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Self-Organized Criticality and Earthquakes.

A. SORNETTE and D. SORNETTE

Laboratoire de Physique de la Matière Condensée, CNRS UA190
Faculté des Sciences, Parc Valrose, 06034 Nice Cedex, France

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Abstract. – We suggest that the concept of self-organized criticality (SOC) is relevant for understanding the processes underlying earthquakes. Earthquakes are an important part of the relaxation mechanism of the crust which is submitted to inhomogeneous increasing stresses accumulating at continental-plate borders. The SOC concept then implies that earthquakes in turn organize the crust both at the spatial and temporal levels. This idea allows to rationalize observations on occurrences and magnitudes of earthquakes. Variants of SOC as well as a novel type of dynamics based on «waiting times» are discussed within a mean-field-like approach which shows the existence of $1/f$ noise in the time gap between large earthquakes. The corresponding long-time correlations have important implications for the statistical long-time forecasting of earthquakes.

1. General ideas.

In a recent series of papers, Bak, Tang and Wiesenfeld [1] proposed that a spatio-temporal nonlinear dynamical system, with quasi-static incoming and outcoming fluxes localized, for instance, at the borders, evolve spontaneously towards a stationary self-organized critical state (SOCS) with no length or time scales others than those deduced from the size of the system and that of the elementary cell. They argue that the SOCS is general and robust and might be the underlying mechanism for the apparition of $1/f$ noise in many different systems (1).

In this letter, we suggest that the concept of self-organized criticality is particularly well suited for rationalizing observations on occurrences and magnitudes of earthquakes. In this goal, we explore the idea that, if earthquakes are the natural consequences of the stationary dynamical state of the crust submitted to steady increasing stresses, they also organize the crust in a self-consistent way. We consider mainly the continental collision situation where stresses are created at the border of continental plates and diffuse slowly inland, according to current ideas on «indentation» [2].

(1) Such as sandpiles, hourglasses, light from quasar, motion of dislocations in resistors, charge density wave systems, sliding vortex lattices in magnetic fields, glassy systems, turbulence, water flow, traffic, interactive economical systems.

According to current plate tectonics [2], it is believed that continental-plate motions, induced by mantlewide convection, create on-going «plate-collisions» and thus stresses which accumulate at the borders of the plates. This can be viewed as an incoming stress flux at the border of the system (we consider here a single continental plate). When the land plate, which is stressed, sheared and/or compressed and/or pulled up or down, is deformed so strongly that the deformation exceeds a certain limit (which may change from place to place), rupture occurs [3, 4] and an earthquake (avalanche in the terminology of [1]) follows. In this case, the land plate rebounds because it becomes free and the accumulated strain energy is radiated in the form of seismic waves (which are the deadly agents for humanity).

Since earthquakes are highly complex events involving significant irregularities of geometry as well as stresses and strengths of materials, the common wisdom is to assume that the complexity and scaling of earthquakes stem from the built-in complexity of the crust. For instance, a given distribution of faults [5] or of stresses [6] is assumed from the start. This common procedure is often justified from *a priori* empirical observations or *a posteriori* verification of its predictions. In this way, several attempts have recently been made for predicting the cumulative frequency-magnitude statistics from an assumed self-similar distribution of stresses and material strengths [6]. Also, the inter-earthquake time distribution has been analysed on the assumption that the stresses evolve with time according to a random walk [7]. In short, power laws are used to derive the existence of other power laws!

In contrast, we make here an attempt to unravel the basic phenomena at the origin of the power laws found in earthquakes. Our basic assumption is that *not only the earthquake features are a consequence of the organization of the crust and its array of faults, but also the large-scale structure of the crust emerges from the whole history of previous earthquakes, which have organized the crust* and deteriorated it according to the current state of observation. In other words, the complexity and scaling of the crust indeed controls that of the earthquakes but also *results* from the large time scale dynamics of the crust submitted to the incoming stress or elastic potential energy flux. We thus view earthquakes as being part of the overall large-scale organization of the crust spontaneously evolving towards a SOCS. The different inhomogeneities are then believed to be irrelevant [1, 8] relatively to the global dynamics (but of course not for the local behaviour).

