Physics 4A
Lecture 6: Jan. 29, 2015

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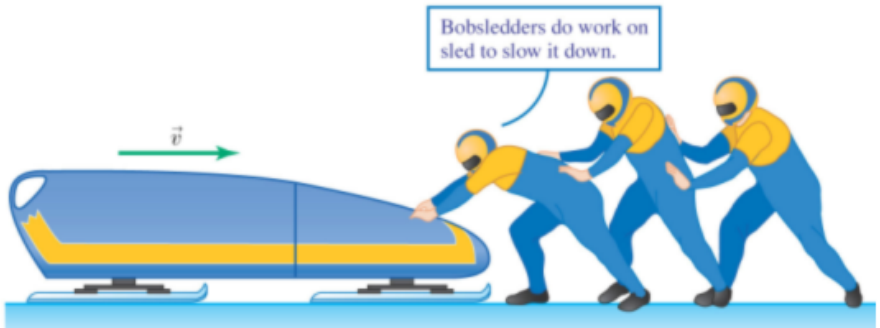
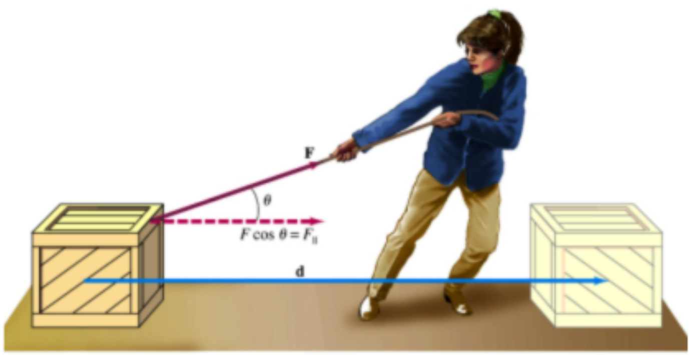
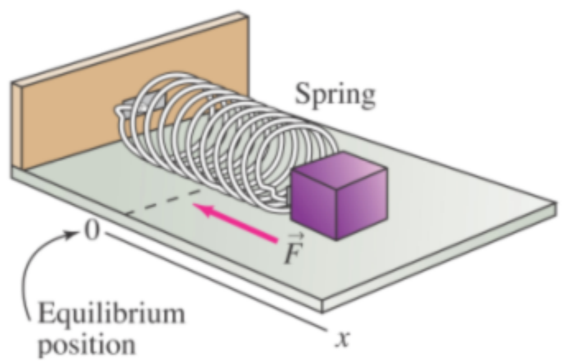
UCSD Physics

WORK and ENERGY

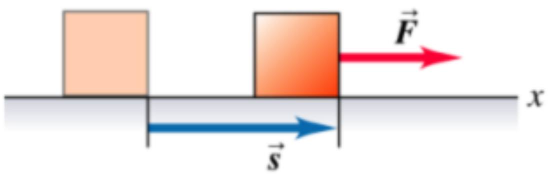






Work is done when a force is applied on a body

Work Done By a Constant Force



When a constant force $\vec{F}$ acts in same direction as displacement $\vec{s}$, work done by force W=Fs

SI Unit of Work: 1 Joule = (1N) (1m) = 1N.m

When a constant force $\vec{F}$ acts at angle φ to the displacement $\vec{s}$

work done $W = \vec{F} . \vec{s}$

Work Done is a Scalar Quantity

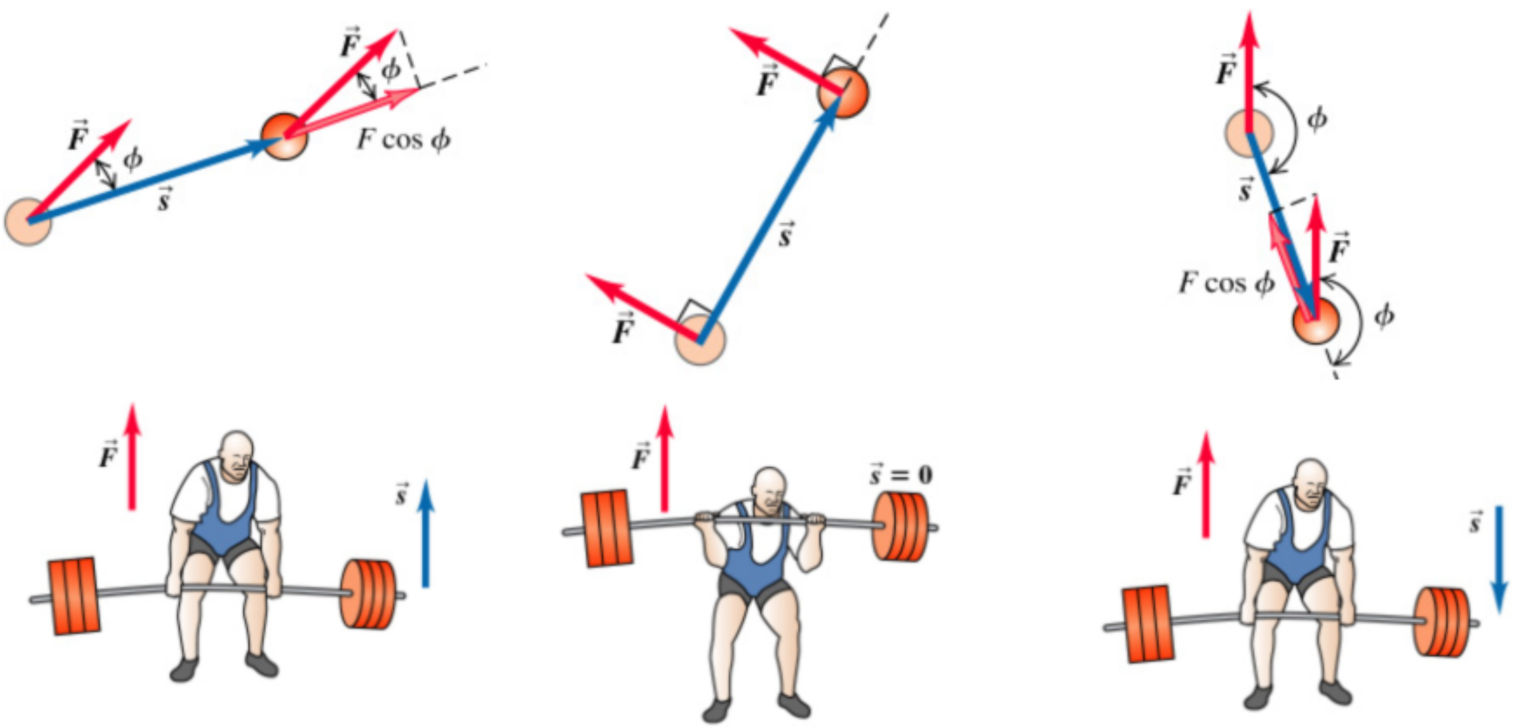
Three Shades of Work

$W = \vec{F}.\vec{s} = |F||s|\cos\varphi \Rightarrow$ Relative direction between $\vec{F}$ & $\vec{s}$ Important

Force has component in direction of displacement: Work done is positive

Force is perpendicular to displacement: Work done is zero

Force has component opposite to displacement: Work done is negative

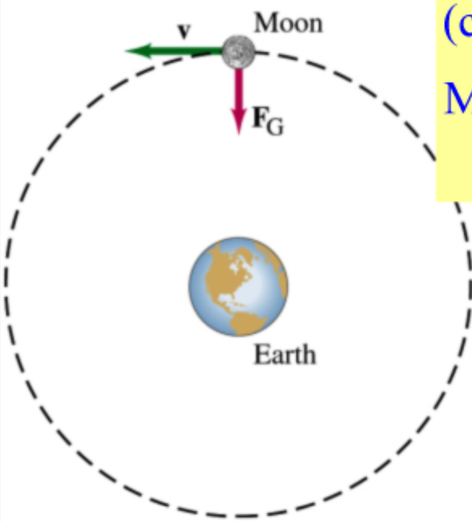


Quick Question

- A man is twirling a stone tied to a string in a circle around his head of radius R. The tension in the string is 80 N. The work done by the man in one revolution is:
- (A) $2\pi R$. 80
- (B) R.80
- (C) Zero
- (D) mv^2/R where v is the stone's speed

Does Earth Do Work On The Moon ?

Moon revolves around earth in nearly circular orbit, kept there by the attractive force exerted by earth. Does Earth do +,0,- work ?



Gravitational force $\vec{F}$ on moon acts towards earth

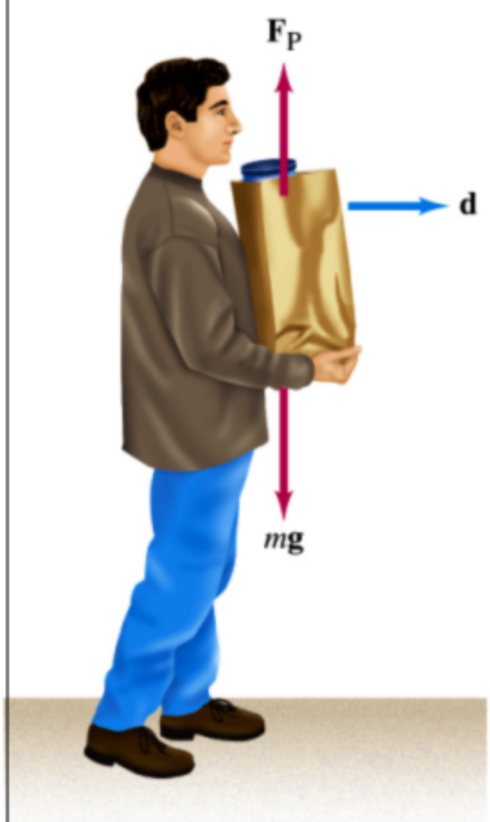
(centripetal force) inward along radius of moon's orbit

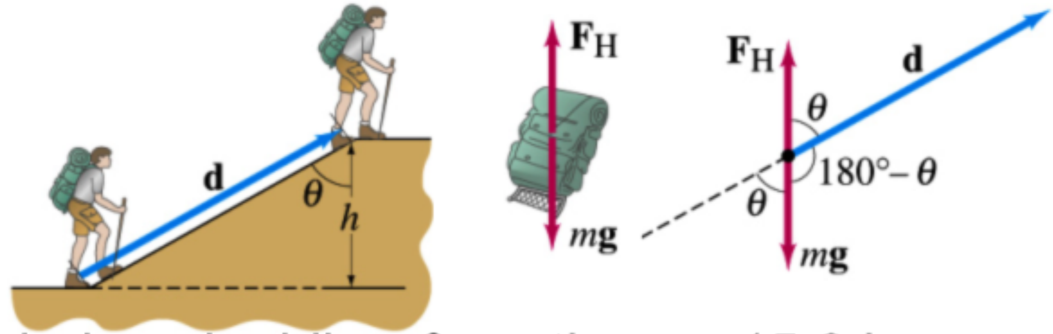
Moon's displacement $\vec{s}$ always along a circle & $\perp \vec{F}$

$$W = \vec{F} . \vec{s}$$

$\Rightarrow W_{earth} = 0$. This is why moon and artificial satellites can stay in orbit without expenditure of fuel, since no work is done against gravity

Doing Work (or Not)





Work done by hiker & earth on a 15.0 kg backpack carried up a height of 10.0m?

