CHAPTER 39

RADIATION III

The Half-Wave Antenna, Antenna Arrays, and the Magnetic Dipole Antenna

The half-wave antenna is a long, straight conductor, one-half wavelength long, that carries a standing wave of current. Its radiation pattern is similar to that of an electric dipole. However, for a given current, it radiates much more energy. This is the building block for assembling arrays of antennas. We deduce its field from that of an electric dipole.

The directivity of a half-wave antenna is hardly better than that of an electric dipole. However, arrays of such antennas, with the proper spacings and the proper phases, can be highly directive. Some arrays comprise a few antennas, but others comprise thousands.

We also calculate $\boldsymbol{E}$ and $\boldsymbol{B}$ in the field of a magnetic dipole, and we discuss briefly electric and magnetic dipoles as receiving antennas.

This chapter ends our study of electromagnetic fields and waves. Obviously we have not exhausted the subject! Indeed, we have done no more than establish a base from which you can explore on your own.

39.1 RADIATION FROM A HALF-WAVE ANTENNA

Figure 39-1 shows a half-wave antenna connected to a transmitter through a parallel-wire line. The half-wave antenna is essentially a pair of wires, each $\lambda/4$ long, fed with a current $I_m \cos \omega t$ at the junction. Here λ is the wavelength of a uniform plane wave in the medium of propagation.

At short wavelengths one can fold back a length $\lambda/4$ of the outer conductor of a coaxial line, as in Fig. 38-6 to obtain a half-wave antenna.

Roof antennas for automobiles are only one-quarter wavelength long; the other half is a reflection in the sheet metal of the roof. Transmitting antennas for AM waves are similarly $\lambda_0/4$ towers standing on conducting ground.

The antenna carries a standing wave of current, with a maximum at the center and nodes at the end. The current at l is thus

$$I = I_m \cos \frac{l}{\lambda} \exp j\omega t. \tag{39-1}$$

Each element of length dl radiates as an electric dipole.

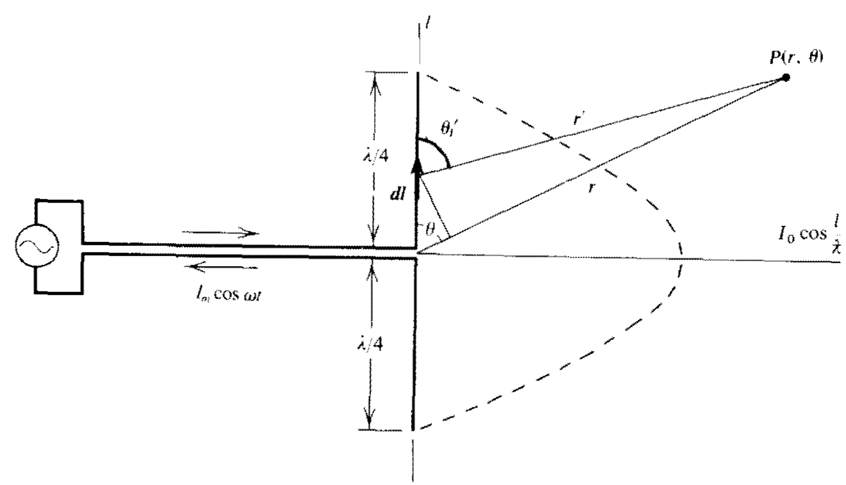


Fig. 39-1. Half-wave antenna. The broken line shows the standing wave of current at $\cos \omega t = 1$.

This description of the half-wave antenna is contradictory because the standing wave along the conductor can be truly sinusoidal only if there is zero energy loss, hence no radiation. In a real antenna the current distribution is not quite sinusoidal, but the distortion hardly affects the field.

