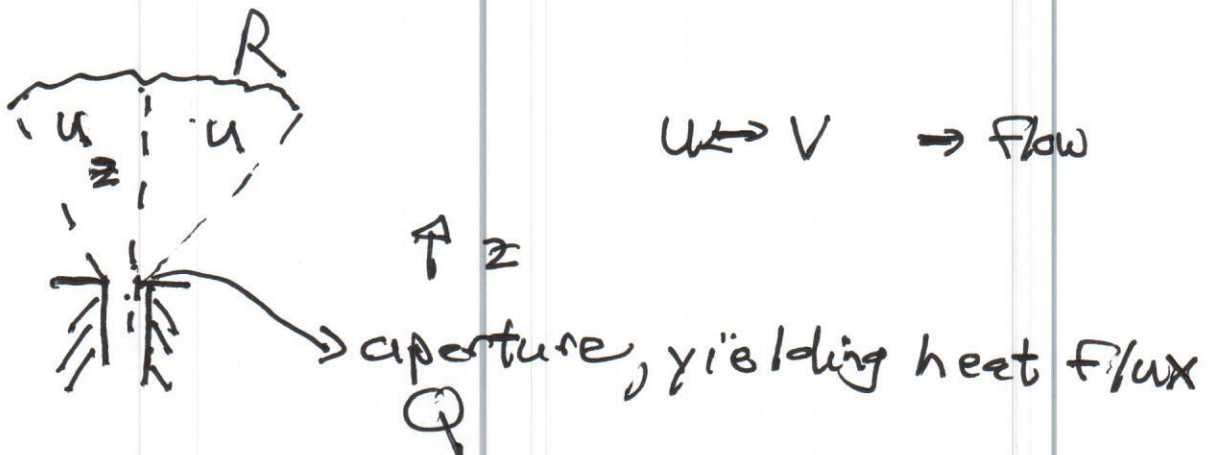


## Problem Set VI:

Due Tuesday, March 7

- 1) Tapas: Keep your answers here brief and focused.
  - a) Estimate the number of modes needed to simulate a high Re flow, assuming you resolve the dissipation scale (many simulations don't!).
  - b) Estimate the Nusselt number for turbulent boundary layer flow. What is the ratio of Nu to that for the corresponding laminar case?
  - c) Briefly describe the layer structure of turbulent flow over a rough surface with roughness scale  $y_0$ .
  - d) Does the continuum model of a fluid improve or decline in validity as Re increases? Hint: compare the mean free path to the dissipation scale. Take the flow subsonic.
  - e) Calculate the spectrum of Burgers turbulence, assuming the flow may be written as a superposition of uncorrelated shocks. Hint: This is a 'three liner'. Consider a convenient way to write the flow  $v$ .
- 2) Derive the coupled equations for the spectrum density  $|v_k|^2$  and response function for Burgers turbulence, which is randomly forced at large scale. The posted notes on closures and the posted articles by Kraichnan, and Forster et.al. may be useful. Your answer should include:
  - a) a spectral energy transfer equation for  $\partial_t |v_k|^2$ , including propagators and triad coherence time.
  - b) a response function, with amplitude dependent effective (turbulent) viscosity
  - c) a triad (3 mode) coherence time, which appears in the  $\partial_t |v_k|^2$  equation and which depends upon the turbulent viscosity.
- 3) Continuing with the system from Problem 2):
  - a) Show your result for the spectral energy equation conserves energy. Is this a discriminating test of the renormalized theory?
  - b) What physically, does the  $k$  dependency of the eddy viscosity mean?
  - c) What is the eddy viscosity at large scales? Give a simple derivation.
  - d) Are the behavior and dependency of the triad coherence time physically reasonable? How might you improve them?
  - e) If one introduces a coupling coefficient  $\lambda$  in the nonlinear term:  $v \partial_x v \rightarrow \lambda v \partial_x v$ , what does Galilean invariance imply about corrections to  $\lambda$ ?
- 4) Consider a plume of hot gas, rising as shown below.



The heat flux  $Q$  is carried by convection, and the plume spreads with height.

- a) Argue that:  $Q = \rho c_p \tilde{T} v R^2 \sim \text{constant}$ , and so determine the relation between  $\tilde{T}$ , height  $z$  and vertical velocity  $v$ .
  - b) Now use the balance of fluid inertia and buoyancy force for stationary convection to eliminate  $v(z)$  and calculate  $\tilde{T}(z)$ .  $\tilde{T}(z)$  should scale as  $\tilde{T}(z) \sim z^{-5/3}$ .
  - c) Calculate the scalings of  $v(z)$ .
  - d) Calculate  $Ra(z)$  and  $Re(z)$ , i.e.: Rayleigh and Reynolds number as functions of  $z$ . For what range of  $z$  is it valid to treat the plume as turbulent?
- 5) More Small Plates:
- a) Estimate the largest water droplet size that will survive suspended in a turbulent air flow of Reynolds number  $Re$  and integral scale  $L$ . Don't worry about gravity – i.e. assume the experiment is done in the space shuttle.
  - b) Estimate the temperature of a body of size  $R$  in a flow  $v_0$  with temperature  $T_0$ . Hint: consider viscous heating effects.
  - c) What are the limiting forms of the dependence of Nusselt number on Prandtl number in a laminar boundary layer when  $Re \gg 1$  and  $Pr \gg 1$ ?
- 6) This problem asks you to synthesize your recently acquired knowledge of fluids and turbulence with your well-developed understanding of waves and radiation so as to understand the acoustic emission of turbulence. [Ref: Landau and Lifshitz.]
- a) Calculate the pressure field radiated by a patch of turbulence by solving formally the inhomogeneous wave equation with source  $\rho \partial^2 T_{ik} / \partial x_1 \partial x_k$  where  $T_{ik} = v_i v_k$ . Show where this set up comes from.
  - b) Specialize your result to the far field, and use the retarded time dependence to relate space to time derivatives.
  - c) Decompose the pressure field into parts, related to:  $\ddot{T}_{ik} = \left( \ddot{T}_{ik} - \frac{1}{3} \ddot{T}_{ll} \delta_{ik} \right) + \frac{1}{3} \ddot{T}_{ll} \delta_{ik} = Q_{ik} + Q \delta_{ik}$ , and calculate the emitted intensity. What are the multipole moments of the acoustic radiation field?
  - d) Use the now-standard simplifying tricks to show that the acoustic energy emitted per mass and per time scales as  $\epsilon_s \sim v^8 / l c_s^5$ . From this estimate  $\epsilon_s / \epsilon$ . What can one deduce concerning the efficiency of acoustic emission from subsonic turbulence? Here  $\epsilon$  is the turbulent dissipation rate, as usual.
  - e) Briefly discuss how the story might change if one were considering convection driven turbulence. To do this, consider how buoyancy effects enter.
  - f) Extra Credit: Do the calculation implied in e).