Draw freebody diag for Backpack & use 2nd law ⇒

$\sum \vec{F}_y = 0 = F_H - m_B g \Rightarrow F_H = 147N$

Work done by hiker on backpack:

$W_H = \vec{F}.\vec{d} = F$dcosè $= Fh = mgh = 1470J$

Earth pulls on backpack: $\vec{F}_E = -mg$

Object displaced at angle $\alpha = 180^o -$ è

Work done by Earth on backpack:

$W_H = \vec{F}.\vec{d} = Fd\cos\alpha = Fd\cos(180 - \theta) = -1470J$

No work is done on Groceries since $\vec{F}_P \perp \vec{d}$

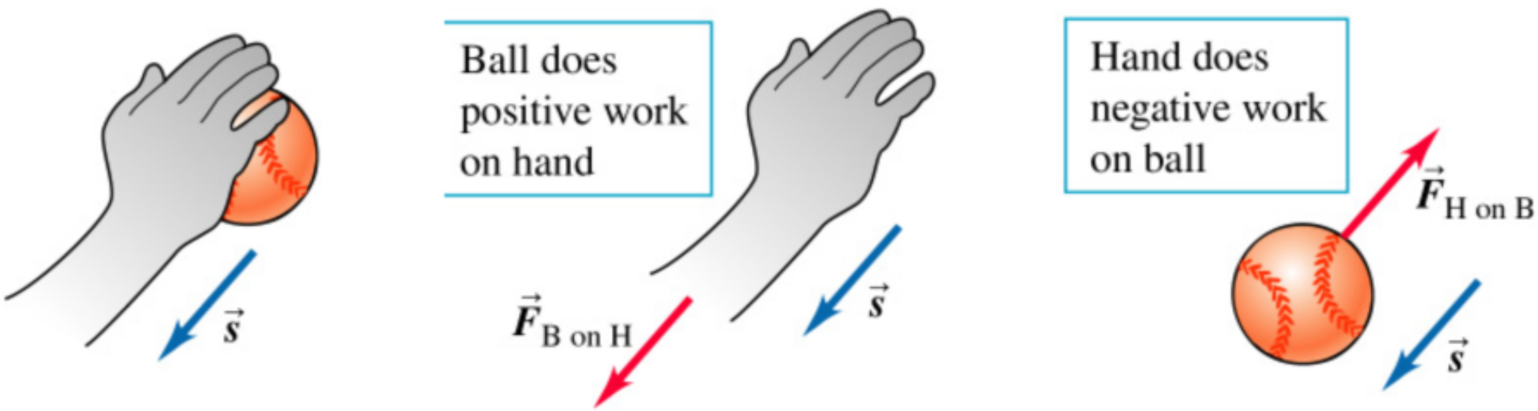
Understanding *Negative* Work

Artifact of 3rd law: e.g. when hand catches ball, H & B have same displacement $\vec{s}$

Ball exerts force $\vec{F}_{BonH}$ in direction of $\vec{s}$. Ball does positive work on hand.

Hand exerts equal & opposite force $\vec{F}_{HonB} = -\vec{F}_{BonH}$ in direction oppo. to $\vec{s}$.

Hand does *negative* work on ball.

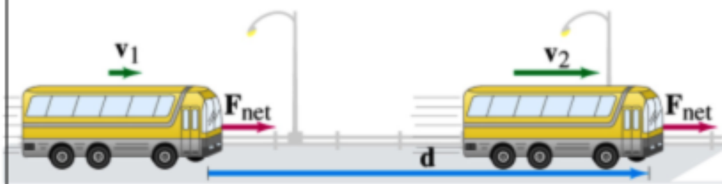


Total work done on a body is ALGABRAIC sum of work done by individual forces in displacing it. Alternately $W_{tot} = \sum W = \left(\sum \vec{F}\right).\vec{s}$

Kinetic Energy & Work Energy Theorem

Energy is the ability of a body to do work ← Working definition

Body in motion has ability to do work and thus has Energy.
Energy of motion is called Kinetic Energy (from *Kinetikos=motion*)



Bus of mass m moving in straight line with speed = v_1. Its accelerated to speed = v_2 by applying force $\vec{F}_{net}$ over distance d

Work done on bus $W_{net} = \vec{F}_{net}.\vec{d}$. Using $F_{net} = ma$ & $v_2^2 = v_1^2 + 2ad$

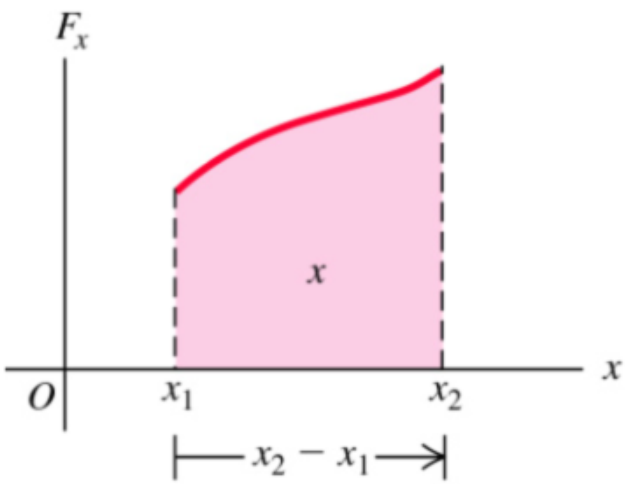
$$\Rightarrow W_{net} = Fd = mad \quad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d \;\Rightarrow W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

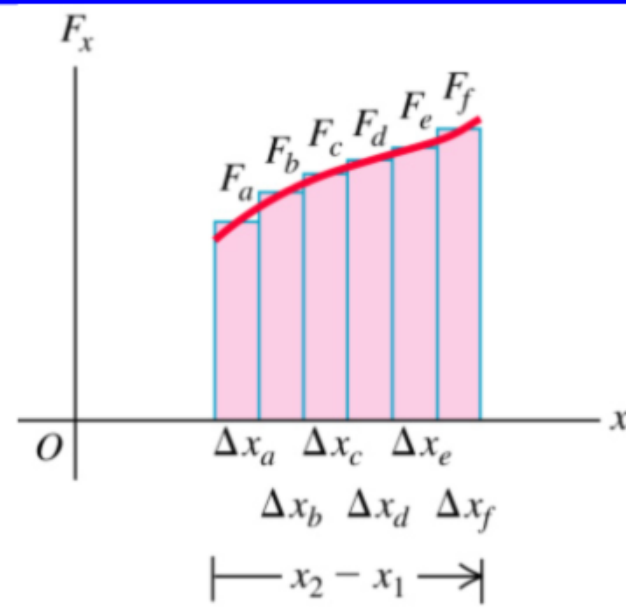
Define Translational kinetic energy $K = \frac{1}{2}mv^2 \Rightarrow$ $\boxed{W_{net} = K_2 - K_1 = \Delta K}$

work-energy theorem

net work done on an object = change in its kinetic energy

Work Done By a *Variable* Force Along x

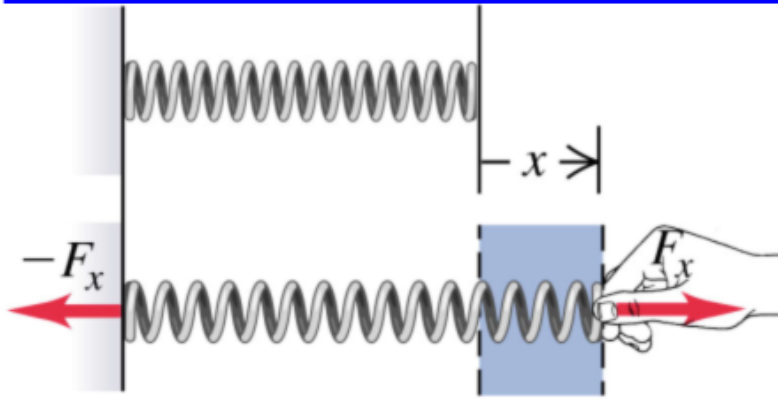


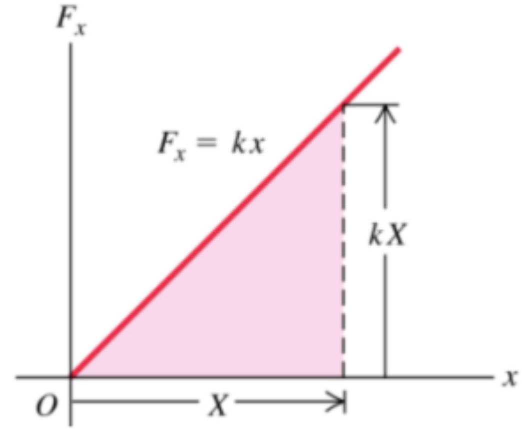


$$W = \int_{x_1}^{x_2} dW = \int_{x_1}^{x_2} F_x dx$$ general expression

On F vs x graph, total work done by variable force is represented by area under curve between $x = x_1$ & $x = x_2$.

Example of Variable Force → On a Spring





Force F_x stretches spring by small distance x.

Hooke's "Law" $\Rightarrow$ $F_x = kx$; k=spring constant

To stretch a spring work must be done on it

$$W = \int_0^X F_x dx = \int_0^X kx dx = \frac{1}{2}kX^2$$

$$\text{In general: } W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2}k(x_2^2 - x_1^2)$$

Work-Energy Theorem: General Derivation

If Force F varies along x. What is the Work-Kinetic Energy relation?

$$W = \int F_x dx = \int m a_x dx$$

rewrite: $a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} . \frac{dx}{dt} = v_x \frac{dv_x}{dx}$

$$W_{net} = \int_{x_1}^{x_2} m a_x dx = m \int_{x_1}^{x_2} v_x \frac{dv_x}{dx} dx = m \int_{v_1}^{v_2} v_x dv_x$$

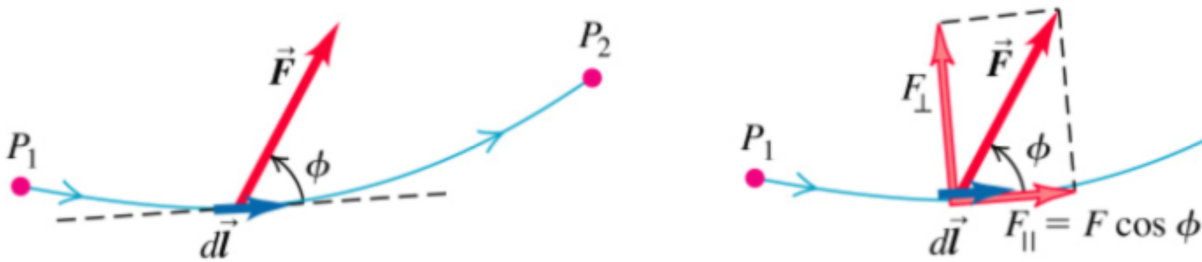
$$\Rightarrow \boxed{W_{net} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}$$

Pop Quiz

- When I catch a ball falling from above in my hand
- (A) I do work on the ball
- (B) the ball does work on me
- (C) the ball does work on gravity

Motion Along Curve & W-E Theorem

Particle moves from P_1 to P_2 under a variable force in a curved path



Force $\vec{F}$ is constant over infinitismal displacement $d\vec{l}$

Work done by $\vec{F}$ during displacement $d\vec{l}$: $dw = F_{\parallel} dl = \vec{F}.d\vec{l}$

$$\text{Net Work done } W = \int_{P_1}^{P_2} F\cos\varphi \, dl = \int_{P_1}^{P_2} F_{\parallel} dl = \int_{P_1}^{P_2} \vec{F}.d\vec{l}$$

Work Done in Displacement Along Curved Path

Throckmorton (!) of weight w sits on swing, with chain of radius R Man pushes *Throcky* by exerting horizontal force F that starts at 0 and gradually increases such that swing moves very slowly and always remains in equilibrium. In the end swing makes angle θ_0 w.r.t vertical. (a) work done by ALL forces on Throcky? (b) work done by tension T of swing on the kid (c) work done by man ?