The standing wave of current is the sum of two waves, one in the positive direction of l and the other in the negative direction, each of amplitude $I_m/2$:†

$$I = \frac{I_m}{2}\left\{\exp j\left(\omega t - \frac{l}{\lambda\!\!\!^{-}}\right) + \exp j\left(\omega t + \frac{l}{\lambda\!\!\!^{-}}\right)\right\}. \tag{39-2}$$

39.1.1 The Electric Field Strength *E*

We set $r \gg \lambda\!\!\!^{-}$. Then $\theta' \approx \theta$, Eq. 38-16 applies, and

$$\boldsymbol{dE} = -\frac{[dp]}{4\pi\epsilon_0 \lambda\!\!\!^{-2} r'}\sin\theta\,\hat{\boldsymbol{\theta}} = -\frac{\omega^2[dp]}{4\pi\epsilon_0 c^2 r'}\sin\theta\,\hat{\boldsymbol{\theta}} \tag{39-3}$$

$$= \frac{\mu_0 j\omega[I]\,dl}{4\pi r'}\sin\theta\,\hat{\boldsymbol{\theta}}, \tag{39-4}$$

where

$$r' \approx r - l\cos\theta, \tag{39-5}$$

as in Fig. 39-1, or

$$\boldsymbol{dE} = \frac{\mu_0 j\omega I_m}{8\pi r'}\left\{\exp j\left(\omega[t'] - \frac{l}{\lambda\!\!\!^{-}}\right) + \exp j\left(\omega[t'] + \frac{l}{\lambda\!\!\!^{-}}\right)\right\}\sin\theta\,dl\,\hat{\boldsymbol{\theta}}. \tag{39-6}$$

We now integrate over the length of the antenna to find $\boldsymbol{E}$ at r, θ. We can replace the r' in the denominator by r since $r \gg \lambda\!\!\!^{-}$, hence $r \gg l$. However, we must *not* replace the r' by r in

$$[t'] = t - \frac{r'}{c} \tag{39-7}$$

because the phases of the exponential terms vary rapidly with r'. So we set

$$[t'] \approx t - \frac{r - l\cos\theta}{c} = [t] + \frac{l\cos\theta}{c}. \tag{39-8}$$

† As in the previous two chapters, we reserve brackets for quantities evaluated at $t - r/c$.

At a given point in space, the $\boldsymbol{dE}$'s thus all have about the same amplitude and direction, but their phases differ. All these $\boldsymbol{dE}$'s point in the direction of the local unit vector $\hat{\boldsymbol{\theta}}$. Then

$$\boldsymbol{E} = \frac{\mu_0 j\omega I_m}{8\pi r}\sin\theta\exp j\omega[t]\int_{-\lambda/4}^{+\lambda/4}\left\{\exp j\frac{l(\cos\theta - 1)}{\lambda\!\!\!^{-}} + \exp j\frac{l(\cos\theta + 1)}{\lambda\!\!\!^{-}}\right\}dl\,\hat{\boldsymbol{\theta}}. \tag{39-9}$$

Integrating yields

$$\boldsymbol{E} = \frac{jI_m}{4\pi c\epsilon_0 r}\sin\theta\exp j\omega[t]\left(\frac{\sin\{\pi(\cos\theta - 1)/2\}}{\cos\theta - 1} + \frac{\sin\{\pi(\cos\theta + 1)/2\}}{\cos\theta + 1}\right\}\hat{\boldsymbol{\theta}}, \tag{39-10}$$

where

$$\sin\frac{\pi(\cos\theta - 1)}{2} = -\cos\left(\frac{\pi}{2}\cos\theta\right), \qquad \sin\frac{\pi(\cos\theta + 1)}{2} = +\cos\left(\frac{\pi}{2}\cos\theta\right). \tag{39-11}$$

Thus

$$\boldsymbol{E} = \frac{j}{2\pi c\epsilon_0 r}\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta}[I]\hat{\boldsymbol{\theta}} \approx 60.0j\frac{\cos\{(\pi/2)\cos\theta\}}{r\sin\theta}[I]\hat{\boldsymbol{\theta}}. \tag{39-12}$$

This expression is indeterminate at $\theta = 0$ and at $\theta = \pi$. But, according to L'Hospital's rule, the limiting value of such a ratio is equal to the limiting value of the ratio of the derivatives. So $\boldsymbol{E}$ is zero on the axis of a half-wave antenna, in agreement with the fact that the elementary dipoles do not radiate along the axis.

