PHYS 273, Winter 2016, Homework 3

Due date: Thursday, February 18th, 2016

1. *Capacity of the carrier pigeon channel.* Consider a commander of an army besieged in a fort for whom the only means of communication to his allies is a set of carrier pigeons. Assume that each carrier pigeon can carry one letter (8 bits), that pigeons are released once every 5 minutes, and that each pigeon takes exactly 3 minutes to reach its destination.

a. Assuming that all the pigeons reach safely, what is the capacity of this link in bits/hour?

b. Now assume that the enemies try to shoot down the pigeons and that they manage to hit a fraction α of them. Since the pigeons are sent at a constant rate, the receiver knows when the pigeons are missing. What is the capacity of this link?

c. Now assume that the enemy is more cunning and that every time they shoot down a pigeon, they send out a dummy pigeon carrying a random letter (chosen uniformly from all 8-bit letters). What is the capacity of this link in bits/hour?

Set up an appropriate model for the channel in each of the above cases, and indicate how to go about finding the capacity.

2. *Information and motor control.* An important task in motor control is tracking. When objects move smoothly across our visual field, we track their motion by moving our eyes, which is called smooth pursuit.

a. The simplest version of the tracking problem is that we have an observable x, which is generated according to a probability distribution $P(x)$ and we try to generate a variable y that is as close as possible to x, in the sense of the mean-square error $\epsilon = \langle (y-x)^2 \rangle$. Develop a variational principle for the choice of $P(y|x)$, in which you aim to find the minimum amount of mutual information $I(x,y)$ needed to reach a certain value of ϵ. (**Hint:** You may try the method of Lagrange multipliers. Ensure that all constraints (a continuum of them) are taken into account.)

b. Solve the problem formulated in (a), i.e. find an expression for $P(y|x)$ and the derive the corresponding consistency conditions.

c. Assume that $P(x)$ is Gaussian. Solve the consistency conditions and plot the rate distortion curve $I(x,y)$ vs ϵ.

3. *Finite time resolution.* Suppose that signals experience a delay as they are detected and this delay fluctuates. The output $x(t)$ is then related to the signal $s(t)$ as $x(t) = s(t-\tau(t))$, where $\tau(t)$ is the fluctuating delay.

a. Assume that fluctuations are small and slow that this is equivalent to $x(t) = s(t - \bar{\tau}) + \eta(t)$, where $\bar{\tau}$ is the average delay and η is noise. Find the noise correlation function in terms of the delay correlation.

b. Fluctuations in the decay are slow so that they extend over times much longer than those of the s correlation. Derive the corresponding expression of correlation and power spectrum of the noise.

c. Show that the signal-to-noise ratio falls at high frequencies, no matter what the spectrum of the input signal is.