Motion is along a curve $\Rightarrow W=\int \vec{F}.d\vec{l}$

Total work done=work done by $\vec{F}_{net}$

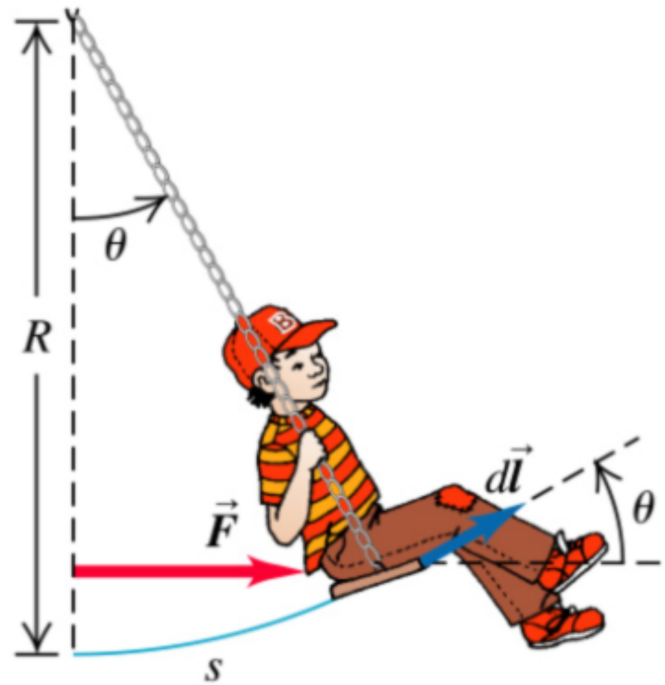
Since (Throcky+swing) in equilibrium

$\Rightarrow$ Net force on him: $\vec{F}_{net}=0$

$\Rightarrow W_{net}=\int \vec{F}_{net}.d\vec{l}=0$

Chain tension $\vec{T} \perp d\vec{l}$ always!

$\Rightarrow W_T=\int \vec{T}.d\vec{l}=\int T dl \cos 90^0=0$

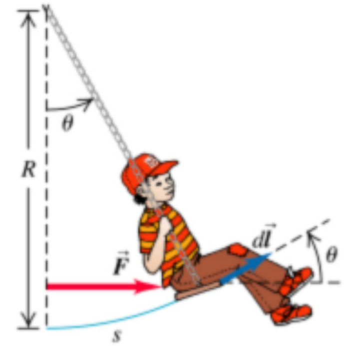


Work Done by Man on Throcky+Swing

Man applies force varying force $\vec{F}$

at any instant during displacement:

System in equilibrium: Apply 1st law



$\sum F_x$ =0=F-Tsinè , $\sum F_y$ =0=Tcosè-w

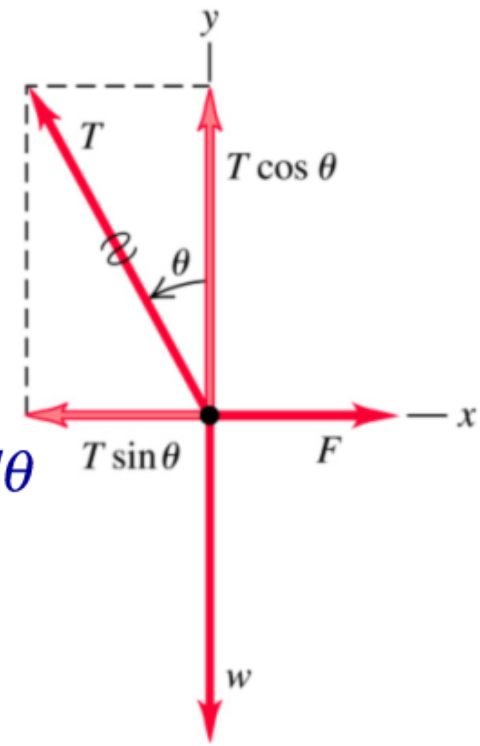
Eliminate T from above ⇒ F = w tanè

Points where $\vec{F}$ applied=arc s = Rè

⇒ Infinitismal displacement $dl = Rd\theta$

$$W_{man} = \int \vec{F}.d\vec{l} = \int F\cos\theta.ds = \int_{\theta=0}^{\theta_0} (w\tan\theta)(\cos\theta)Rd\theta$$

$$\Rightarrow W_{man} = wR\int_{\theta=0}^{\theta_0} \sin\theta d\theta \Rightarrow wR(1-\cos\theta_0)$$



Force & Power

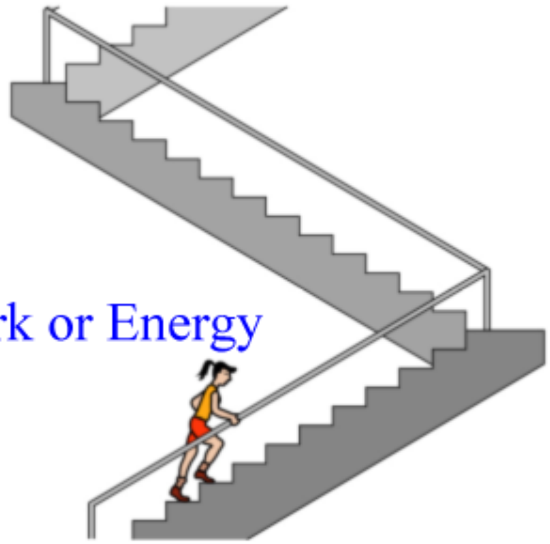
Work done is force applied over a distance. No mention of time in def.

Power is time rate at which work is done. When a work ΔW is done over time Δt

define average Power $P_{ave} = \frac{\Delta W}{\Delta t}$ and Instantaneous power $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$

SI Unit of power: 1W = 1 J/s

More useful unit is $KW = 10^3$ W or $MW = 10^6$ W



1 hp = 550ft.lb/s= 746W

Unit of power can be used to define new unit of Work or Energy

Kilowatt-hour (kWh)

And finally, $P_{ave} = \frac{\Delta W}{\Delta t} = \frac{F_{\parallel}\Delta s}{\Delta t} = F_{\parallel} . V_{ave}$

$$P = \vec{F}.\vec{v}$$

(Girl) Power !

Red Rocks, Nevada

Woman, m=50kg, climbs up cliff h=400m in 15 minutes ! What's her power output in Watts & hp?

$$P_{ave} = \frac{\text{Work Done}}{\text{Time Taken}} = \frac{\text{ÄW}}{\text{Ät}}$$

$$\text{ÄW} = \vec{F}.\vec{d} = (mg)h = (50.0\text{kg})(9.8\text{m/s}^2)(400\text{m})$$

$$\Rightarrow \text{ÄW} = 1.96?10^5\text{J} \ \& \ P_{ave} = \frac{\text{ÄW}}{\text{Ät}} = 0.218\text{kW}$$

$$\text{Since } 1\text{hp} = 0.746\text{kW} \Rightarrow \boxed{\text{Girl power} = 0.29\text{hp}}$$

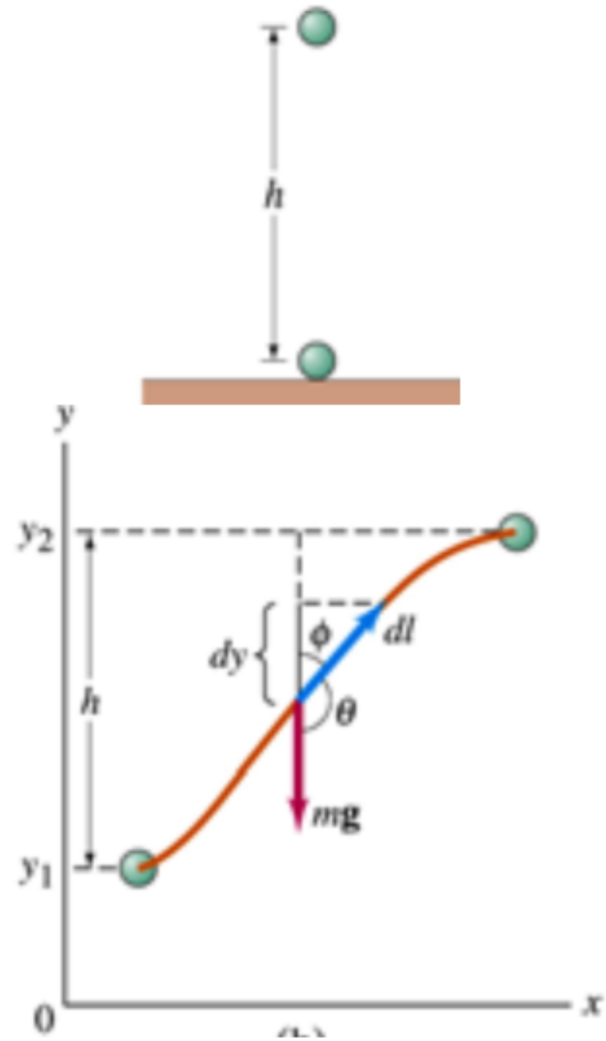
Conservative Force

Work done by a conservative force on an object moving from one point to another **depends only** on the initial and final positions and is independent of path taken. e.g: Gravity