Why should the magnitude of $\boldsymbol{E}$ be independent of the frequency? The explanation is that the $\boldsymbol{E}$ of an elementary dipole, for a given current, is proportional to $1/\lambda\!\!\!^{-}$, but the antenna is $\lambda/2$ long.

Figure 39-2 shows that the radiation pattern for a half-wave antenna is similar to that of a dipole. This is because the phase differences between the $\boldsymbol{dE}$'s from the elements of current along the antenna are small near $\theta = \pi/2$, where the $\boldsymbol{dE}$'s are large, and are large only near the polar axis where the $\boldsymbol{dE}$'s tend to zero.

39.1.2 The Magnetic Field Strength *H*

The value of $\boldsymbol{H}$ follows immediately. We found in Sec. 38.1.2 that, for the electric dipole, $\boldsymbol{H}$ is azimuthal and

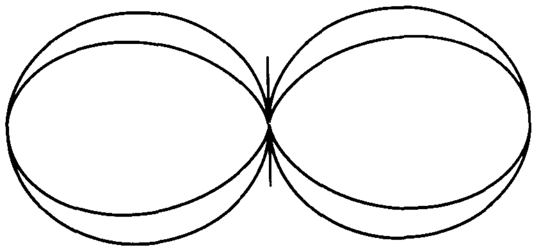


Fig. 39-2. Polar diagrams of the functions $\cos\{(\pi/2)\cos\theta\}$ (outer pair of curves) and of its square (inner pair). On the outer pair, the distance from the origin to a point on a curve is proportional to the magnitude of E, or of H at a fixed distance from a half-wave antenna, in that direction. On the inner pair, this distance is proportional to $\mathscr{S}_{av}$. Compared to the electric dipole, the half-wave antenna radiates a somewhat larger fraction of its power in the region of the equatorial plane.

$$\frac{E}{H} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \approx 377 \text{ ohms} \qquad (r \gg \lambda\!\!\!^-). \tag{39-13}$$

Therefore, in the field of a half-wave antenna,

$$\boldsymbol{H} = \frac{j}{2\pi r}\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta}[I]\hat{\boldsymbol{\phi}}. \tag{39-14}$$

39.1.3 The Poynting Vector $\boldsymbol{E}\times\boldsymbol{H}$

The time-averaged Poynting vector is

$$\mathscr{S}_{av} = \tfrac{1}{2}\operatorname{Re}(\boldsymbol{E}\times\boldsymbol{H}^*) \tag{39-15}$$

$$= \frac{1}{\pi c\epsilon_0}\frac{\cos^2\{(\pi/2)\cos\theta\}}{\sin^2\theta}\frac{I_{rms}^2}{4\pi r^2}\hat{\boldsymbol{r}} \tag{39-16}$$

$$= 9.543\frac{\cos^2\{(\pi/2)\cos\theta\}}{\sin^2\theta}\frac{I_{rms}^2}{r^2}\hat{\boldsymbol{r}} \qquad \text{watts/meter}^2. \tag{39-17}$$

See Fig. 39-2.

39.1.4 The Radiated Power P and the Radiation Resistance

To obtain the radiated power, we integrate over a sphere of radius r:

$$P = \frac{I_{rms}^2}{4\pi^2 c\epsilon_0 r^2}2\pi\int_0^\pi \frac{\cos^2\{(\pi/2)\cos\theta\}}{\sin^2\theta}r^2\sin\theta\,d\theta. \tag{39-18}$$

The integral is equal to 1.2188267, and

$$P = 73.083 I_{rms}^2 \approx 73 I_{rms}^2 \qquad \text{watts}. \tag{39-19}$$

The radiation resistance of a half-wave antenna is about 73 ohms.

39.2 ANTENNA ARRAYS

The electric dipole and the half-wave antenna are omnidirectional in the equatorial plane: at a given distance, the amplitude of the field is the same in all directions in that plane. Omnidirectional antennas have their uses, but for most applications the radiation field of an antenna should be maximum in a given direction. This is achieved with arrays of half-wave antennas that are properly spaced and properly phased.

