1. Suppose a flat matter dominated Universe for this problem (makes calculations easier).
(a) What was the size of the particle horizon at $z = 1100$ when the CMB decoupled and last scattered? (give answer in Mpc). This is the maximum size of a region at that time that could be in causal contact and therefore in thermal equilibrium.
(b) Using the angular diameter distance, what is the angular size on the sky today of such a causally connected region? (give answer in both radians and degrees)
(c) How many such causally connected patches are there on the sky today?
(d) Consider CMB photons coming from the opposite sides of the sky. Using the above results explain why they cannot be causally connected and therefore cannot have been in thermal equilibrium at the time the CMB photons left.
(e) Given the above, speculate why CMB photons coming from opposite sides of the sky are measured to be at the same temperature!

2. What was the density of (a) baryons, (b) dark matter, (c) radiation, and (d) vacuum energy at the time of the CMB last scattering? [Hint, you might want to look up Ω_{DM} in our text book.]

3. Muons have masses of 106 MeV ($c = 1$). They stay in equilibrium via reactions such as $\mu^+\mu^- \leftrightarrow e^+e^-$.
(a) At roughly what temperature did the bulk of the muons "annihilate" into electrons and positrons, etc? Give the answer in MeV (very easy), and degrees Kelvin.
(b) At what time did this occur (in seconds after the big bang)? [Hint: approximate the Universe as flat, matter dominated from radiation-matter equality until today. Earlier than radiation-matter equality assume the Universe is flat, radiation dominated. We did this is class.]
(c) What was the density of the Universe at this time? (give answer in gm/cm^3)
(d) What was the size of the Universe at that time (i.e. the size of the particle horizon)? (give answer in kilometers)
(e) How much mass was inside the horizon at that time? (give answer in grams and in solar masses.)