Show all steps in your calculations. Justify all answers. Write clearly.

Some constants: \( \hbar c = 12,400 eV \text{A}, \quad k_B = 1/11,600 eV / K, \quad m_e c^2 = 511,000 eV \)

**Problem 1 (10 pts)**
In an ideal gas at temperature \( T \) the molecules of mass \( m \) are confined to move only in two dimensions, \( x \) and \( y \), with normalized velocity distribution

\[
F(v_x,v_y) = \left( \frac{m}{2\pi k_B T} \right)^{1/2} e^{-m(v_x^2 + v_y^2)/(2k_B T)}
\]

(a) Verify that this distribution is normalized, using that

\[
\int_{-\infty}^{\infty} dx e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}.
\]

(b) Find the normalized speed distribution \( g(v) \), where \( g(v)dv \) is the probability that a molecule has speed between \( v \) and \( v+dv \). Show explicitly that the \( g(v) \) that you find is normalized.

(c) The average speed \( \langle v \rangle \) is given by

\[
\langle v \rangle = \sqrt{\frac{k_B T}{m}}, \quad \text{where A is a number. Find A.}
\]

(d) The rms speed is given by \( v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{k_B T}{m}}, \quad \text{where B is a number. Find B,}
\]

using either \( F(v_x,v_y) \) or \( g(v) \).

**Hints:**

\[
\int_0^{\infty} dx x^n e^{-\lambda x^2} = -\frac{d}{d\lambda} \int_0^{\infty} dx x^n e^{-\lambda x^2}; \quad \int_0^{\infty} dx xe^{-\lambda x^2} = \frac{1}{2\lambda}
\]

**Problem 2 (10 pts)**
A black body at temperature \( T = 4.674 K \) emits 1\( \mu \)W of power in the wavelength range 620,000\( \text{A} \) to 620,001\( \text{A} \).

(a) What is the maximum power that this body emits within a wavelength range of 1\( \text{A} \)? For what wavelength is that?

(b) At which wavelength (approximately) larger than the one found in (a) does this body emit 16\( \mu \)W of power within a wavelength range of 1\( \text{A} \)?

(c) Find a wavelength that is smaller than the one found in (a) where the body emits less than 1\( \mu \)W of power in a wavelength range of 1\( \text{A} \) around that wavelength.

**Problem 3 (10 pts)**
In a Compton scattering experiment the maximum kinetic energy of the scattered electron is 248 eV.

(a) What is the wavelength of the incident photon, in \( \text{A} \)?

(b) What is the minimum kinetic energy of the scattered electron, in eV?

(c) When the scattered electron has kinetic energy 100 eV, at what angle is the photon scattered? Give your answer in degrees.

(d) For extra credit (3 pts): When the scattered electron has the largest possible momentum in direction perpendicular to the incident direction, at what angle is the photon scattered? Give your answer in degrees.