winding wristwatch utilizes random motion of the user to power itself. A number of other various rectifiers produce similar effects in macroscopic systems, but whether this is possible in a microscopic system is another question. Brownian noise, in some nanoscale system, should be able to produce meaningful work in a similar manner, yet this would seem a violation of the Second Law of Thermodynamics. It turns out that if we are clever enough, we can indeed create devices that produce work from thermal noise, without violating the laws of thermodynamics. These are called thermal ratchets.

A 'thermal ratchet' refers to a broad range of systems where thermal noise, or Brownian motion, is rectified and harnessed to do useful work. This is a directed transport phenomenon where thermal noise plays the dominating role. But directed transport in a system with a single thermal heat bath is forbidden by the second law of thermodynamics. The effect of this noise must be symmetric, and no features of our device can bias the Brownian motion. Thus a first requirement for a thermal ratchet is breaking of thermal equilibrium [1]. This will come in various forms; different heat baths, perturbations, periodic driving, etc. It turns out that another requirement is breaking of spatial inversion symmetry [1], for example in the form of a periodic and asymmetric potential, or perturbation. With these two conditions, directed transport can emerge to produce work.

**Feynman's Thought Experiment**

The question of why these requirements are necessary was addressed by Feynman in the famous *Feynman Lectures on Physics* in 1962 [2]. In this thought experiment, Feynman proposed a machine, small enough to be influenced by moving particles, with a small paddle

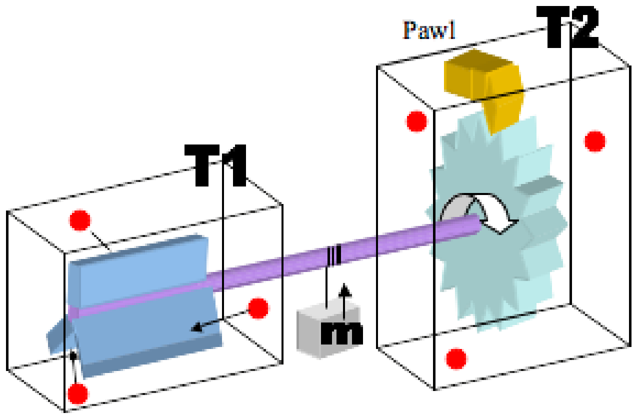


**Figure 1: Feynman's ratchet and pawl. [3]**

wheel in one thermal bath and a ratchet and pawl in another (Fig. 1). This would be connected to a mass that could be lifted and produce work. The paddle wheel in the first bath, at temperature T1, is in contact with molecules whose random Brownian motion can turn the paddle. Since the noise is symmetric there is no preferential direction of the wheel's rotation. However, the ratchet on the opposing side allows rotation only in one preferential direction. It seems that this can produce useful work from Brownian motion, which violates the second law. But Feynman showed that if the whole device is at the same temperature it will only rotate back and forth randomly. This is because the ratchet mechanism is also influenced by random Brownian motion, which allows the wheel to slip backwards intermittently. Now, if we break thermal equilibrium, with T2<T1, the ratchet will indeed only move forward and produce work. Before examining real systems, we'll briefly consider the theory of a thermal ratchet.

**Theory**

The theory of thermal ratchets can be expressed by a Langevin equation, which describes the time evolution of a particle's position with respect to some faster varying stochastic variable (in this case the thermal noise). Consider a Brownian particle at temperature T in a potential V(x) with a load force F. Then the Langevin equation for the particle's position is:

$$\dot{x} = k[-V'(x,t) + F + \xi(t)]$$

where ξ(t) represents the thermal fluctuations as white Gaussian noise with zero mean [4]. From this Langevin equation, we can write the corresponding Fokker-Planck equation for the evolution of the probability density. This is given as:

$$\partial_t \rho(x,t) = -\partial_x J(x,t)$$

where the particle current J is:

$$J(x,t) = -k\rho(x,t)[\partial_x \mu(x,t) - F]$$

[4]. The particle current refers to what is used to do work, whether thermal particles made to diffuse in a preferential direction, electrons in a circuit, etc. This general approach can be applied to the many specific Brownian motor systems that exist, which we will now discuss.

