HW problem week 2

Your turned in assignment should be clearly written and easy to follow! Learning how to explain your work in a way that is as easy as possible to follow is an important part of your training as a physicist. An incoherent mess of equations with a correct final answer could receive less points than a solution which is clearly explained at every step but has an algebra mistake somewhere. Once you’ve solved the problem, you can rewrite it on a new piece of paper for clarity if you need to.

Problem 0, ungraded  Redo the derivation of the number of modes of oscillation in a cavity for the situation where the boundary condition on the electric field is that the derivative vanishes at the boundary, rather than that the field vanishes on the boundary (this is what’s done in the supplementary pdf posted on the class website). Show that the conclusion about the number of modes in a range $d\lambda$ is unchanged.

Problem 1  A 2000 W incandescent bulb radiates from a tungsten filament. The distribution of energy radiated per wavelength has a maximum at $\lambda_{max} = 483$ nm.

1. By expressing Planck’s law in terms of frequency (see problem 3-14 in the book), and using that a photon of frequency $f$ has energy $hf$, obtain the distribution function $n(f)$ for number of photons with frequency between $f$ and $f + df$ emitted per second by the filament. You will need to use the analog of equation 3-7:

   $$ R(\lambda) = \frac{c}{4} u(\lambda) $$

   for $u$ as a function of $f$, and you will need the surface area of the filament.

2. Consider calculating the total number of photons emitted by the wire by integrating the distribution $n(f)$ over all frequencies.

   $$ N_{tot} = \int_{0}^{\infty} df \ n(f) $$

   By adimensionalizing, show that the total number of photons goes like $T^3$. (You don’t have to actually do the integral to compute $N_{tot}$, but if you want to, I recommend using mathematica or wolfram alpha to help).

3. How many photons are emitted between frequencies $4 \times 10^{14}$ and $4.01 \times 10^{14}$ Hz?
4. The peak of the energy distribution $u(f)$ occurs near $f \approx 2.82 \ k_b T/h$, but the peak of the number distribution $n(f)$ occurs near $f \approx 1.59 \ k_b T/h$. Why are these numbers different?