Here, our goal is not to discuss in details models which can be defined as variants of the paradigm of SOC introduced in [1]. Inspired by the earthquake problem, one can, for instance, consider models taking into account the local redistribution of stresses in the crust which are either conservative or dissipative and depending on space and time .... Also, the role of the avalanches (earthquakes) of [1] can be taken to redistribute the accumulated stress conservatively and (or) to dissipate and relax it. The analogy between SOC and earthquakes thus suggests many interesting variations on the theme! The investigation of some of these variants will be presented in a separate communication [8]. For our purpose, we keep the essential ingredients believed to control the occurrence of SOC, namely the existence of a spatio-temporal nonlinear dynamical system, with on the average stationary incoming and outcoming fluxes spatially localized. We now make use of the scaling laws which are consequences of the SOC hypothesis and which are relevant for the earthquakes and analyse the different consequences of our basic assumption.

2. The frequency-magnitude distribution.

The number N of earthquakes of magnitude M can be expressed by the quasi-universal empirical Gutemberg-Richter law [2, 9] valid over a broad range of magnitudes (typically

$2 \leqslant M \leqslant 8$)

$$\log N = a - bM, \tag{1}$$

where a and b are two constants which may depend a little upon the particular region of observation. b is remarkably universal ($0.8 \leqslant b \leqslant 1.1$) for all earthquakes with $2 \leqslant M \leqslant 6.5$ anywhere in the world [2]! The exponent b seems however to depend a little upon the focal depth. For larger magnitudes, important fluctuations and deviations from this law are observed [10, 11].

In the following, we focus our attention to the case $2 \leqslant M \leqslant 6.5$ for which relation (1) is well defined. This leads to the existence of a quasi-universal power law, since the earthquake magnitude M is related to the energy released at the earthquake source by a logarithmic relationship [2]

$$\log E \approx c + dM, \tag{2}$$

where $c \approx 11$ and $d \approx 1$ for $M \leqslant 7$ (and $d \approx 1.5$ for $M \geqslant 7$) [12], with E expressed in ergs (2). Thus, the two eqs. (1) and (2) imply the following scaling law relating the number N of earthquakes of energy E:

$$N \sim E^{-\tau+1} \tag{3}$$

with $\tau - 1 = b/d$ leading to an observed value $\tau \approx 2$.

Such a scaling law which has no general explanation is indeed expected within the SOCS hypothesis. A plausible link between «avalanches» and earthquakes consists in making the size s of an avalanche of [1] correspond to the energy E released by the earthquakes. It is interesting to note that the numerically determined value for τ in the model of [1] is $\tau \approx 2$ in two dimensions and $\tau \approx 2.3$ in three dimensions which are close to the observed value. However, τ seems to depend upon the precise boundary condition involved in the model [8] thus preventing a really accurate prediction.

3. A mean-field theory of the return period of the same type of earthquake.

In the following, we restrict ourselves to the case $\tau \geqslant 2$ corresponding to a finite integrated number of earthquakes $\int_0^\infty N(E')\,\mathrm{d}E' < +\infty$. In the case where $\tau < 2$, one has to introduce an upper energy cut-off in the power law (3) and with this addition the analysis carried below can be reproduced step by step.

How long must one wait in order to observe an earthquake of energy E after a first earthquake of the same magnitude? In a mean-field spirit, we propose to forget the spatio-temporal complexity of the SOC problem and focus on a global variable, namely the energy E released by an earthquake occurring at time t anywhere within the plate. This is of course a terrible approximation which, however, allows one to develop a basis for more elaborate theories which could take fully account of spatio-temporal fluctuations. By using the single condition of stationarity, we derive below the mean-field value $\tau_{\mathrm{MF}} = 3$.

(2) The seismic moment is often considered as the best parameter for measuring the size of an earthquake corresponding to a given fault motion. Since the seismic moment is proportional to the released energy, expression (2) is unchanged when relating M to S apart from a different value for c.