**The Flashing Ratchet**

The simplest Brownian motor that we can consider is the so called 'flashing' ratchet. Consider a particle in an asymmetric and periodic potential that is flashing on and off (Fig. 2) [5]. There is some external force, from the right to left, which the particles can do work against. When the potential is off the particle is free to diffuse, and when it is on it becomes trapped.

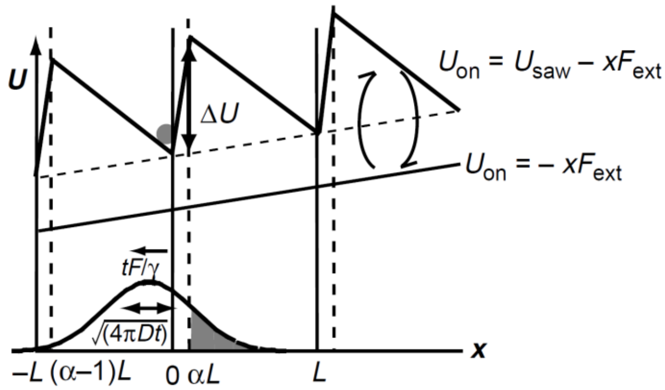


**Figure 3: Flashing potential. While off, the particle diffuses with Gaussian probability, seen on the bottom of the figure. When on, the particle becomes trapped. It moves to the right despite an external force to the left [5].**

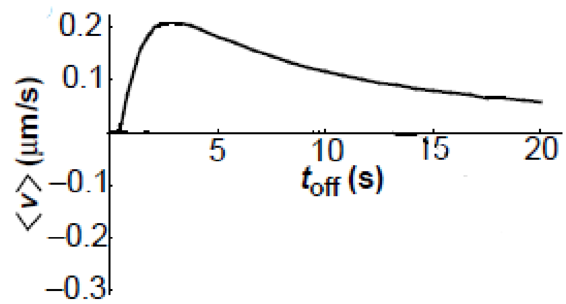


**Figure 2: Velocity distribution of particle current with respect to the time the potential stays off [5].**

Without thermal noise, the particle would move to the left in the presence of this force. But with it, the particle's position is given by a probability distribution based on a random walk. Therefore when the potential is off the particle starting at iL diffuses around the region, and because of the asymmetry of the potential it is more likely to be trapped in the well at (i+1)L than (i-1)L when the potential turns on again. Thus despite a force to the left, the particle moves to the right as the potential is turned on and off. In this case, we can write the Langevin equation with this periodic potential as

$$\dot{x} = k[-\zeta(t)V'(x) + F + \xi(t)],$$

where ζ(t) is noise of values 0 and 1, switching the potential on and off [4].

It turns out that the velocity of particle current depends nonmonatonically on the switching frequency (Fig. 3) [5]. This is because while the potential is off, the particle must diffuse at least a distance αL but not longer than (1-α)L. Therefore we can tune the switching frequency to specific particles and devices. The energy that we extract from this system, as a particle current, now comes not from thermal diffusion but from the potential being turned on. Thus we do not violate the second law, yet use thermal noise to yield directed transport and work.

**The Rocking Ratchet**

A typical "flashing" thermal ratchet involves no changes in the spatial average of the force on the particle, but instead simply changes the landscape of the potential surface locally. Another common type of ratchet involves applying a fluctuating net force across the entire surface while leaving the time averaged potential unchanged. This approach essentially involves tilting a standard sawtooth potential back and forth between two limits by adding a linear factor to the potential $\pm F_{max}$. If $F_{max}$ is sufficiently high, the potential surface at this limit will decrease monotonically and the particles will all flow to the left preferentially (See figure 4 A)[5]. However, when the $-F_{max}$ limit is reached, the minima which exist in the standard sawtooth potential will remain, and trap particles, preventing them from flowing more than one period to the right.

