HW problem week 8

A 3d quantum harmonic oscillator is described by the potential

$$V(x) = \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2 + \frac{1}{2}m\omega_z^2 z^2.$$

1. In cartesian coordinates, what would be the first four energy eigenstates if all of the ω were equal? For unequal ω, what are the corresponding wave functions and energies?

2. In words (i.e., something other than $\omega_x = \omega_y$), what is the condition on the potential for L_z to be a sharp observable? Show by explicit calculation the the ground state is an eigenstate of $L_z = -\mathbf{i}\hbar\partial_\phi$ only if $\omega_x = \omega_y$.

3. If $\omega_x = \omega_y = \omega_z$, the potential has full spherical symmetry and can be solved in spherical polar coordinates. In this case, give the angular dependence of the ground state and the three first excited states. For each of these states, what is their total angular momentum, and what is their L_z?

4. Your answers for the first excited wave functions in part 1 were different than your answer in part 3. By taking linear combinations of the wave functions from part 1 you can put the wavefunctions into the form you found in part 3. Using the table on page 282 of the textbook, find explicitly the linear combinations which produce the three $l = 1$ states. What (up to some overall constant) are the functions $R_{2,l,m}$ for the harmonic potential?

5. For the spherically symmetric case, find the next state (above the ground state) which has zero total angular momentum. What is its energy?