Problem 1

\[ \Phi_1 = \frac{1}{\sqrt{3}} S + \sqrt{\frac{2}{3}} P_x \]
\[ \Phi_2 = \frac{1}{\sqrt{3}} S - \frac{1}{\sqrt{6}} P_x + \frac{1}{\sqrt{2}} P_y \]
\[ \Phi_3 = \frac{1}{\sqrt{3}} S - \frac{1}{\sqrt{6}} P_x - \frac{1}{\sqrt{2}} P_y \]
\[ \Phi_4 = P_z \]

1) check that they are perpendicular. Obviously, \( \langle \Phi_i | \Phi_1 \rangle = 0 \) for \( i = 1, 2, 3 \)
\[ \langle \Phi_1 | \Phi_2 \rangle = \frac{1}{3} - \sqrt{\frac{2}{18}} = 0 \]
\[ \langle \Phi_1 | \Phi_3 \rangle = \frac{1}{3} + \frac{1}{6} - \frac{1}{2} = 0 \]
\[ \langle \Phi_1 | \Phi_4 \rangle = \frac{1}{3} - \sqrt{\frac{2}{18}} = 0 \]

2) check that \( \Phi_1, \Phi_2, \Phi_3 \) are in the \( x-y \) plane and at 120°

\( \Phi_1 \) points along the \( x \) axis.

\( \Phi_2 \) points along the \( y \) axis with \( \tan \alpha = -\frac{1}{\sqrt{6}} \), \( \theta = 120° \)

\( \Phi_3 \) points along the \( z \) axis with \( \tan \alpha = \frac{1}{\sqrt{2}} \), \( \theta = 240° \)

Problem 2

Superposition of single and double bonds \( \implies \) length is in between

\[ \begin{array}{c}
\text{Single bond} \\
\text{Double bond}
\end{array} \]
Problem 3

\[
\frac{d \vec{u}}{dt} = -\frac{e}{m} \vec{E} \Rightarrow \vec{u} = \vec{u}_0 - \frac{e}{m} t \vec{E}
\]

Gain in line of energy:

\[
\Delta E = \frac{1}{2} m (u^2 - u_0^2) = \frac{1}{2} m \left( -\frac{2 e t \vec{u}_0 \cdot \vec{E} + \frac{e^2 t^2 \vec{E}^2}{m^2} \right)
\]

On the average, \(<\vec{u}_0 \cdot \vec{E}> = 0 \Rightarrow (a) \quad \Delta E = \frac{(e E t)^2}{2m} \quad \text{energy lost in collisions at time t}

(b) Probability that collision occurs at time \( t + dt \) in interval 1)

\[
P(+) dt = \frac{dt}{6} e^{-t/6}
\]

The mean time between collisions is \(<+> = \int_0 \int d t + P(+) = \frac{1}{6}

The average energy lost in a collision

\[
<\Delta E> = \frac{e^2 E^2}{2m} <t^2> = \frac{e^2 E^2}{2m} \int_0^{t^2} d t + P(+) = \frac{e^2 E^2}{2m} . 2 \frac{1}{6}^2
\]

so the average energy lost per unit time is \( \frac{<\Delta E>}{6} = \frac{e^2 E^2}{m} \)

If there are \( N \) collisions per unit volume, energy transferred per unit volume per unit time is \( N \cdot \frac{e^2 E^2}{m} \) = \( \sigma \cdot E^2 \)

In a wire of cross section \( A \), length \( L \), energy lost per unit time \( \rho \text{ per cm} = \)

\[
\rho = \sigma \cdot E^2 \cdot A \cdot L = \sigma \frac{A}{L} \cdot (E L)^2 = \frac{V^2}{R} \text{ with } R = \frac{\rho L}{A} \text{, } S = \frac{1}{S}
\]

\[V = E \cdot L \text{. Now } J = \sigma E \Rightarrow J = JA = \sigma \frac{A}{L} (E L) = \frac{V}{R} \Rightarrow \]

\[
= 1 \rho = J^2 R
\]
Problem 4

\[ J_x = n_i e_i u_{ix} + n_2 e_2 u_{2x} = \sum n_i e_i^2 \frac{\xi}{m_i} E_x = \sum n_i \frac{le_i}{m_i} \mu_i E_x \]

\[ U_{ix} = \frac{e_i \xi}{m_i} E_x \quad \mu_i = \frac{le_i}{m_i} \xi \]

\[ J_y = n_i e_i u_{iy} + n_2 e_2 u_{2y} = \sum n_i e_i u_{iy} \]