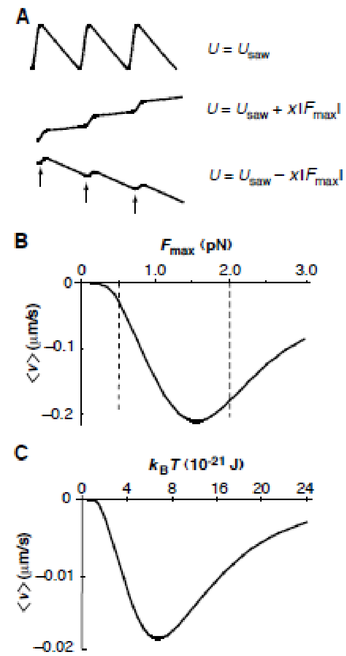


Fig 4. The mechanism of a rocking ratchet. (a) the unperturbed, sawtooth potential with a constant force applied. (b) particle velocity as a function of $F_{max}$. (c) particle velocity as a function of thermal noise

It is interesting to note that in this approach, the thermal ratchet is perfectly functional even in the absence of thermal noise, provided that the period of oscillation is sufficiently low. With a high frequency, it is possible that the particles will not have time to move across one of the sawteeth in the $+F_{max}$ position, before they flow back to the right in the $-F_{max}$ position, and thus no long distance transport will occur. In the absence of thermal noise, condition on $F_{max}$ is trivially $\Delta U/[(1-\alpha)L]<F_{max}$, but with thermal noise, this condition can be relaxed depending on the maximum magnitude of the forces due to thermal fluctuations. For a slow square wave modulation, the average rate of flow can be calculated analytically calculated [6]. In figure 4 the average velocity of the particles are plotted against $F_{max}$ in figure 4B and against thermal noise at

$F_{max}$=0.4pN in figure 4C. Notably, we see that increasing the noise can actually increase flow suggesting that in some applications, it may be useful to add noise to a system in order to increase transport.

One example of a rocking ratchet demonstrated by D. Perez de Lara, et al. Involves purely magnetic manipulation of the potentials in order the achieve directionally preferred transport of superconducting vortices [7]. In this work, researchers used a superconducting Nb film in contact with a structurally symmetric array of Ni nano-rings. The superconducting vortices play the role of the nonequilibrium particle, and the driving force of the rocking potential is induced by A.C. currents in the sample. The goal is to demonstrate that unlike similar superconductor vorticy thermal ratchets whose function is derived from structural asymmetry, this thermal ratchet is driven by magnetic effects alone. In this situation, the magnetic state of the ring arrays is measured in several cases: one with in-plane field along the axis of the square array, field along the long axis of the elliptical rings, and field along the short axis of the elliptical rings. In all cases, the data suggests that the ring form onion states; that is, where the magnetization of the ring along the circumfrence of the ring, but half of the ring is polarized clockwise, and the other half counter-clockwise (see figure 5).

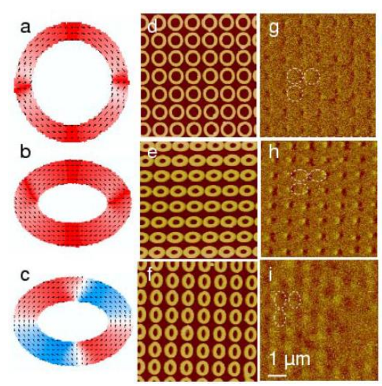

Fig 5. Circular and eliptical rings with “onion” state

Structural defects are often effective potential wells for superconducting vortices. Close to the critical temperature of the superconductor, magnetoresistance generates a periodic potential minima when the vortex density is an integer multiple of the site density. Thus, when the number of vortices per Ni ring is controlled by an externally applied magnetic field, it can be characterized simply by making magnetoresistence measurements. In these experiments, samples with elliptical rings are structurally symmetric, and thus display no ratchet effect in the absence of magnetization. But when the rings are magnetized in parallel onion states, the ratchet effect is clear (See figure 6). Thus, the ratchet effect can be attributed purely to magnetic asymmetry. Further confirmation of this purely magnetic phenomena can be found in the fact that the magnitude of the vortex current does not change with the a.c. input driving current, and thus there is no reversal of the vortex ratchet for a purely magnetically induced ratchet. Thus a net current of superconducting vortices can be driven via a zero-average A.C. current in the presence of conducting rings, which become magnetized in onion states, confirming that the ratchet effect in this system is purely magnetically driven.