$$W_G = \int_1^2 \vec{F}_g . d\vec{l} = \int_1^2 mg \cos\theta \, dl$$

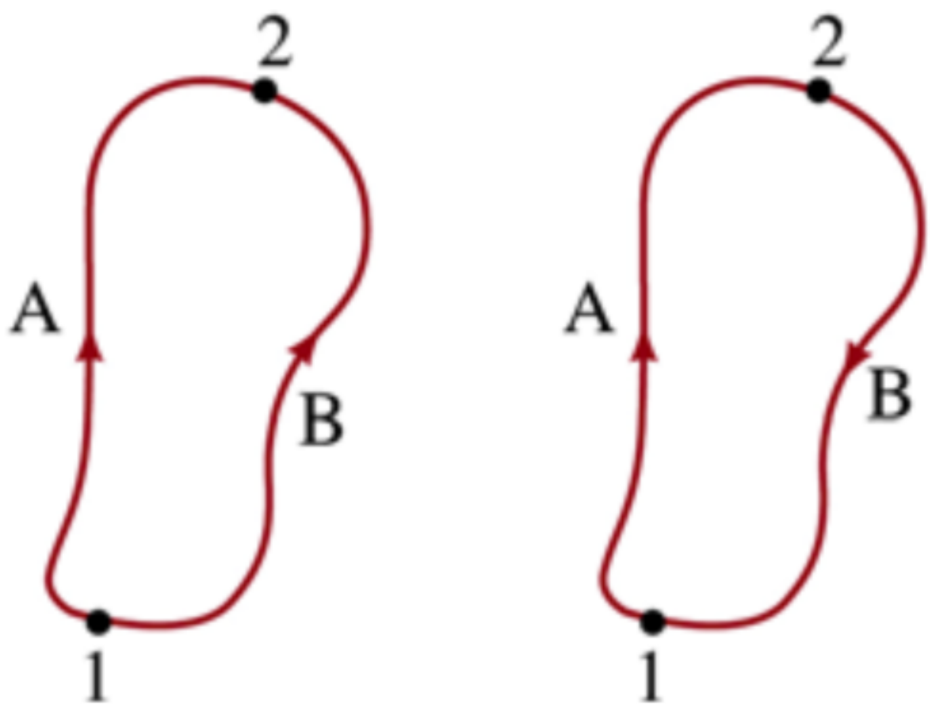
Use $dy = dl \cos\phi = -dl \cos\theta$

$$\Rightarrow W_G = -\int_{y_1}^{y_2} mg\,dy = -mg(y_2 - y_1) = -mgh$$

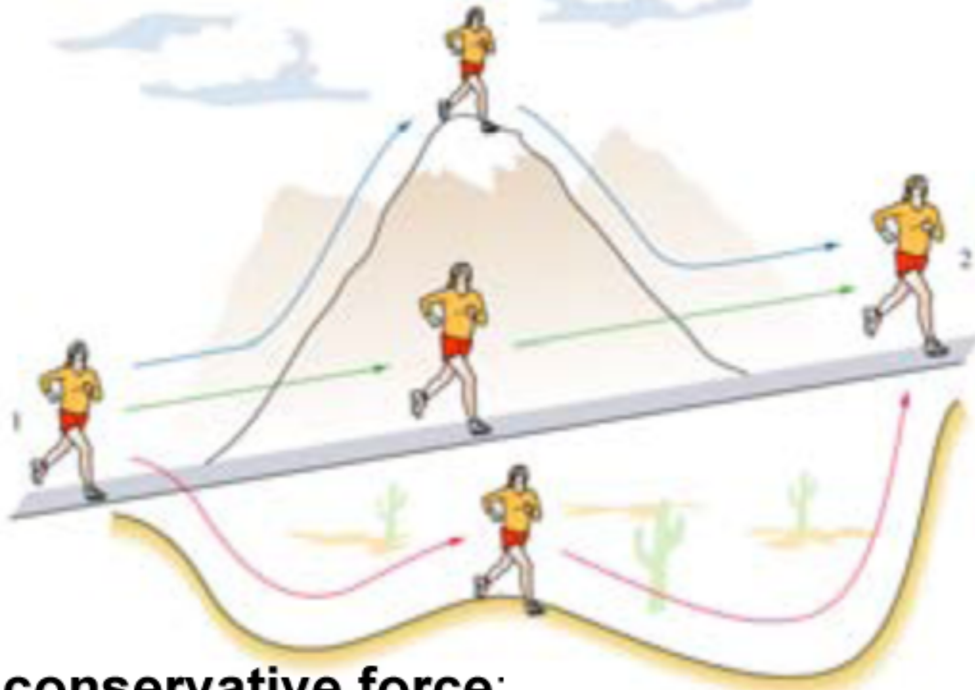


Conservative Force: Alternate Def.

A force is a conservative force IF the net work done by the force on the object moving around any *closed* path is zero



Work Done By Conservative Forces

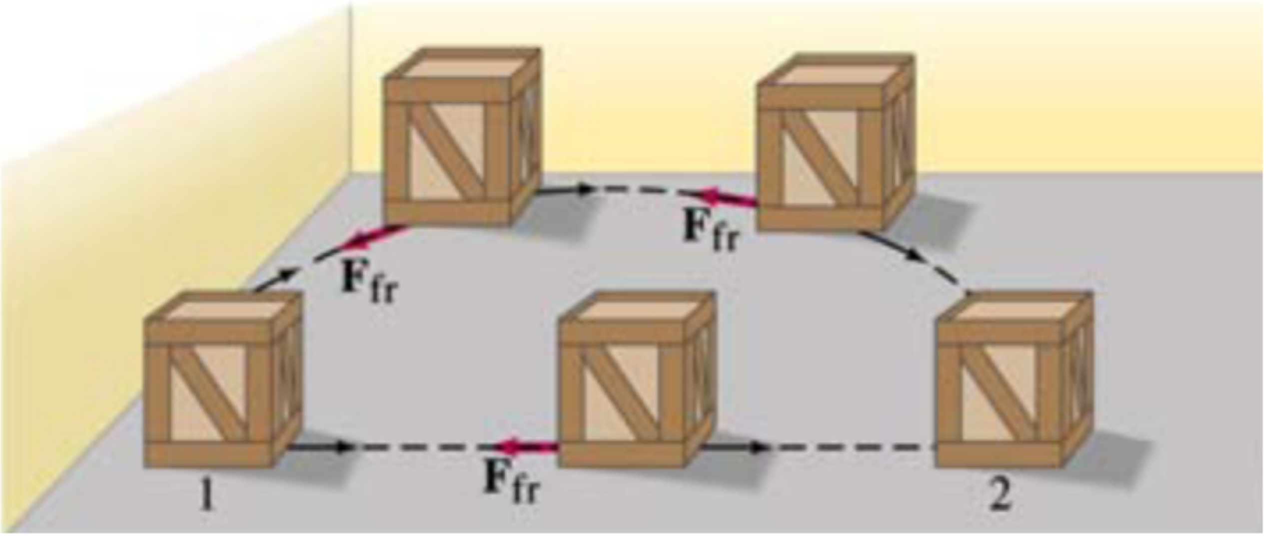


Work done by **conservative force**:

1. Can be expressed as difference between initial and final values of the potential energy functions
2. It is reversible
3. Independent of path of body, depends on start/end point only
4. When start/end point are same, work done is zero

Non-Conservative Force

Example: Force of Friction



work done in moving a crate across a level floor depends on the total distance travelled, since friction force is directed opposite to direction of motion $\Rightarrow W_F \propto$ path length.

Since friction is oppo. to motion dir., $W_F < 0$ always, indep. of path

Work done by non-conservative forces not recoverable

Pop quiz

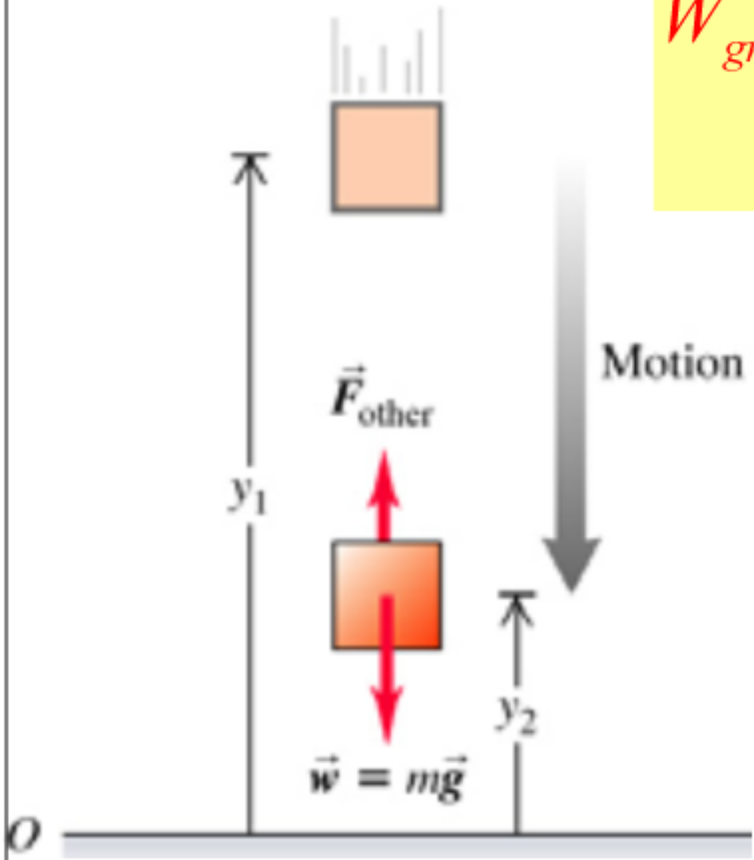
- The figure on the board shows a frictionless slide. The two ends A and B are at the same height. The speed of an object at the bottom C of the slide is
- (A) greater when it is released at A
- (B) greater when it is released from B
- (C) the same in either case.

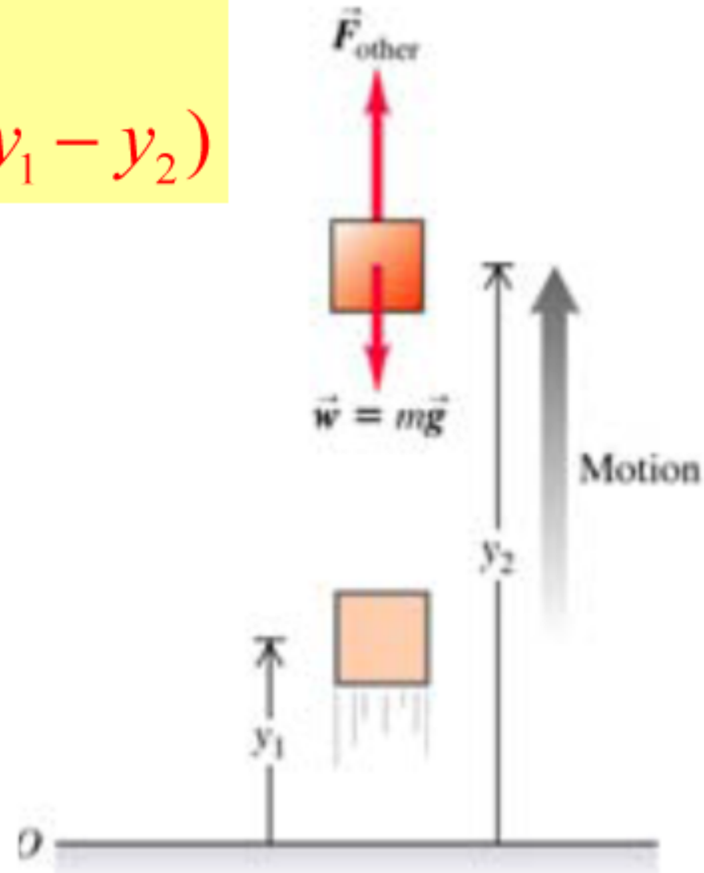
Potential Energy: U

- Potential Energy: Energy U associated with the position or configuration (arrangement) of objects that exert forces on each other
- Various kinds of Potential energy can be defined and each type is associated with a particular *conservative force*
- Most common example of Potential energy
  - gravitational potential energy
    - associated with body's weight and height above ground level or another reference point

