PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #1

(1) Compute the information entropy in the Fall 2012 Physics 140A grade distribution. See http://www-physics.ucsd.edu/students/courses/fall2012/physics140a/index.html.

(2) Study carefully problem #11 from the worked examples to chapter 1 of the lecture notes. Suppose I have three bags. Initially, bag #1 contains a quarter, bag #2 contains a dime, and bag #3 contains two nickels. At each time step, I choose two bags randomly and randomly exchange one coin from each bag. The time evolution satisfies $P_i(t+1) = \sum_j Y_{ij}\, P_j(t)$, where $Y_{ij} = P(i\,,\,t+1\,|\,j\,,\,t)$ is the conditional probability that the system is in state i at time $t+1$ given that it was in state j at time t.

(a) How many configurations are there for this system?

(b) Construct the transition matrix Y_{ij} and verify that $\sum_i Y_{ij} = 1$.

(c) Find the eigenvalues of Y (you may want to use something like `Mathematica`).

(d) Find the equilibrium distribution $P_i^{\rm eq}$.

(3) A system consists of N 'molecules'. Each molecule consists of four 'spins': σ, μ_1, μ_2, and μ_3, where each spin polarization can takes values ± 1. The molecular Hamiltonian is

$$\hat{h} = J\sigma(\mu_1 + \mu_2 + \mu_3) - H\big(3\sigma - \mu_1 - \mu_2 - \mu_3\big) \ .$$

(a) Enumerate all the molecular energy states along with their degeneracies.

(b) Find the molecular partition function $\zeta(T, H)$.

(c) Compute the magnetic susceptibility $\chi(T, H = 0)$.

(4) Consider a system of identical but distinguishable particles, each of which has a non-degenerate ground state with $\varepsilon_0 = 0$, and a $g-$fold degenerate excited state with energy $\varepsilon > 0$. Study carefully problems #1 and #2 from the worked example problems for chapter 4 of the lecture notes, where this system is treated in the microcanonical and ordinary canonical ensembles. Here you are invited to work out the results for the grand canonical ensemble.

(a) Find the grand partition function $\Xi(T, z)$ and the grand potential $\Omega(T, z)$. Express your answers in terms of the temperature T and the fugacity $z = e^{\mu/k_{\rm B}T}$.

(b) Find the entropy $S(T, z)$.

(c) Find the number of particles, $N(T, z)$.

(d) Show how, in the thermodynamic limit, the entropy agrees with the results from the microcanonical and ordinary canonical ensembles.