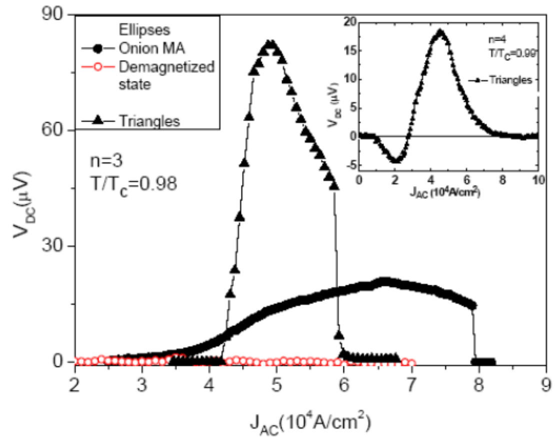


Fig 6. Vortecy current as a function of the applied AC field. Shown for onion magnetization, demagnetized states, and triangular nanostructures

## The Plasmonic Brownian Ratchet

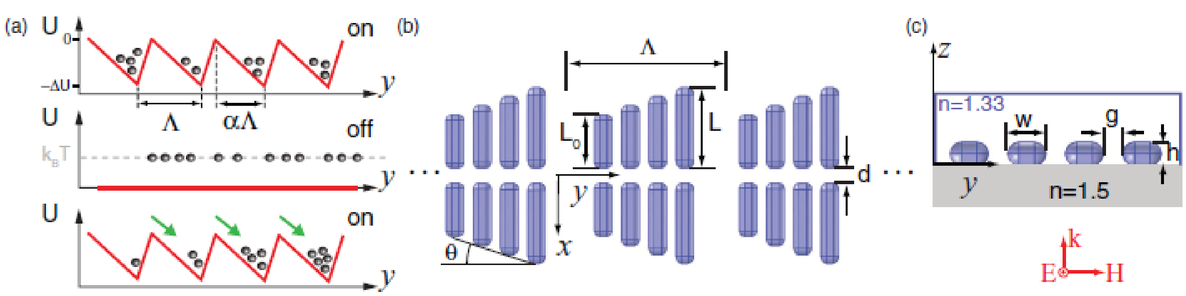


Fig 7. The basic operation of a flashing ratchet (a), an overhead schematic of the plasmonic structures (b), and a planar view of the strucutres with the direction of the fields indicated (c)

The plasmonic Brownian ratchet is another interesting simulated example of a flashing ratchet [8]. In this example, researchers utilize an array of plasmonic nanostructures to bias Brownian motion of dielectric beads. The goal is to produce geometrically asymmetric, anisotropic traps for the particles, and then modulate the time of interaction with the traps by turning on an off illumination of the surface. Each cell of the lattice in this structure consists of four metallic dipole antennas who's length gradually varies, and spaced at a dramatically subwavelength spacing (See figure 7b). The length of the series is given by $L_o$=L-3(ω+g)tan(θ). The angle θ characterizes the degree of asymmetry in the structure. Each unit cell is repeated with a period of Λ and the structures lie on a glass substrate which is immersed in water containing well dispersed dielectric beads at room temperature. In order to modulate the potential, the system is illuminated with a p lane wave normal to the surface and with the electric field polarized along the dipole antenna axis (figure 7c).

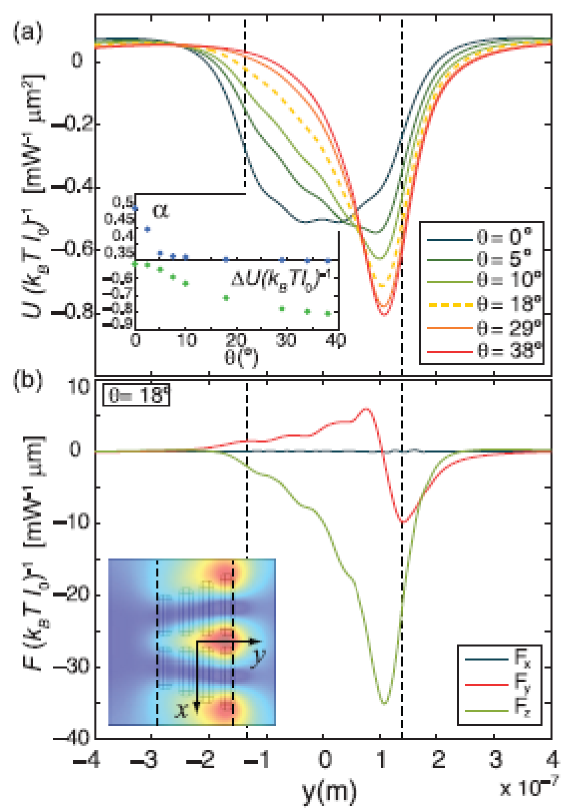


Fig 8. U(y) for a variety of vaues of θ (a). Maximum potential depth and disorder parameter as a function of θ (a inset). Force experienced by a particle (b) and the norm of the electric field (b inset).