Work Done By Gravity in Vertical Motion

$$W_{grav} = Fs$$
$$= w(y_1 - y_2)$$



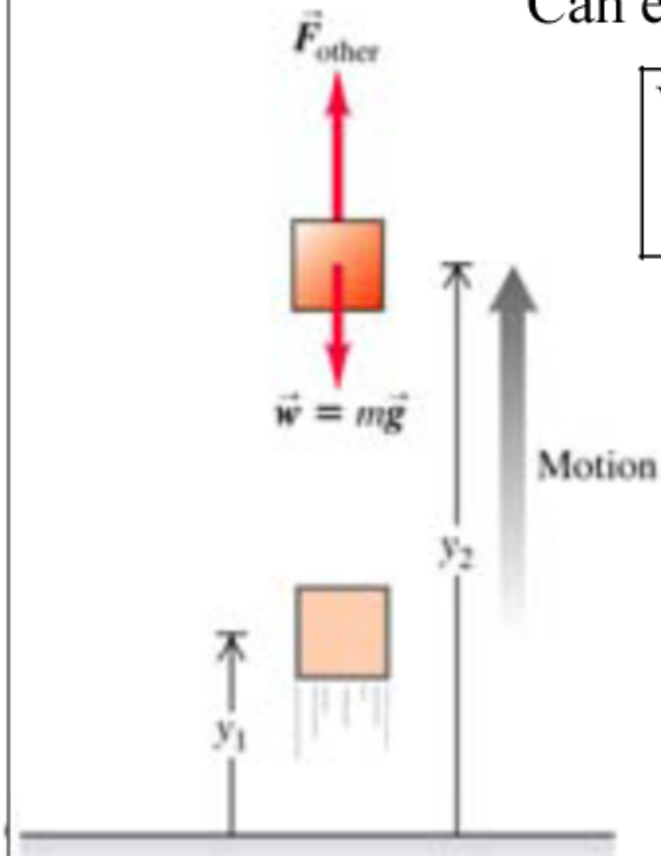


Gravitation Potential Energy

Define: Gravitational Pot. Energy U = mgy

Unit of Pot. energy U = Joule (J)

Can express work done in terms of change in U



$$W_{grav} = mg(y_1 - y_2) = U_1 - U_2 = -(U_2 - U_1) = -\Delta U$$

definition

If body moves up (y increases),

$\Rightarrow W_{grav} < 0, \ \& \ \Delta U > 0$

If body moves down (y decreases),

$\Rightarrow W_{grav} > 0, \ \& \ \Delta U < 0$

Conservation of Mechanical Energy

Body continues to fall under gravitational force only

y_1, v_1, K_1, U_1

Work-Energy theorem ⇒
relation between total work done & kinetic energy

$$\Rightarrow W_{tot} = \ddot{A}K = K_2 - K_1$$

also $W_{tot} = W_{grav} = -\ddot{A}U = U_1 - U_2$

$$\Rightarrow K_2 - K_1 = U_1 - U_2 \text{ or } K_1 + U_1 = K_2 + U_2$$

Define: E=K+U=Total mech. energy of system

y_2, v_2, K_2, U_2

$$E = K + U = \text{constant}$$

⇒ Conservation of Mechanical Energy

Effect Of Other Forces On Body (F_{other})

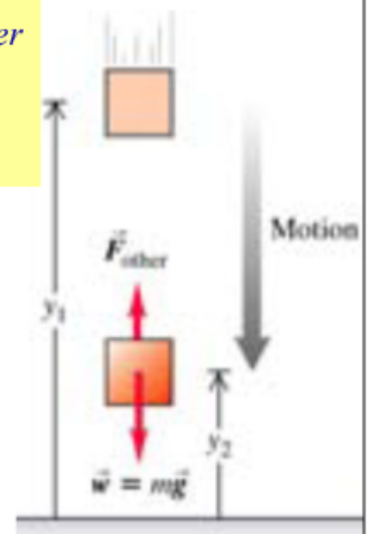
When other forces act on body, in addition to weight force then $\vec{F}_{other} \neq 0$ & it does work W_{other}

$$\Rightarrow \boxed{W_{tot} = W_{other} + W_{grav} = K_2 - K_1}$$

Nevertheless, $W_{grav} = U_1 - U_2$

$$\Rightarrow W_{tot} = W_{other} + U_1 - U_2 = K_2 - K_1$$

$$\Rightarrow \boxed{K_1 + U_1 + W_{other} = K_2 + U_2}$$



master equation:

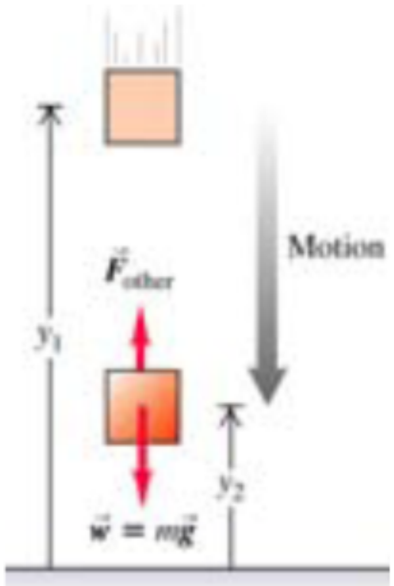
$$\boxed{\frac{1}{2}mv_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + mgy_2}$$

Conservation Of Mechanical Energy

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + mgy_2$$

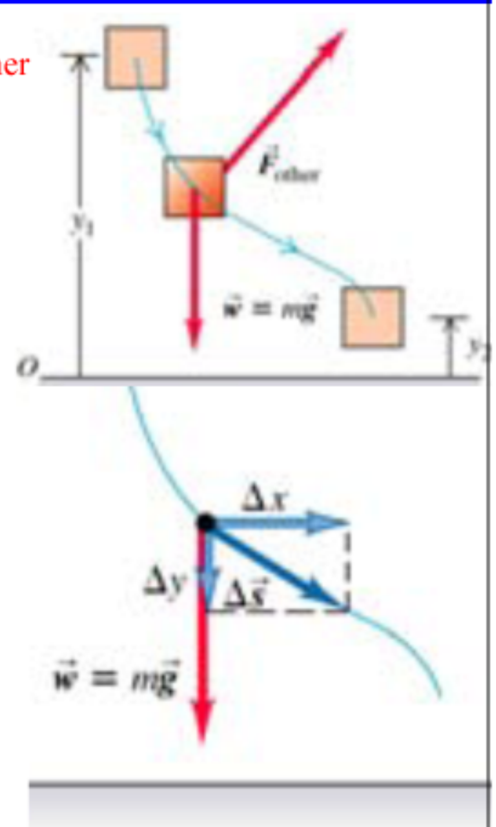
Work done on a body by all forces on the body other than gravity equals change in the total mechanical energy E=K+U of the system



If $W_{other} > 0$, E increases & Vice Verca

Gravitational Potential When Motion Along Curved Path

Body in curved trajectory under gravity & $\vec{F}_{other}$



Divide trajectory in smaller elements $\Delta\vec{s}$

$$\Delta\vec{s} = \Delta x\ \hat{i} + \Delta y\ \hat{j}$$

$\therefore dW_{grav} = \vec{f}_{grav}$.Ä$\vec{s}$ = $-mg\Delta y$

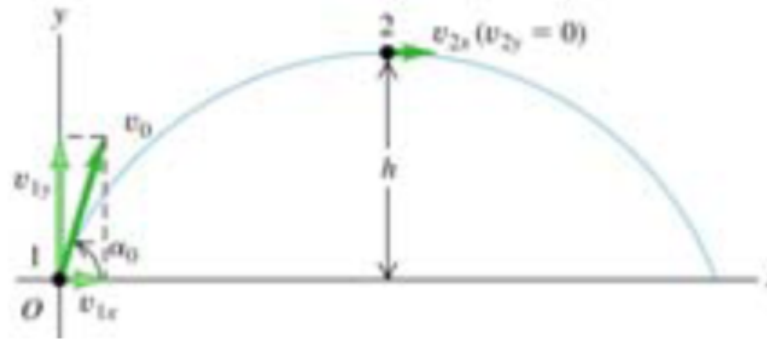
$$\therefore W_{grav} = -mg\sum \Delta y = -mg(y_1 - y_2)$$

$\Rightarrow$ Work done by gravity is same whether path is curved or straight !

& $\Delta U = -W_{grav} = mg(y_1 - y_2)$

Projectile Motion & Energy Conservation

Use energy conservation to find the maximum height h of projectile launched with initial speed v_0 and at angle α_0



Determine K & U at 1 & 2 and apply energy conservation

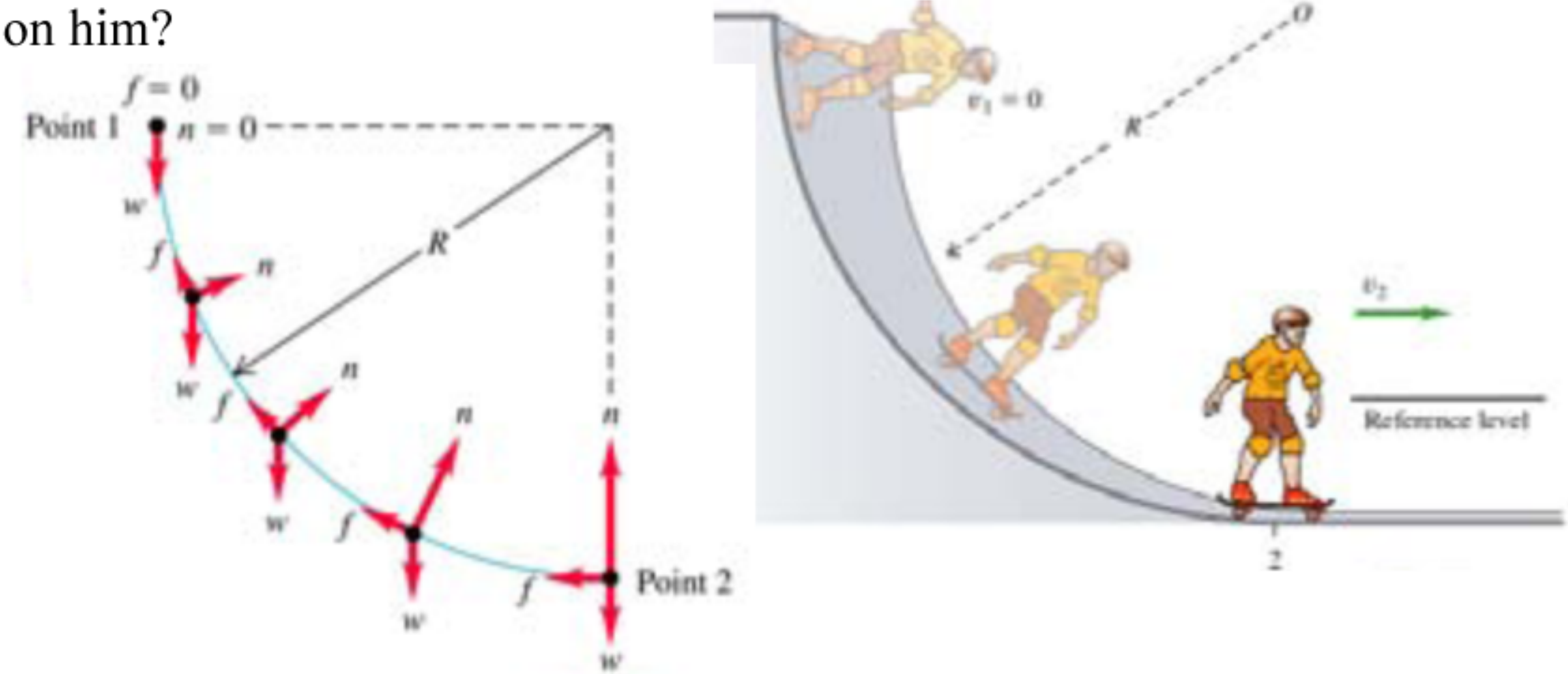
$$K_1 + U_1 = K_2 + U_2 \Rightarrow$$

$$\frac{1}{2}m(v_{1x}^2 + v_{1y}^2) + 0 = \frac{1}{2}m(v_{2x}^2 + v_{2y}^2) + mgh \Rightarrow v_{1x}^2 + v_{1y}^2 = v_{2x}^2 + v_{2y}^2 + 2gh$$

Since in projectile motion: $v_{1x} = v_{2x}$ & $v_{2y} = 0$

$$v_{1y}^2 = 2gh \Rightarrow h = \frac{v_1^2 \sin^2\alpha}{2g}$$ (as calculated before!)