Linear arrays comprise several parallel half-wave antennas disposed along a straight line. *Planar arrays* operating at wavelengths of the order of 1 centimeter comprise many more, often thousands, disposed over a rectangular or circular plane surface. Usually the individual antennas are identical, equally spaced, and oriented similarly. Beam steering and pattern control are achieved nearly instantaneously by means of phase shifters next to each element.

The radiation patterns of arrays are typically like the one shown in Fig. 39-3, with one main lobe and several smaller side lobes.

An *adaptive receiving array* adjusts its pattern automatically to optimize the signal-to-noise ratio in the presence of identifiable noise sources.

We illustrate the principle involved in antenna arrays by calculating the field of two half-wave antennas spaced by $\lambda/2$, first when they are in phase and then when they are in opposite phases.

Example

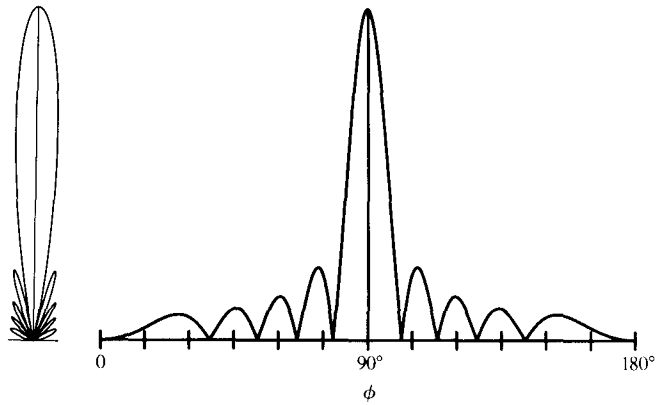
PAIR OF PARALLEL ANTENNAS SEPARATED BY ONE-HALF WAVELENGTH

Figure 39-4 shows a pair of parallel half-wave antennas separated by a distance $\lambda/2$. We assume that $r \gg \lambda\!\!\!^-$.

The antennas are in phase

If the antennas are in phase, then $\boldsymbol{E}$ at point P is the sum of two terms like that of Eq. 39-12, except that one wave travels a distance $r + (\lambda/4)\cos\psi$ and the other a distance $r - (\lambda/4)\cos\psi$. Therefore one wave leads, relative to an imaginary antenna at the center, by the phase angle

$$\frac{2\pi}{\lambda}\frac{\lambda\cos\psi}{4} = \frac{\pi}{2}\cos\psi, \tag{39-20}$$



(a) (b)

Fig. 39-3. The radiation pattern of an antenna is a plot of E as a function of θ. This is the radiation pattern of a 10-element linear array of in-phase half-wave antennas. (a) Polar diagram. (b) Cartesian diagram.

the other lags by the same amount, and

$$\boldsymbol{E} = 60j\frac{\cos\{(\pi/2)\cos\theta\}}{r\sin\theta}\left\{\exp\left(j\frac{\pi}{2}\cos\psi\right) + \exp\left(-j\frac{\pi}{2}\cos\psi\right)\right\}[I]\hat{\boldsymbol{\theta}} \tag{39-21}$$

$$= 120j\frac{\cos\{(\pi/2)\cos\theta\}}{r\sin\theta}\cos\left(\frac{\pi}{2}\cos\psi\right)[I]\hat{\boldsymbol{\theta}}. \tag{39-22}$$

The angle ψ is awkward to use, but we can express it in terms of θ and ϕ, since

$$r\cos\psi = r\sin\theta\cos\phi. \tag{39-23}$$

Then

$$\boldsymbol{E} \approx 120j\frac{\cos\{(\pi/2)\cos\theta\}}{r\sin\theta}\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)[I]\hat{\boldsymbol{\theta}}. \tag{39-24}$$