In order to characterize researchers performed simulations on the structures and determined the structure of the potential well of a single antenna array as a function of parameters such as θ in figure 8a,b. It is clear from these plots that they asymmetric geometry of the antennas yields an anisotropy in the trap. The field pattern reveals that within the gap between the longest antenna, there is a dramatic field enhancement whereas the neighboring, shorter antennas leads to a continuous field profile along the y axis. By changing physical parameters of the array, it is possible to change features such as the depth of the potential well, and its degree of asymmetry. However, at sufficiently large values of θ, the size and thus resonant frequencies of neighboring antennas is too great, and the antennas begin to decouple.

To demonstrate the function of the Brownian ratchet, researchers carried out simulations for diffusive beads in the presence of their proposed plasmonic structure with the incident trapping field periodically turned on and off. For particles of mass m, radius σ, temperature T, and subject to an external force $F_y$, the motion is governed by the Langevin equation of motion:

$$m\ddot{\mathbf{r}} = F_y - \gamma m\dot{\mathbf{r}} + \sqrt{2\gamma m k_B T}\,R(t).$$

Where we see the standard drag coefficient γ and diffusion constant $D=k_BT/m\gamma$. By solving this equation for N particles at 300K=T, researchers simulated the statistics of the system. In this case, Λ=800nm because for this value, U(y) contains no local maxima, $I_0$=75mW/μm$^2$and the process is simulated in 1D. The results of this simulation can be seen in the histograms in Figure 9. The simulation confirms the possibility for this type of device to cause biased brownian motion, which could have exciting results in the fields where precise manipulations of small particles are currently a great challenge.

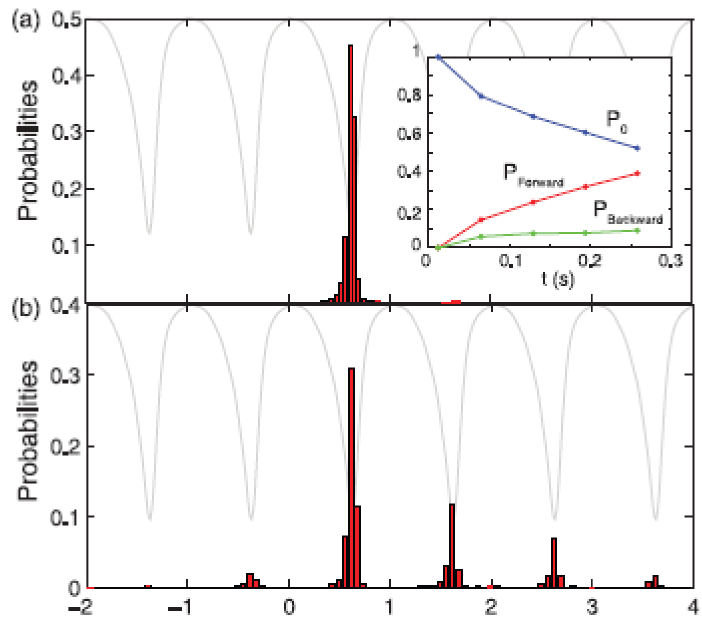


Fig 9. Simulation of diffusion probabilities for 4000 particles in the proposed structure. (a) shows the initial configuration of the system, and (b) demonstrates the distribution after 16 on-off cycles.

## Conclusion

Thermal ratchets provide a way to get meaningful work out of random thermal fluctuations, provided there is some energy input to break thermal equilibrium and spatial symmetry. These thermal ratchets have applications in nanoscale and biological technology, for example in sorting various particles based on how they diffuse through the ratchet. We have investigated a wide number of different forms these systems can take, from a simple asymmetric and periodic potential, to more modern magnetic and plasmonic systems.

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