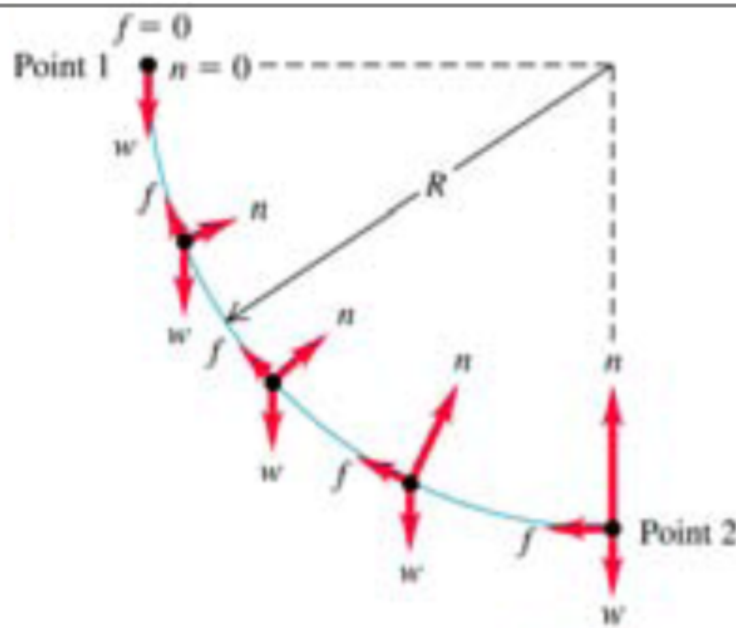
Boy+board (m=25kg) starts from rest & skates down curved (R=3m) ramp with friction. At the bottom, his speed=6m/s. What work is done by friction on him?

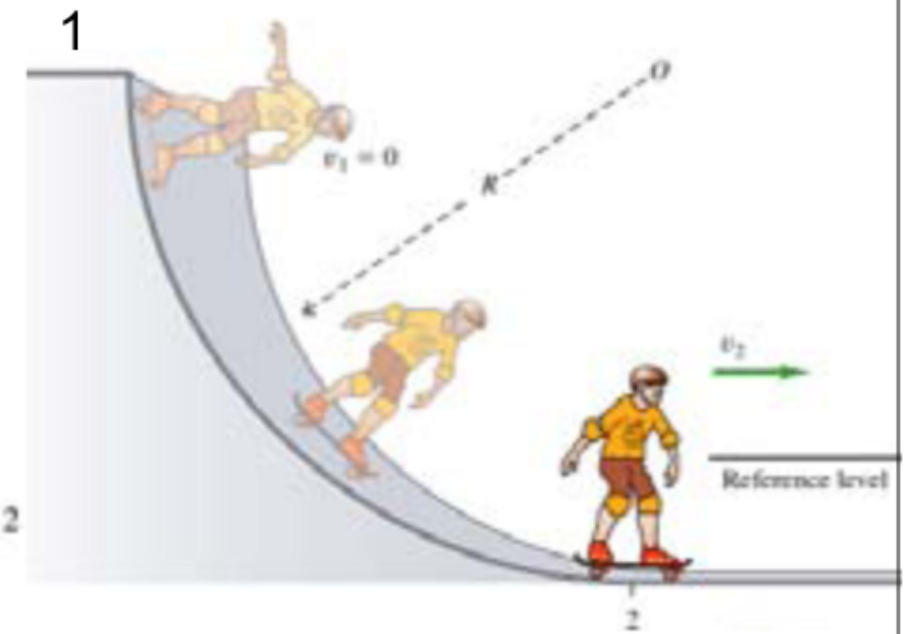


Forces on boy: weight, normal and *friction* $\vec{f}$

$\vec{n}$ always $\perp$ to path $\Rightarrow$ does no work done on boy!

But weight (mg) force and $\vec{F}_{other} = \vec{f}$ do work W_{other} on boy





$$K_1 = 0,\ U_1 = mgR = 735J \ \ \&\ \ K_2 = \frac{1}{2}mv_2^2 = 450J,\ U_2 = 0$$

Conservation of Mechanical Energy

$$\Rightarrow \boxed{K_1 + U_1 + W_{other} = K_2 + U_2}$$

$$\Rightarrow W_{other} = W_f = (K_2 + U_2) - (K_1 + U_1)$$

$$= -285J$$

did not need to know μ_K or f !

Quick Question

A girl throws a stone from a bridge. Consider the following ways she might throw the stone. The speed of the stone as it leaves her hand is the same in each case.

- (A) Thrown straight up.
- (B) Thrown Straight down
- (C)Thrown out at an angle of 45º to the horizontal
- (D) thrown straight out horizontally

In which case will the speed of the stone be greatest when it hits the water below?

Quick Question

- A pendulum in a grandfather clock swings back and forth.
- (A) Gravity does positive work on the pendulum when it is swinging from a high point to a low point.
- (B) gravity does positive work on it when it swings from a low to a high point.
- (C)Gravity does no work on the pendulum.
- (D) gravity does positive work in both cases.

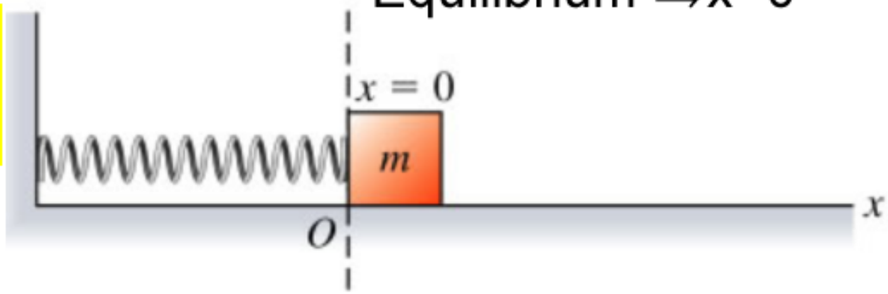
Elastic Potential Energy

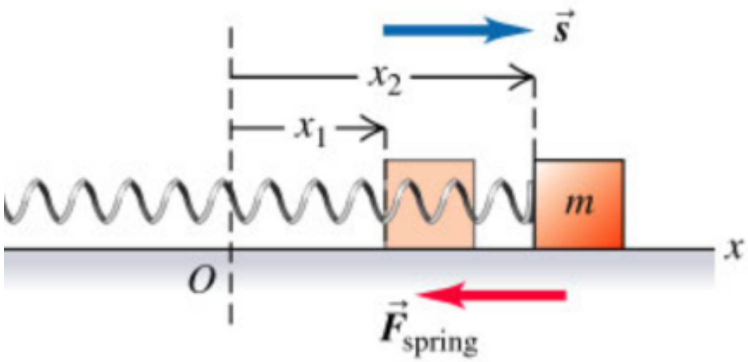
an extends spring+block from equilibrium position

Equilibrium $\Rightarrow x=0$

ce by man to stretch

ng by distance x $\Rightarrow F_x = kx$



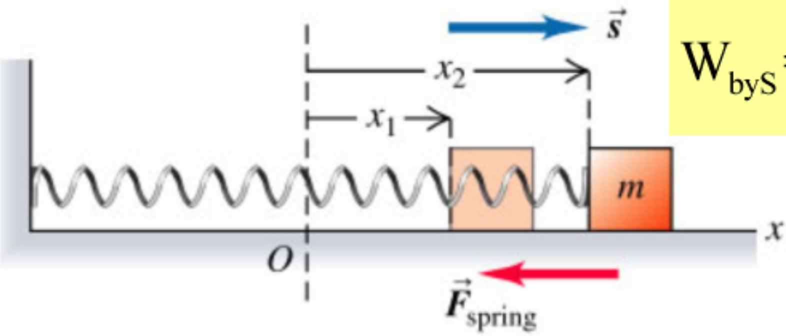


Work done by man, on spring, to stretch

it from $x_1 \rightarrow x_2$: $W_{onS} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

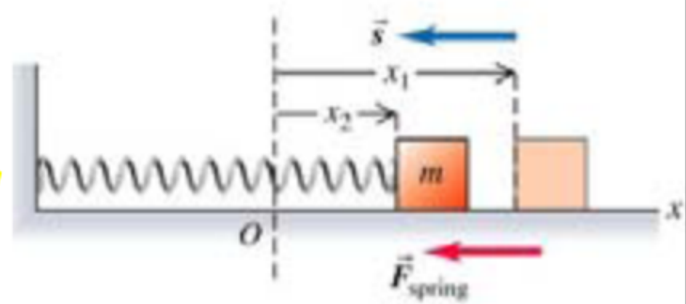
Force & Work Done By A Stretched Spring

3rd law ⇒ Spring exerts restoring force on block
⇒ does negative work when displaced from $x_1 \rightarrow x_2$



$$W_{byS} = -W_{onS} = W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 < 0$$

At another instant when spring "relaxes" (unstretched) then pulls on block
$x_1 > x_2$ & spring does positive work on block which speeds up

