Problem 1: AH 4.1
(a) Base centered cubic

YES BL: simple tetragonal

\[ \vec{a}_1 = \frac{a}{2} (x + \frac{1}{2}) \]
\[ \vec{a}_2 = \frac{a}{2} (x - \frac{1}{2}) \]
\[ \vec{a}_3 = a \hat{z} \]

(b) Side-centered cubic

Note a Bravais lattice

BL is simple cubic

Basis:

\[ \vec{d}_1 = 0 \]
\[ \vec{d}_2 = \frac{a}{2} (x + \frac{1}{2}) \]
\[ \vec{d}_3 = \frac{a}{2} (\frac{1}{2} + \frac{1}{2}) \]

(c) Edge-centered cubic

Not a BL

BL is simple cubic

Basis:

\[ \vec{d}_1 = 0 \]
\[ \vec{d}_2 = \frac{a}{2} \hat{x} \]
\[ \vec{d}_3 = \frac{a}{2} \hat{y} \]
\[ \vec{d}_4 = \frac{a}{2} \hat{z} \]
Problem 2: ANY.5

(a) 
A point \( x \) is equidistant from the vertices of the triangle.

Distance to a vertex \( = d \)

\[
\cos 30^\circ = \frac{a}{2d} = \frac{\sqrt{3}}{2} \implies d = \frac{a}{\sqrt{3}}
\]

The point \( x \) in the direction \( \vec{c}/2 \) from \( x \) and distance \( a \) from the vertices:

\[
\left(\frac{c}{2}\right)^2 + d^2 = a^2 = \frac{c^2}{4} + \frac{a^2}{3} = a^2 \implies \frac{c^2}{4} = \frac{2}{3} a^2 = \sqrt{\frac{8}{3}} a
\]

(b) For hexagonal lattice

\( \vec{a}_1 = a \hat{x} \), \( \vec{a}_2 = \frac{a}{2} \hat{x} + \frac{\sqrt{3}}{2} \hat{y} \), \( \vec{a}_3 = c \hat{z} \)

Volume of unit cell: \( V = (\vec{a}_1 \times \vec{a}_2) \cdot \vec{a}_3 = \frac{\sqrt{3}}{2} a^2 c = \sqrt{2} a^3 \frac{c}{2} \)

There are 2 atoms in unit cell \( N = \frac{2}{\sqrt{2} a^3} \)

For bcc, with spacci \( a' = 4.23 \), has 2 atoms in unit cell \( N = \frac{2}{a'^3} = \frac{2}{4.23^3} \)

\[ a = \frac{1}{\sqrt[6]{2}} a' = 3.77 \]
Problem 3: AH 4.6

Let \( d \) be nearest neighbor distance, then radius of sphere \( R = \frac{d}{2} \). Let \( N_e \) be number of atoms in conventional cubic unit cell. The packing fraction \( \rho \)

\[
\rho = \frac{\frac{4}{3} \pi R^3 N_e}{a^3} = \frac{1}{6} \frac{\pi d^3}{a^3} N_e
\]

<table>
<thead>
<tr>
<th>Structure</th>
<th>( d )</th>
<th>( N_e )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>fcc</td>
<td>( a^{2}/\sqrt{2} )</td>
<td>4</td>
<td>( \frac{\pi}{3\sqrt{2}} ) = 0.740</td>
</tr>
<tr>
<td>bcc</td>
<td>( \frac{\sqrt{3}}{2} a )</td>
<td>2</td>
<td>( \frac{\sqrt{3}\pi}{8} ) = 0.680</td>
</tr>
<tr>
<td>sc</td>
<td>( a )</td>
<td>1</td>
<td>( \frac{\pi}{6} ) = 0.524</td>
</tr>
<tr>
<td>diamond</td>
<td>( \frac{\sqrt{3}}{4} )</td>
<td>8</td>
<td>( \frac{\sqrt{3}\pi}{16} ) = 0.340</td>
</tr>
</tbody>
</table>
Problem 4:
Consider a rotation axis O. We can assume there exists a lattice plane perpendicular to O if \( n \geq 3 \) because: simply find a plane \( \perp O \) that contains a lattice point that is not on the axis, by rotating \( n \) times, we generate \( n \) non-collinear points equal a plane. (\( n \)-fold)

Consider then the \( \perp O \) perpendicular to the paper, and let \( P \) be a point \( P \) that is closer to \( O \) than any other point on the paper.

![Diagram]

Let \( a = \text{distance } OP \). Let \( \xi = 2\pi/n \)

1) By rotating around \( O \) with angle \( \xi \), \( P \) goes into \( P' \), another lattice point.
2) Translate \( O \) by vector \( PP' \) to get point \( O' \). \( O' \) is also an \( n \)-fold rotation axis.
3) Rotate around \( O' \) by angle \( -\xi \) and bring \( P' \) to a point \( P'' \).
4) Use geometry to calculate the distance \( PP'' \). You should find \( PP'' = 2a(1 - \cos \xi) \)
5) Because we assumed \( P \) is closest to \( O \) than any other point, we must have either:
   (i) \( PP'' = a \), \( \Rightarrow \xi = 60^\circ \) or \( n = 6 \) (here, \( P'' = O \))
   (ii) \( PP'' > 2a \), \( \Rightarrow \xi > 60^\circ \), \( n = 4 \) or \( n = 3 \)

Finally, \( n = 2 \) is clearly possible. So the above shows that \( n = 5 \) and \( n > 7 \) not.