and $b$ without going into the details of the determination of the path. [The following derivation was first given by M. M. Gorden, *American Journal of Physics*, 23, 247 (May, 1955).]

In Figure 4–7, $P_1$ is the initial momentum of the $\alpha$ and $P_2$ the final momentum. It is evident from the vector diagram that the total change in momentum $\Delta P = P_2 - P_1$ is along the $z'$ axis. The magnitude of $P_1$ and $P_2$ is $MV$. From the isosceles triangle
Rutherford scattering geometry. The nucleus is at O. The α particle has initial momentum $MV$ parallel to line COA and final momentum of the same magnitude (by conservation of energy) parallel to line OBA. The distance $b$ is called the impact parameter. The change in momentum is along the symmetry axis $y'$. The scattering angle $\theta$ can be related to the impact parameter by setting this change in momentum equal to the component of the impulse in the $y'$ direction $\Delta P = \int F \cos \phi \, dt$.

formed by $P_1$, $P_2$, and $\Delta P$, we find the magnitude of $\Delta P$ to be $\frac{1}{2} \Delta P/MV = \sin \frac{1}{2} \theta$, or $\Delta P = 2MV \sin \frac{1}{2} \theta$. We now write Newton's law for the α particle:

$$F = \frac{dP}{dt}$$

or

$$dP = F \, dt$$

The force $F$ is given by Coulomb's law, $Kq_\alpha Q/r^2$, and is in the radial direction. Taking components along the $y'$ axis and integrating, we have

$$\int (dP)_{y'} = \Delta P = \int F \cos \phi \, dt = \int F \cos \phi \frac{dt}{d\phi} \, d\phi \quad (4-5)$$

where we have changed the variable of integration from $t$ to $\phi$. We can write $dt/d\phi$ in terms of the angular momentum of the α about the origin. Since the force is central (i.e., it acts along the line joining the α and the origin), there is no torque about the origin, and the angular momentum is conserved. Initially, the angular momentum is $MVb$. At a later time, it is $Mr^2 \, d\phi/dt$. Thus conservation of angular momentum implies

$$Mr^2 \frac{d\phi}{dt} = MVb \quad (4-6)$$
Using Eq. (4-6) for $d\phi/dt$ in Eq. (4-5) and $Kq_{\alpha}Q/r^2$ for $F$, we have

$$\Delta P = \int \frac{Kq_{\alpha}Q}{r^2} \cos \phi \frac{r^2}{V_b} \, d\phi = \frac{Kq_{\alpha}Q}{V_b} \int \cos \phi \, d\phi$$

or

$$\Delta P = \frac{Kq_{\alpha}Q}{V_b} (\sin \phi_2 - \sin \phi_1)$$

where $\phi_1$ and $\phi_2$ are the initial and final values of $\phi$. From Figure 4-6 we see that $\phi_1 = -\phi_0$, $\phi_2 = +\phi_0$, where $2\phi_0 + \theta = 180^\circ$. Thus $\sin \phi_2 - \sin \phi_1 = 2 \sin (90 - \frac{1}{2}\theta) = 2 \cos \frac{1}{2}\theta$. Writing $\phi$ in terms of $\theta$ and using our previous result for the net momentum change, $\Delta P = 2MV \sin \frac{1}{2}\theta$, we have, finally,

$$2MV \sin \frac{1}{2}\theta = \frac{Kq_{\alpha}Q}{V_b} \cdot 2 \cos \frac{1}{2}\theta$$

or

$$b = \frac{Kq_{\alpha}Q}{MV^2} \cot \frac{1}{2}\theta \quad (4-7)$$

Of course, it is not possible to choose or to know the impact parameter for any $\alpha$ particle; however, all such particles with impact parameters less than, or equal to, a particular $b$ will be scattered through an angle $\theta$ greater than or equal to that given by Eq. (4-7). Let the intensity of the incident $\alpha$-particle beam be $I_0$ particles per

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**Figure 4-7** Momentum diagram for Rutherford scattering. The magnitude of the momentum change $\Delta P$ is related to the scattering angle $\theta$ by $\Delta P = 2MV \sin \frac{1}{2}\theta$. 

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$\Delta P$