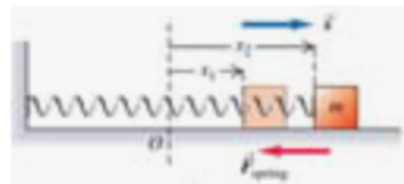


$\vec{F}_{spring}$ & displacement $\vec{s}$ are ||

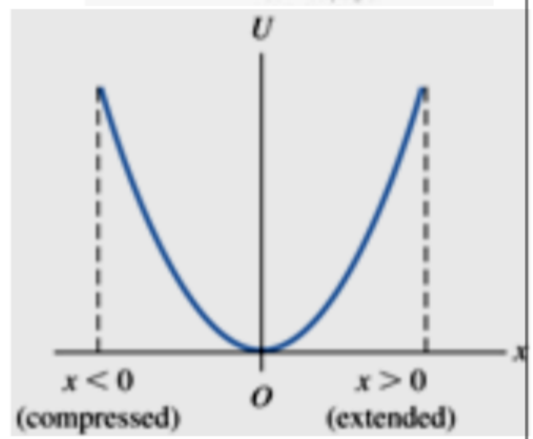
Elastic Potential Energy

Define Elastic Potential Energy U = $\frac{1}{2}kx^2$

Work done W_{el} by spring on block



$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = U_1 - U_2 = -\Delta U$$



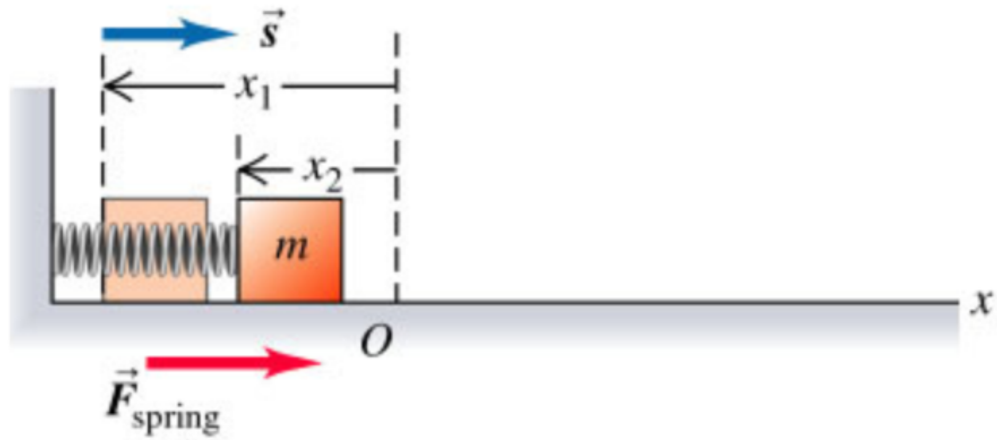
Stretched spring $\Rightarrow x_2 > x_1 \Rightarrow W_{el}$ decreases & potential U increases

Compressed spring $\Rightarrow x_2 < x_1$

$\Rightarrow W_{el}$ increases & potential U decreases

When compressed spring compressed more, $W_{el} < 0$

If spring is compressed or stretched more, its potential energy U increases

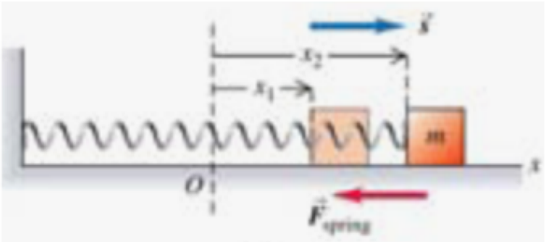


Compressed spring $\Rightarrow$ displacements negative with respect to reference x = 0

$$|x_2| < |x_1|$$

Potential energy $U_{el} = \frac{1}{2}kx^2$ is ALWAYS positive !

W-E Theorem $\Rightarrow W_{tot} = K_2 - K_1$



If spring force is the *only force* that does work on block

$\Rightarrow W_{tot} = W_{el} = U_1 - U_2$ Combining this with W-E theorem$\Rightarrow$

$$W_{tot} = U_1 - U_2 = K_2 - K_1 \text{ or } K_1 + U_1 = K_2 + U_2$$

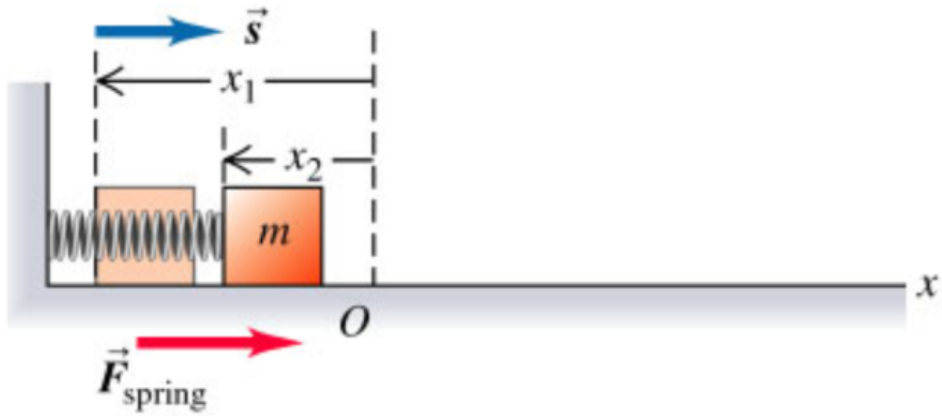
Conservation of Mechanical Energy E=K+U

Explicitly: $$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

in case of frictionless, massless spring

Quick question

- If a mass M falls with velocity v on a spring of force constant k and compresses it by x:
- (A) All the kinetic energy is converted to gravitational potential energy
- (B) The spring's stored elastic potential energy increases by ½ M v^2
- (C)gravitational potential energy is converted to elastic potential energy
- (D) all the kinetic energy plus gravitational potential energy is converted to stored elastic energy in the spring.

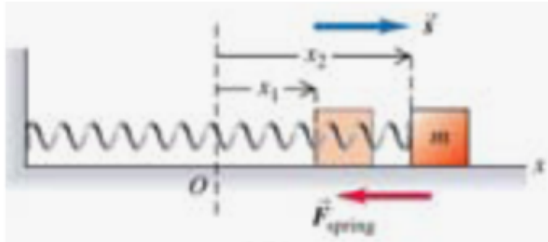


Compressed spring $\Rightarrow$ displacements negative with respect to reference $x = 0$

$$|x_2| < |x_1|$$

Potential energy $U_{el} = \frac{1}{2}kx^2$ is ALWAYS positive !

W-E Theorem $\Rightarrow W_{tot} = K_2 - K_1$



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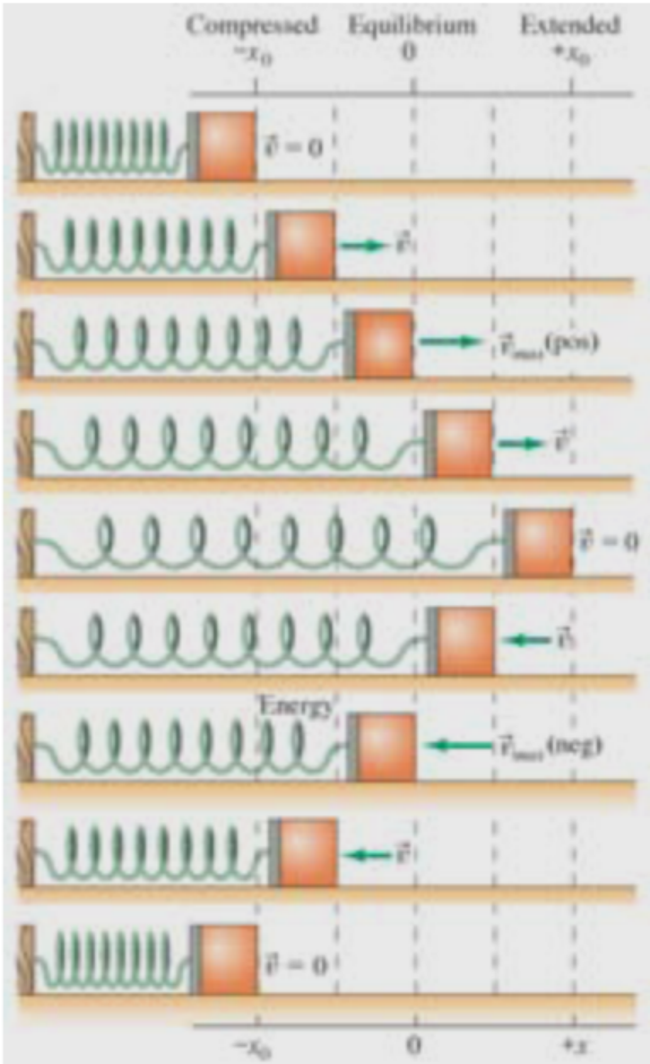
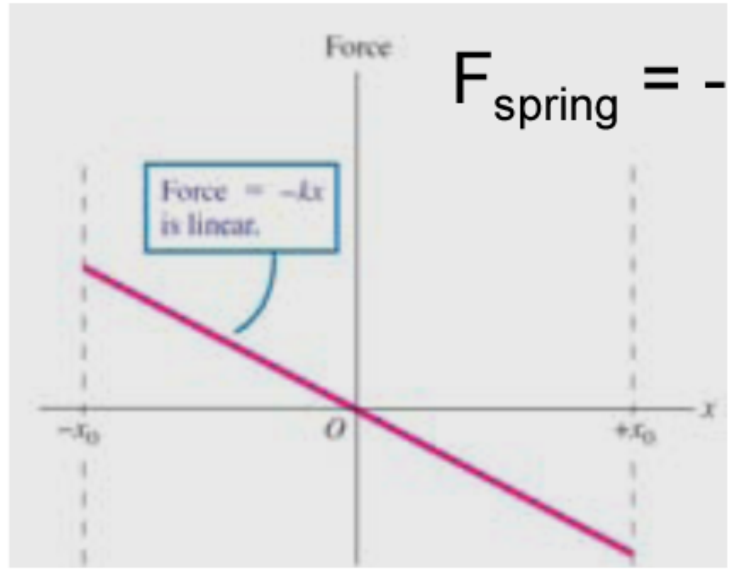
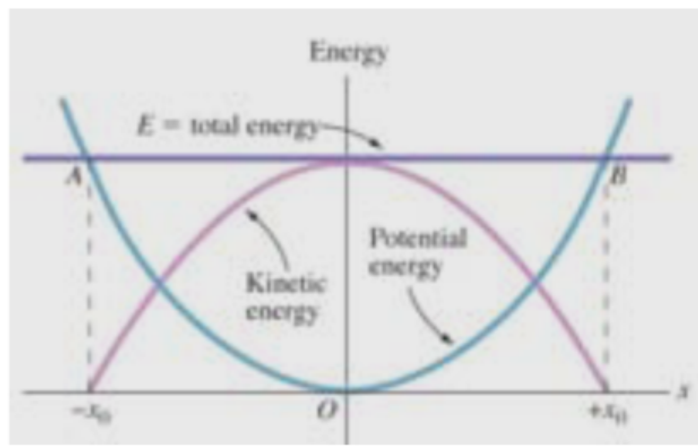
Conservation of Mechanical Energy In Spring-Block syst