In this goal, let us now introduce a «gap» dynamics which is different from the «duration» dynamics discussed in [1]. We assume that, on the average, the increase of energy in the system is made step by step, say one step ε_0 per unit time t_0. During a time t, there will be an increase of stored energy of order $(t/t_0)\,\varepsilon_0$. Since earthquakes have a typical duration (minutes) much shorter than the time gap (months or years) between two earthquakes, they can thus be considered as instantaneous. Since $N(E) \sim E^{-\tau+1}$ is the probability of observing an earthquake of energy E, $\int_{E_{\max}}^{\infty} N(E')\,\mathrm{d}E'$ is the probability that an earthquake has an energy larger than $E_{\max}$. During a time t, suppose there are a total of $n(t)$ earthquakes of arbitrary energy. Then, the condition $n(t)\int_{E_{\max}}^{\infty} N(E')\,\mathrm{d}E' \sim 1$ means that at most one earthquake of energy of order $E_{\max}(t)$ has occurred in the time t. With the power law (3), we obtain

$$n(t) \sim E_{\max}(t)^{\tau-2}. \tag{4}$$

Equation (4) gives the value of the typical largest earthquake observed after a waiting time t: $E_{\max}(t) \sim n(t)^{1/(\tau-2)}$. We can also derive the probability $P(t)$ that all earthquakes occurring between time 0 and t have an energy release less than E:

$$P(t, E) \sim \exp\left[-n(t)\int_{E}^{\infty} N(E')\,\mathrm{d}E'\right] \sim \exp\left[-Cn(t)\,E^{-(\tau-2)}\right], \tag{5}$$

where C is a constant. Knowing $n(t)$, the corresponding «waiting time» or return period as a function of magnitude is deduced from a condition of conservation on the average energy flux $J(t)$ defined by

$$J(t) \sim \{n(t)/t\}\,\langle E\rangle(t). \tag{6}$$

The average earthquake energy released over a time t is defined by

$$\langle E\rangle(t) = \int_{1}^{E_{\max}(t)} EN(E)\,\mathrm{d}E. \tag{7}$$

We have taken as a unit of energy the energy scale ε_0. Equation (6) means that the total energy released during a time t, which is $J\cdot t$, scales as the number of events multiplied by the average energy $\langle E\rangle(t)$ released per earthquake. Due to the long tail (3), $\langle E\rangle$ depends on time according to

$$\langle E\rangle(t) \sim n(t)^{(3-\tau)/(\tau-2)} \tag{8}$$

Now, from our general hypothesis, the system is at a global stationary stable SOCS and therefore the energy flux entering the system must be on the average dissipated by the earthquakes which implies the condition of stationarity $J \sim$ const. From eq. (6) with (7), this leads to the scaling law for $n(t)$:

$$n(t) \sim t^{\tau-2}. \tag{9}$$

The condition of stationarity also implies that $n(t)$ must be proportional to t since the average number of earthquakes $n(t)/t$ per unit time is constant. This imposes the mean-field value for τ in our «gap model»: $\tau_{\mathrm{MF}} = 3$. It is different from the exponent obtained numerically or from geological observations which takes account of spatio-temporal fluctuations. Note that it implies $\langle E \rangle(t) = \text{const}$ which is also consistent with the stationary condition.

The condition $n(t) \sim t$ with eq. (4) yields

$$E_{\max} \sim t. \tag{10}$$

This divergence of $E_{\max}(t)$ at long times is bound to saturate due to finite-size effects which act as cut-offs. Therefore, one must consider eq. (10) and the other scaling laws valid only in a certain range of values constrained by the finite size of the system. The existence of a finite incoming stress flux also acts as a finite-size cut-off (see the discussion of ref. [1]).

