5-2 The issue is: Can we use the simpler classical expression $p = (2mK)^{1/2}$ instead of the exact relativistic expression $p = \frac{K\left(1 + \frac{2mc^2}{K}\right)^{1/2}}{c}$? As the relativistic expression reduces to $p = (2mK)^{1/2}$ for $K \ll 2mc^2$, we can use the classical expression whenever $K \ll 1\text{ MeV}$ because mc^2 for the electron is 0.511 MeV.

(a) Here $50\text{ eV} \ll 1\text{ MeV}$, so $p = (2mK)^{1/2}$

$$\lambda = \frac{h}{p} = \frac{h}{\left[(2)\left(\frac{0.511\text{ MeV}}{c^2}\right)(50\text{ eV})\right]^{1/2}} = \frac{hc}{\left[(2)(0.511\text{ MeV})(50\text{ eV})\right]^{1/2}}$$

$$= \frac{1\,240\text{ eV nm}}{\left[(2)(0.511 \times 10^6)(50)(\text{eV})^2\right]^{1/2}} = 0.173\text{ nm}$$

(b) As $50\text{ eV} \ll 1\text{ MeV}$, $p = (2mK)^{1/2}$

$$\lambda = \frac{hc}{\left[(2)\left(\frac{0.511\text{ MeV}}{c^2}\right)(50 \times 10^3\text{ eV})\right]^{1/2}} = 5.49 \times 10^{-3}\text{ nm}.$$

As this is clearly a worse approximation than in (a) to be on the safe side use the relativistic expression for p: $p = K\frac{\left(1 + \frac{2mc^2}{K}\right)^{1/2}}{c}$ so

$$\lambda = \frac{h}{p} = \frac{hc}{\left(K^2 + 2Kmc^2\right)^{1/2}} = \frac{1\,240\text{ eV nm}}{\left[(50 \times 10^3)^2 + (2)(50 \times 10^3)(0.511 \times 10^6\text{ eV})\right]^{1/2}}$$

$$= 5.36 \times 10^{-3}\text{ nm} = 0.005\,36\text{ nm}$$

5-7 A 10 MeV proton has $K = 10\text{ MeV} \ll 2mc^2 = 1\,877\text{ MeV}$ so we can use the classical expression $p = (2mK)^{1/2}$. (See Problem 5-2)

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(2)(938.3\text{ MeV})(10\text{ MeV})\right]^{1/2}} = \frac{1\,240\text{ MeV fm}}{\left[(2)(938.3)(10)(\text{MeV})^2\right]^{1/2}} = 9.05\text{ fm} = 9.05 \times 10^{-15}\text{ m}$$

5-8

$$\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}} = \frac{h}{(2meV)^{1/2}} = \left[\frac{h}{(2me)^{1/2}}\right]V^{-1/2}$$

$$\lambda = \left[\frac{6.626 \times 10^{-34}\text{ Js}}{\left(2 \times 9.105 \times 10^{-31}\text{ kg} \times 1.602 \times 10^{-19}\text{ C}\right)^{1/2}}\right]V^{-1/2}$$

$$\lambda = \left[\frac{1.226 \times 10^{-9}\text{ kg}^{1/2}\text{m}^2}{sC^{1/2}}\right]V^{-1/2}$$

5-10 As $\lambda = 2a_0 = 2(0.052\,9)\text{ nm} = 0.105\,8\text{ nm}$ the energy of the electron is nonrelativistic, so we can use

$$p = \frac{h}{\lambda} \text{ with } K = \frac{p^2}{2m};$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.058 \times 10^{-10} \text{ m}\right)^2} = 21.5 \times 10^{-18} \text{ J} = 134 \text{ eV}$$

This is about ten times as large as the ground-state energy of hydrogen, which is 13.6 eV.

5-11 (a) In this problem, the electron must be treated relativistically because we must use relativity when $pc \approx mc^2$. (See problem 5-5). the momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-14} \text{ m}} = 6.626 \times 10^{-20} \text{ kg}\cdot\text{m/s}$$

and $pc = 124 \text{ MeV} >> mc^2 = 0.511 \text{ MeV}$. The energy of the electron is

$$E = \left(p^2c^2 + m^2c^4\right)^{1/2}$$
$$= \left[\left(6.626 \times 10^{-20} \text{ kg}\cdot\text{m/s}\right)^2\left(3 \times 10^8 \text{ m/s}\right)^2 + \left(0.511 \times 10^6 \text{ eV}\right)^2\left(1.602 \times 10^{-19} \text{ J/eV}\right)^2\right]^{1/2}$$
$$= 1.99 \times 10^{-11} \text{ J} = 1.24 \times 10^8 \text{ eV}$$

so that $K = E - mc^2 \approx 124 \text{ MeV}$.

(b) The kinetic energy is too large to expect that the electron could be confined to a region the size of the nucleus.

5-12 Using $p = \frac{h}{\lambda} = mv$, we find that $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1 \times 10^{-10} \text{ m}\right)} = 7.27 \times 10^6 \text{ m/s}$.

From the principle of conservation of energy, we get

$$eV = \frac{mv^2}{2} = \frac{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(7.27 \times 10^6 \text{ m/s}\right)^2}{2} = 2.41 \times 10^{-17} \text{ J} = 151 \text{ eV}.$$

Therefore $V = 151 \text{ V}$.