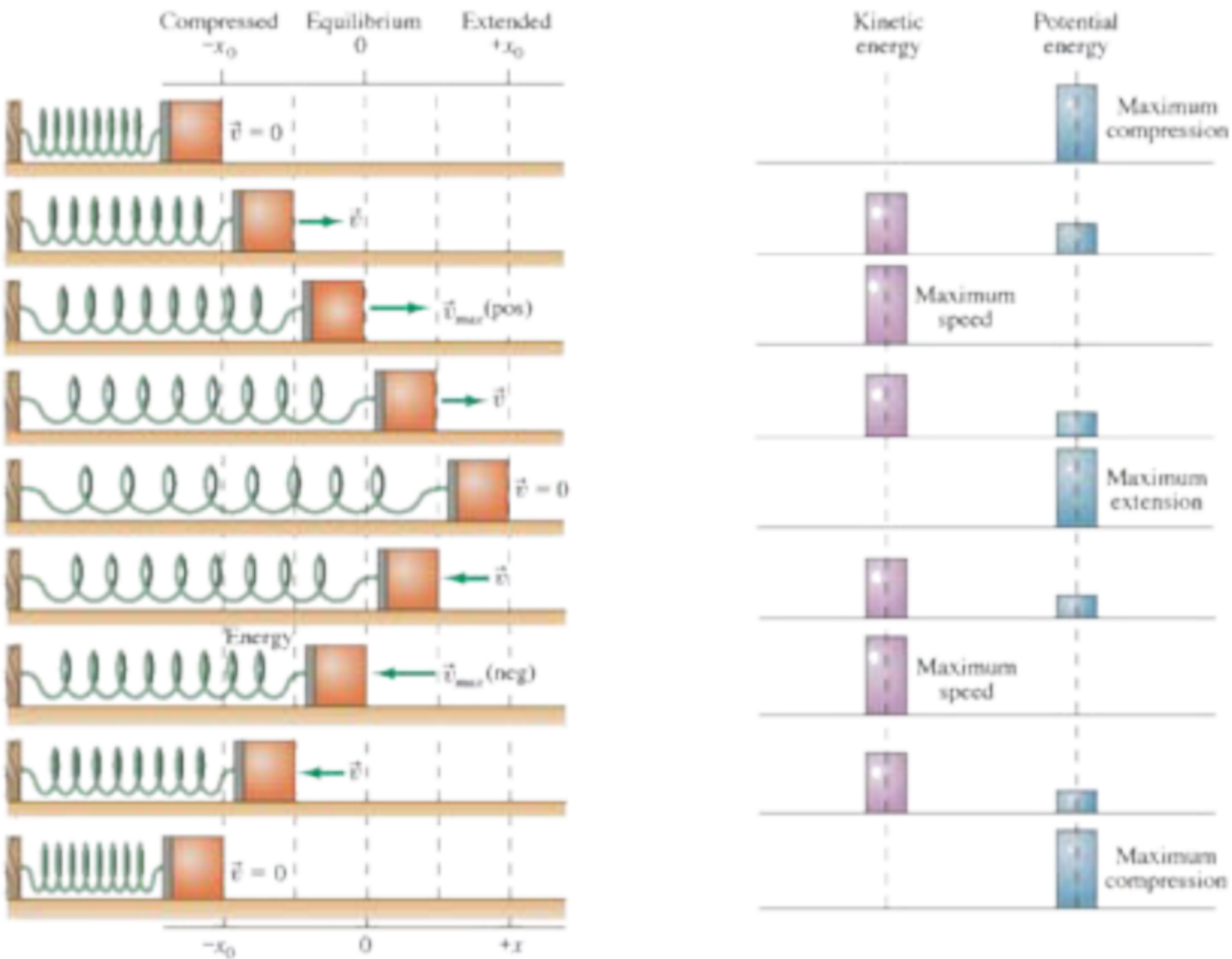




$$F_{spring} = - kx$$



Conservation of Mechanical Energy In Spring-Block syst



In Presence of Additional Forces On Block

In presence of additional forces, which do work W_{other}

$\therefore W_{tot} = W_{el} + W_{other}$ & W-E Theorem $\Rightarrow W_{tot} = K_2 - K_1$

Since $W_{el} = U_1 - U_2 \Rightarrow K_1 + U_1 + W_{other} = K_2 + U_2$

or
$$\boxed{\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 + W_{other} = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2}$$

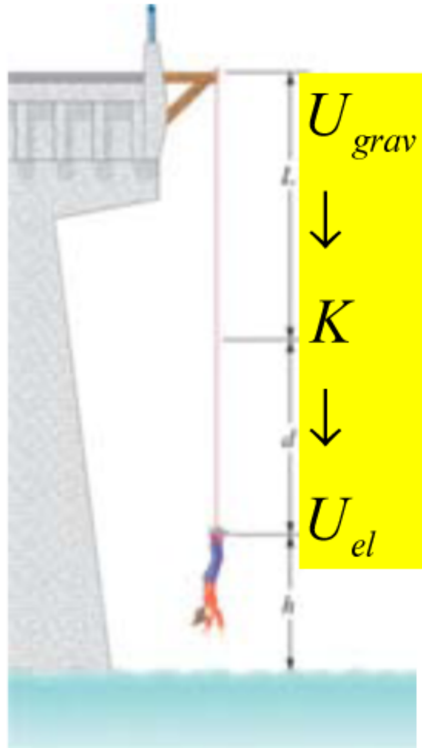
Work done by all forces other than elastic force equals change in the total mechanical energy E=K+U of system

System= spring + block

If $W_{other} > 0 \Rightarrow$ Energy E increases

If $W_{other} < 0 \Rightarrow$ Energy E decreases

Situation With Both Gravitational & Elastic Potential Energy: Bungee Jumping !



Woman tied to an elastic rope of length L, jumps off a cliff !!

Survives because of conservation of mechanical energy

⇒the joy of bungee jumping !

Situation With Both Gravitational & Elastic Potential Energy: General Expression

$$K_1 + U_{g,1} + U_{el,1} + W_{other} = K_2 + U_{g,2} + U_{el,2}$$

Work done by all forces except gravitational and elastic force equals change in the total mechanical energy E=K+U

In bungee jumping as woman falls U_{grav} decreases, is converted into her increased kinetic energy K

Beyond a certain point in the fall, woman's speed decreases so that both K and U_{grav} are converted into **elastic potential energy U_{el}** .
The "Other force" in bungee jumping could be gusting of wind …up or down!...

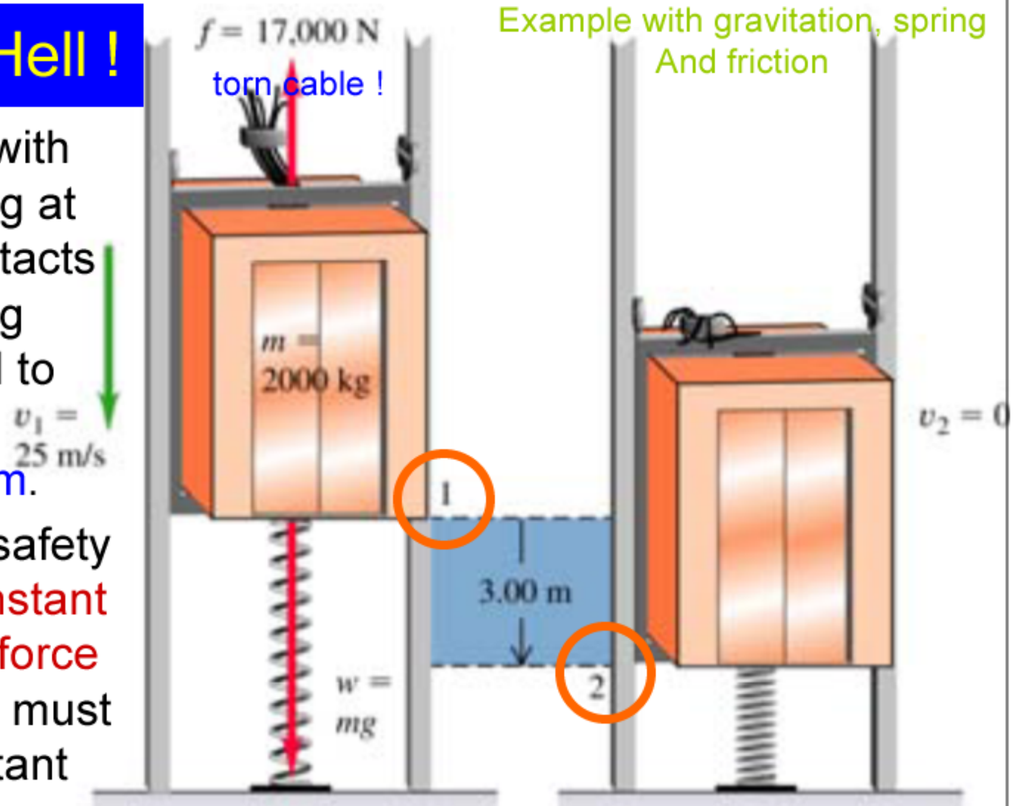
makes it an adventure to die for!

Elevator To Hell !

2000 Kg elevator with broken cable falling at 25m/s when it contacts a cushioning spring which is supposed to stop the fall by compressing by 3m.

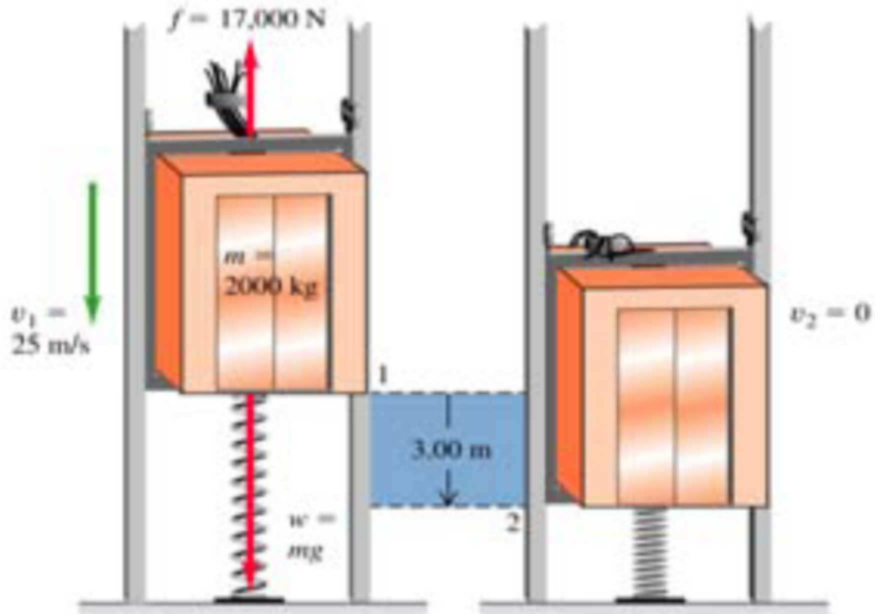
During motion, a safety clamp applies constant 17000N frictional force on elevator. What must be the force constant k of the spring ?

Example with gravitation, spring And friction



⇒ Use most general form of energy conservation

$$K_1 + U_{g,1} + U_{el,1} + W_{other} = K_2 + U_{g,2} + U_{el,2}$$



Apply Energy Conservation at points 1 & 2

$$\Rightarrow K_1 + 0 + W_{other} = 0 + (mgy_2 + \frac{1}{2}ky_2^2)$$

$$\therefore \text{Spring constant } k = \frac{K_1 + W_{other} - mgy_2}{y_2^2}$$