(1) Consider an ultrarelativistic ideal gas in three space dimensions. The dispersion is $\varepsilon(p) = pc$.

(a) Find $T$, $p$, and $\mu$ within the microcanonical ensemble (variables $S$, $V$, $N$).
(b) Find $F$, $S$, $p$, and $\mu$ within the ordinary canonical ensemble (variables $T$, $V$, $N$).
(c) Find $\Omega$, $S$, $p$, and $N$ within the grand canonical ensemble (variables $T$, $V$, $\mu$).
(d) Find $G$, $S$, $V$, and $\mu$ within the Gibbs ensemble (variables $T$, $p$, $N$).
(e) Find $H$, $T$, $V$, and $\mu$ within the S-p-N ensemble. Here $H = E + pV$ is the enthalpy.

(2) Consider a system composed of spin tetramers, each of which is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_1\sigma_4 + \sigma_2\sigma_3 + \sigma_2\sigma_4 + \sigma_3\sigma_4) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4).$$

The individual tetramers are otherwise noninteracting.

(a) Find the single tetramer partition function $\zeta$.
(b) Find the magnetization per tetramer $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \rangle$.
(c) Suppose the tetramer number density is $n_t$. The magnetization density is $M = n_t m$. Find the zero field susceptibility $\chi(T) = (\partial M/\partial H)_{H=0}$.

(3) For an ideal gas, find the difference $C_\varphi - C_V$ for the following functions $\varphi$. You are to assume $N$ is fixed in each case.

(a) $\varphi(p, V) = p^3 V^2$
(b) $\varphi(p, T) = p e^{T/T_0}$
(c) $\varphi(T, V) = VT^{-1}$

(4) Find an expression for the energy density $\varepsilon = E/V$ for a system obeying the Dieterici equation of state,

$$p(V - Nb) = Nk_B T e^{-Na/V} k_B T,$$

where $a$ and $b$ are constants. Your expression for $\varepsilon(v, T)$ should involve an integral which can be expressed in terms of the exponential integral,

$$Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt.$$