In the xy-plane, $\theta = \pi/2$ and

$$E \propto \cos\left(\frac{\pi}{2}\cos\phi\right). \tag{39-25}$$

This function is zero at ϕ equal to 0 or π, and maximum at

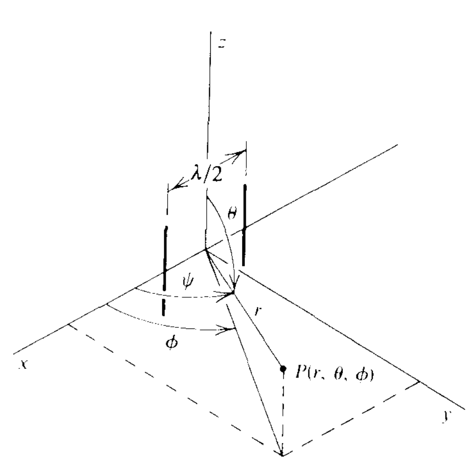


Fig. 39-4. Pair of parallel half-wave antennas separated by a distance of $\lambda/2$. The distances from the centers of the antennas to P are approximately $r - (\lambda/4)\cos\psi$ and $r + (\lambda/4)\cos\psi$.

$\phi = \pi/2$: there is destructive interference along the x-axis and constructive interference along the y-axis.

In the xz-plane, $\phi = 0$ and

$$E \propto \frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta}\cos\left(\frac{\pi}{2}\sin\theta\right). \tag{39-26}$$

The first term on the right is the angular distribution for a single half-wave antenna; it is zero at $\theta = 0$ and maximum at $\theta = \pi/2$. The second term comes from the interference between the two antennas; it is maximum at $\theta = 0$ and zero at $\theta = \pi/2$. The product of the two is zero both at $\theta = 0$ and at $\theta = \pi/2$.

Finally, in the yz-plane, $\phi = \pi/2$ and

$$E \propto \frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta}, \tag{39-27}$$

as for a single half-wave antenna. The two waves are in phase, and the total field is twice that of a single antenna when $r \gg \bar{\lambda}$.

Figure 39-5 shows the radiation pattern.

The antennas are in opposite phases

The antenna at $x = \lambda/4$ now leads by π. Equation 39-21 applies, except that the first term between the pair of braces on the right is negative and

$$\boldsymbol{E} = 120\frac{\cos\{(\pi/2)\cos\theta\}}{r\sin\theta}\sin\left(\frac{\pi}{2}\sin\theta\cos\phi\right)[I]\hat{\boldsymbol{\theta}}. \tag{39-28}$$

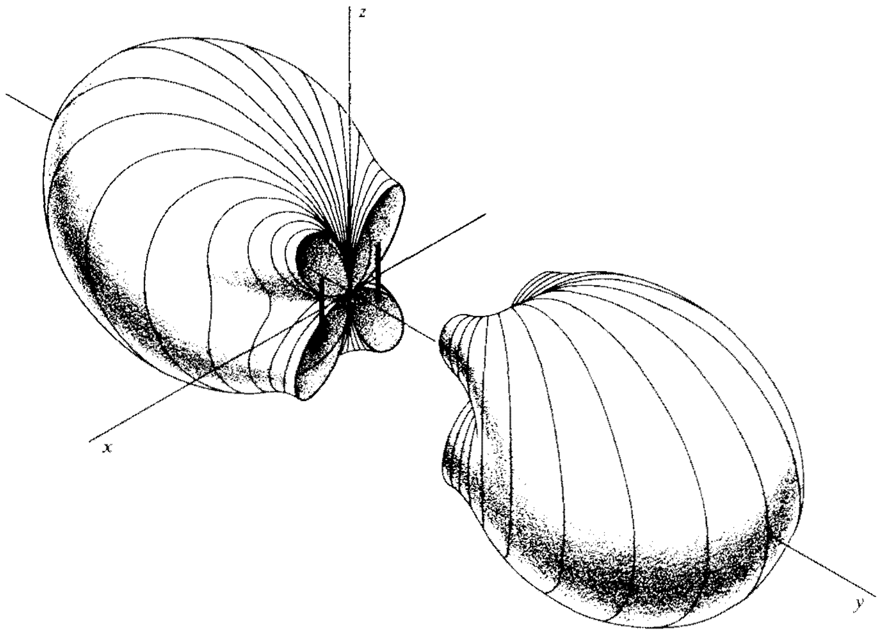


Fig. 39-5. The radiation pattern for the simple antenna array of Fig. 39-4 when the two antennas are excited in phase and for $r \gg \lambdabar$. Here we have plotted the magnitude of E, or of H, radially as a function of θ and of ϕ. We have split the surface into two parts for clarity. In the yz-plane, the field is twice that of a single antenna. Along the x-axis the waves arrive in opposite phases, for $r \gg \lambdabar$, and cancel. There is zero field on the z-axis, again for $r \gg \lambdabar$.