5-15 For a free, non-relativistic electron $E = \frac{m_e v_0^2}{2} = \frac{p^2}{2m_e}$. As the wavenumber and angular frequency of the electron's de Broglie wave are given by $p = \hbar k$ and $E = \hbar\omega$, substituting these results gives the dispersion relation $\omega = \frac{\hbar k^2}{2m_e}$. So $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m_e} = \frac{p}{m_e} = v_0$.

5-17 $E^2 = p^2c^2 + \left(m_e c^2\right)^2$

$E = \left[p^2c^2 + \left(m_e c^2\right)^2\right]^{1/2}$. As $E = \hbar\omega$ and $p = \hbar k$

$$\hbar\omega = \left[\hbar^2 k^2 c^2 + \left(m_e c^2\right)^2\right]^{1/2} \text{ or}$$

$$\omega(k) = \left[k^2 c^2 + \frac{\left(m_e c^2\right)^2}{\hbar^2}\right]^{1/2}$$

$$v_p = \frac{\omega}{k} = \frac{\left[k^2 c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}{k} = \left[c^2 + \left(\frac{m_e c^2}{\hbar k}\right)^2\right]^{1/2}$$

$$v_g = \left.\frac{d\omega}{dk}\right|_{k_0} = \frac{1}{2}\left[k^2 c^2 + \left(\frac{m_e c^2}{\hbar}\right)^2\right]^{-1/2} 2kc^2 = \frac{kc^2}{\left[k^2 c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}$$

$$v_p v_g = \left\{\frac{\left[k^2 c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}{k}\right\}\left\{\left[k^2 c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}\right\} = c^2$$

Therefore, $v_g < c$ if $v_p > c$.

5-24 (a) $\Delta x \Delta p = \hbar$ so if $\Delta x = r$, $\Delta p \approx \frac{\hbar}{r}$

(b) $$K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2}$$

$$U = -\frac{ke^2}{r}$$

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

(c) To minimize E take $\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0 \Rightarrow r = \frac{\hbar^2}{m_e ke^2}$ = Bohr radius = a_0. Then

$$E = \left(\frac{\hbar}{2m_e}\right)\left(\frac{m_e ke^2}{\hbar^2}\right)^2 - ke^2\left(\frac{m_e ke^2}{\hbar^2}\right) = \frac{m_e k^2 e^4}{2\hbar^2} = -13.6 \text{ eV}.$$

5-35 (a)

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}a(k)e^{ikx}\,dk=\frac{A}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{-\alpha^2(k-k_0)^2}e^{ikx}dk=\frac{A}{\sqrt{2\pi}}e^{-\alpha^2k_0^2}\int_{-\infty}^{+\infty}e^{-\alpha^2\left(k^2-\left(2k_0+ix/\alpha^2\right)k\right)}dk$$

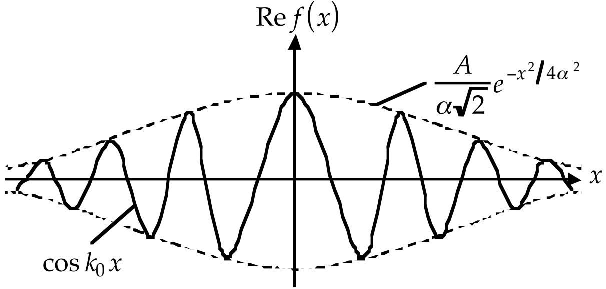
. Now complete the square in order to get the integral into the standard form $\int_{-\infty}^{+\infty}e^{-az^2}dz$:

$$e^{-\alpha^2\left(k^2-\left(2k_0+ix/\alpha^2\right)k\right)}=e^{+\alpha^2\left(k_0+ix/2\alpha^2\right)^2}e^{-\alpha^2\left(k-\left(k_0+ix/2\alpha^2\right)\right)^2}$$

$$f(x)=\frac{A}{\sqrt{2\pi}}e^{-\alpha^2k_0^2}e^{\alpha^2\left(k_0+ix/2\alpha^2\right)^2}\int_{k=-\infty}^{+\infty}e^{-\alpha^2\left(k-\left(k_0+ix/2\alpha^2\right)\right)^2}dk$$

$$=\frac{A}{\sqrt{2\pi}}e^{-x^2/4\alpha^2}e^{ik_0x}\int_{z=-\infty}^{+\infty}e^{-\alpha^2z^2}dz$$

where $z=k-\left(k_0+\frac{ix}{2\alpha^2}\right)$. Since $\int_{z=-\infty}^{+\infty}e^{-\alpha^2z^2}dz=\frac{\pi^{1/2}}{\alpha}$, $f(x)=\frac{A}{\alpha\sqrt{2}}e^{-x^2/4\alpha^2}e^{ik_0x}$. The real part of $f(x)$, $\operatorname{Re}f(x)$ is $\operatorname{Re}f(x)=\frac{A}{\alpha\sqrt{2}}e^{-x^2 4\alpha^2}\cos k_0x$ and is a gaussian envelope multiplying a harmonic wave with wave number k_0. A plot of $\operatorname{Re}f(x)$ is shown below:



Comparing $\frac{A}{\alpha\sqrt{2}} e^{-x^2 4\alpha^2}$ to $Ae^{-(x/2\Delta x)^2}$ implies $\Delta x = \alpha$.

(c) By same reasoning because $\alpha^2 = \frac{1}{4\Delta k^2}$, $\Delta k = \frac{1}{2\alpha}$. Finally $\Delta x \Delta k = \alpha\left(\frac{1}{2\alpha}\right) = \frac{1}{2}$.