Face in y direction: \[ F_{iy} = e_i E_y + e_i \frac{U_{ix}}{c} \]

\[ U_{iy} = F_{iy} / m_i = e_i \xi E_y + e_i \frac{U_{ix}}{m_i} \frac{1}{c} \]

\[ J_y = \sum \left( n_i e_i \frac{\xi}{m_i} E_y + n_i e_i \frac{\xi}{m_i} \frac{e_i}{m_i} \frac{U_{ix}}{c} \right) \rightarrow \]

\[ J_y = \sum \left( n_i \frac{le_i}{m_i} \mu_i E_y + n_i e_i \frac{\mu_i^2}{c} \frac{E_x H}{c} \right) \]

\[ J_y = 0 \Rightarrow E_y = -\frac{E_x H}{c} \frac{n_i e_i \mu_i^2 + n_2 e_2 \mu_2^2}{n_i le_i \mu_i + n_2 le_2 \mu_2} \]

and \[ J_x = \left( n_i le_i \mu_i + n_2 le_2 \mu_2 \right) E_x \rightarrow \]

\[ \frac{R_H}{J_x H} = \frac{E_y}{J_x H} = -\frac{1}{c} \frac{n_i e_i \mu_i^2 + n_2 e_2 \mu_2^2}{(n_i le_i \mu_i + n_2 le_2 \mu_2)^2} \]
(b) \[ R_H = -\frac{1}{\varepsilon} \frac{n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2}{(n_1 e_1 \mu_1 + n_2 e_2 \mu_2)^2} \]

If \( n_2 \neq 0 \), \( n_2 \mu_2 \neq 0 \), \( n_2 e_2 \neq 0 \)

\[ R_H = -\frac{1}{\varepsilon} \frac{n_1 e_1 \mu_1^2}{(n_1 e_1 \mu_1)^2} = -\frac{1}{n_1 e_1 \varepsilon} \]

as in 1 band case

(c) \( R_H = 0 \) \( \Rightarrow \) \( n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2 = 0 \)

so the signs of \( e_1 \) and \( e_2 \) have to be opposite.

Assume \( e_1 = -e \) (electrons), \( e_2 = e \) (holes)

\[ R_H = 0 \) \( \Rightarrow \) \( n_1 \mu_1^2 = n_2 \mu_2^2 \)

This is called a "compensated metal"
(d) The paper says that the density of carriers experiences a fourfold enhancement. This is deduced from the formula

\[ \mathcal{R}_H = - \frac{1}{N_1 \varepsilon C} \]


decrease in the magnitude of \( \mathcal{R}_H \) by a factor of 4.

Assume instead a situation with 2 carriers with charge \(-e\) and \(+e\), and in simplicity \( \mu_1 = \mu_2 \). Then,

\[ \mathcal{R}_H = - \frac{1}{\varepsilon C} \frac{N_1 - N_2}{(N_1 + N_2)^2} \]

If the carrier concentration is changing so that \( N_1 \) and \( N_2 \) become close, i.e. \( \mathcal{R}_H \to 0 \), \( \mathcal{R}_H \) can decrease by a factor 4 with a much smaller change in carrier concentration than a factor of 4, which is more plausible.

For example, \( N_2 = 1 \), \( N_1 \) changing from 1.05 to 1.01 will give a change in \( \mathcal{R}_H \) of a factor 4.8, with a change in carrier concentration of 2% instead of 400%.