The radiation pattern is now that of Fig. 39-6.

These simple arrays are only slightly more directional than a single half-wave antenna.

Clearly, one can obtain a wide range of radiation patterns by varying either the geometry of an antenna array or the phases of the individual antennas, or both. The main beam sharpens as the size of the array increases.

39.3 MAGNETIC DIPOLE RADIATION

Figure 39-7 shows a magnetic dipole that is similar to that of Fig. 37-4. As in that section, we set

$$a^3 \ll r^3 \qquad \text{and} \qquad a^2 \ll 2\lambdabar^2. \tag{39-29}$$

We already know that

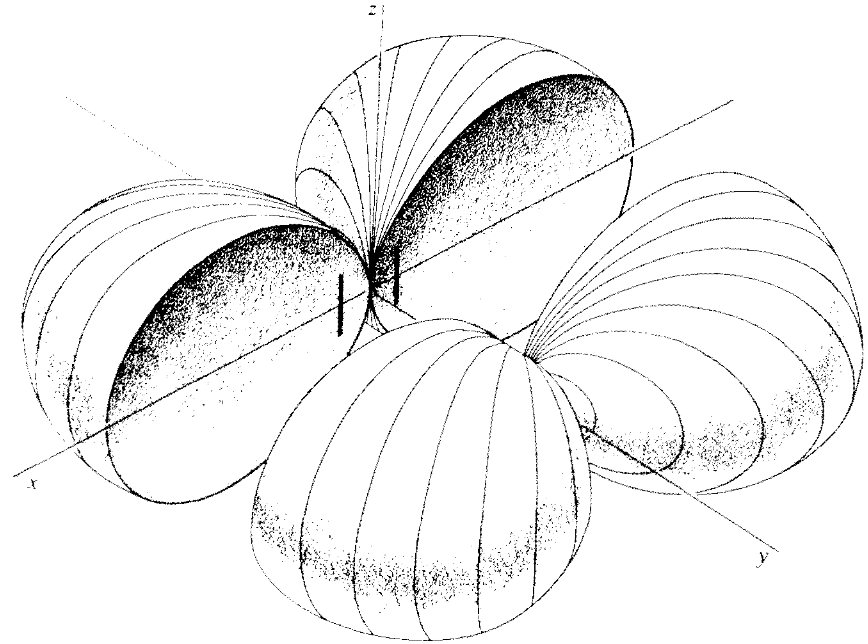


Fig. 39-6. The radiation pattern for the array of Fig. 39-4 with the antennas excited in opposite phases.

$$V = 0, \qquad \mathbf{A} = j\frac{\mu_0[m]}{4\pi\lambdabar r}\left(1 - j\frac{\lambdabar}{r}\right)\sin\theta\,\hat{\boldsymbol{\phi}}, \tag{39-30}$$

from the second example in Sec. 37.4.

39.3.1 The Electric Field Strength $\mathbf{E}$

Since $V = 0$, and since $\omega = c/\lambdabar$,

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -j\omega\mathbf{A} = \frac{\mu_0 c[m]}{4\pi\lambdabar^2 r}\left(1 - j\frac{\lambdabar}{r}\right)\sin\theta\,\hat{\boldsymbol{\phi}}, \tag{39-31}$$

where $\mu_0 c \approx 377$ ohms. Thus $\mathbf{E}$ is azimuthal.

At zero frequency, λ is infinite and $\mathbf{E}$ is zero, as expected.

For $r \gg \lambdabar$,

$$\mathbf{E} = \frac{\mu_0 c[m]}{4\pi\lambdabar^2 r}\sin\theta\,\hat{\boldsymbol{\phi}} \qquad (r \gg \lambdabar). \tag{39-32}$$

Observe that $\mathbf{E}$ is proportional to the time derivative of $\mathbf{A}$, hence to the time derivative of the current, and thus to the azimuthal acceleration