Equation (10) allows the prediction of the typical return period of earthquakes of a given magnitude. With eq. (2), it gives

$$t = t_0 \exp[dM], \tag{11}$$

where $d \approx 1$ for $M \leqslant 7$ and 1.5 for $M \geqslant 7$. To understand the meaning of this result, suppose that earthquakes of $M_1 = 3$ occur frequently, say every month on the average, in a given region of the world: $t_{M_1} = 1$ month. Then, the typical return time of an earthquake of magnitude $M_2 = 7$ will be $t_{M_2} = \exp[1.5M_2 - M_1]\, t_{M_1} \approx 1800$ months $= 150$ years.

This result also enables to rationalize the experimental observation on the relative occurrence of earthquakes with $6 \leqslant M \leqslant 7$ with respect to earthquakes with $M \geqslant 7$. In [13], about 10 earthquakes of $6 \leqslant M \leqslant 7$ occur per decade (with large fluctuations) compared to the order of one per decade for earthquakes with $M \geqslant 7$. With eq. (11), we predict a ratio of the order of 5 which is in reasonable agreement with the observation.

Note, however, that the comparison with observation is difficult due to insufficient sampling and the existence of large fluctuations.

4. $1/f$ noise in the return events.

Our mean-field approach leads directly to the prediction of $1/f$ noise in the return events. Let us come back to eq. (5) and obtain from it the probability, $P(t, E_{\max}(t)) \sim \text{const}$, that all earthquakes during a period of time t have an energy release less than $E_{\max}(t)$. The probability that no earthquake of energy $E_{\max}(t)$ occurs between time 0 and t and that one earthquake of energy $E_{\max}(t)$ occurs between time t and $t + \mathrm{d}t$ is thus

$$p(t) \sim P(t, E_{\max}(t)) \cdot \left\{ \int_{E_{\max}}^{\infty} N(E')\,\mathrm{d}E' \right\} \frac{\mathrm{d}n(t)}{\mathrm{d}t} \sim Ct^{-1}. \tag{12}$$

This power law is characteristic of $1/f$ noise [14, 15], since it is invariant under a rescaling $t \to \lambda t$ and $\mathrm{d}t \to \lambda\,\mathrm{d}t$. This means that the probability of observing an earthquake of magnitude $M \sim d^{-1} \log(t/t_0)$ between one day and two days is the same as, say, that for observing an earthquake of magnitude $M \sim d^{-1} \log(365t/t_0)$ between one year and two years (we have used eq. (2) relating M to E) and so on. This law (13) compares well with empirical observations [3] which suggest the scaling law $p(t) = Ct^{-m}$ with $m \approx 1$. Once C and m are known, the mean return period and its standard deviation can be readily calculated [3].

Note that long periods of observation are needed to predict the maximum magnitude and its return period. Any form of regression analysis applied to general $1/f$ processes in general and earthquake prediction in particular would have to estimate not just the average value of a parameter (magnitude) over the entire extent of data, but also its value averaged over one unit of time, 10 units, 100 units, etc., extending from the shortest to the longest times of interest [15]. In practice, the situation is slightly bettere since $1/f^{\alpha}$ noise with $\alpha > 1$ is expected on the basis of the existence of spatio-temporal fluctuations which change the exponent τ from 3 to 2. As in [1], pure $1/f$ noise corresponds to the mean-field approximation.

5. Conclusion.

The hypothesis of SOC for earthquakes leads to a power law for the temporal fluctuations for earthquake occurrence which rationalize many observations. We thus believe that our picture may constitute a general and powerful framework for the spatio-temporal description of the crust at large scales and times, and for the long-term prediction of earthquakes. We have presented a first description based on mean-field–like ideas of the earthquake return time statistics. However, spatio-temporal fluctuations are important and renormalize the exponent. This should be included in future more sophisticated theories. We hope that the present work will nucleate interest in the physical community for this challenging problem.

The general ideas which have been presented in this work would also be relevant for other problems such as the average seasonal temperature, annual amount of rainfall, rate of traffic flow, etc. In these systems, there is also a competition between an average incoming stationary flux with inhomogeneous boundary conditions which is released by sudden and sometimes catastrophic events. With the appropriate translation, the preceding discussion of the size-frequency relationship and on the return time of extreme events should apply.

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