Physics 4A
Lecture 4: Jan. 15, 2015

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Max. height & the range of projectile depends on the firing angle $\alpha_0$

Projectile at its highest point at time $t_1$ when $v_y = 0$

$\Rightarrow v_y = v_0 \sin\alpha_0 - gt_1 = 0 \Rightarrow t_1 = \frac{v_0 \sin\alpha_0}{g}$

at this time, $y = h = v_0 \sin\alpha_0 \frac{v_0 \sin\alpha_0}{g} - \frac{1}{2} g \left( \frac{v_0 \sin\alpha_0}{g} \right)^2$

$\Rightarrow h = \frac{v_0^2 \sin^2\alpha_0}{2g}$; largest at $\alpha_0 = 90^0$ (vertical launch)
Range of Projectiles

R is the projectile's x location at some \( t = t_2 \)
when \( y = 0 \)

\[
0 = (v_0 \sin \alpha_0) t_2 - \frac{1}{2}gt_2^2 = t_2 \left( v_0 \sin \alpha_0 - \frac{1}{2}gt_2 \right) = 0
\]

Two solutions for \( t_2 \):

- \( t_2 = 0 \)
- \( t_2 = \frac{2v_0 \sin \alpha_0}{g} \)

Range \( R = v_0 \cos \alpha_0 \cdot \frac{2v_0 \sin \alpha_0}{g} = \frac{v_0^2 \sin 2\alpha_0}{g} = R \)

(Using trig. identity: \( 2 \sin \theta \cos \theta = \sin 2\theta \))

\( R = R_{\text{max}} \) when \( \alpha_0 = 45^\circ \)
Uniform Circular Motion

Very different from projectile motion where accel. was const. and **always** in 1 direction

In uniform circular motion the speed of object is constant but velocity is always changing

No component of accel. parallel (or tangent) to path so no change in speed

$$\vec{a} = \vec{a}_\perp = \vec{a}_r$$

Component of accel. **perpendicular** to path causes direction of velocity to change
Radial Component of Acceleration

Particle moving in circle of radius R around O
Particle moves from \( P_1 \rightarrow P_2 \) in time \( \Delta t \)

Velocity changes from \( \vec{v}_1 \rightarrow \vec{v}_2 \) in time \( \Delta t \)

Triangles \( OP_1P_2 \) & \( Op_1p_2 \) are similar \( \Rightarrow \)
Ratio of corresponding sides are equal

\[
\Rightarrow \frac{\Delta \vec{v}}{|\vec{v}_1|} = \frac{\Delta s}{R} \Rightarrow \Delta \vec{v} = \frac{\Delta s}{R} |\vec{v}_1|
\]

\[
\Rightarrow |\vec{a}_{av}| = \frac{\Delta \vec{v}}{\Delta t} = \frac{|\vec{v}_1| \Delta s}{R \Delta t}
\]

\[
|\vec{a}_{inst}| = \lim_{\Delta t \rightarrow 0} \frac{|\vec{v}_1| \Delta s}{R \Delta t} = \frac{|\vec{v}_1|}{R} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{|\vec{v}_1| |\vec{v}_1|}{R} = \frac{v^2}{R}
\]
Radial Component of Acceleration

\[ \vec{a}_{\text{inst}} = |\vec{a}_{\text{rad}}| = \frac{v^2}{R} \]

also called Centripetal Acceleration

Centripetal = “seeking the center”

\( \vec{a}_{\text{rad}} \) is \( \perp \) to \( \vec{v} \) and directed radially \textit{inwards}

Period of Motion : \( T \)

In time \( T \), particle makes one full trip around circle. So its speed

\[ |\vec{v}| = \frac{2\pi R}{T} \]

But since \( |\vec{a}_{\text{rad}}| = \frac{v^2}{R} \) \( \Rightarrow \)

\[ |\vec{a}_{\text{rad}}| = \frac{4\pi^2 R}{T^2} \]
Pop Quiz:

• A satellite is moving in a circular orbit of radius $9 \times 10^3$ Km around the Earth with a speed of $20 \text{ m/s}$. Suddenly it is kicked into an orbit of $18 \times 10^3$ Km with a speed of $40 \text{ m/s}$

Its acceleration towards Earth

• (A) Does not change
• (B) Increases by a factor of 2
• (C) Decreases by a factor of 2
• (D) Decreases by a factor of $\sqrt{2}$
Non-Uniform Circular Motion

Here speed changes along the circular path

\[ a_\parallel = a_{\text{tangent}} = \frac{d |\vec{v}|}{dt} \neq 0 \quad \text{and} \quad a_r = \frac{v^2}{R} \]

\( a_{\text{rad}} \) is largest when speed is largest, smallest when speed is smallest.
\( a_{\text{tan}} \) is \( \parallel \) to \( \vec{v} \) (going downhill) and anti-\( \parallel \) \( \vec{v} \) when object going uphill!
• **Kinematics:**
  – *Language* for describing motion
    • displacement, velocity, acceleration

• **Dynamics:**
  – *Cause of change in motion*
    • New Concepts: Force and Mass
    • Newton’s Three Laws of Motion
      – Obtained from first principles ⇒ Fundamental
      – Based on experimental observations of motion, NOT product of some math derivation
Intuition About Motion May Misguide You!

Motion of a hockey puck on a horizontal table. Puck stops shortly after you stop pushing.

Many, like Aristotle, would conclude:

Bodies in motion naturally come to rest,

&

A force must act on puck to sustain motion.
Idealization of A Motion

Examine motion of a hockey puck on a waxed floor under a similar push (force)

Puck now travels much farther before it stops

What’s different is that the surface has smaller friction the interaction between the belly of puck & floor retards puck’s motion

(b) Waxed floor: puck slides farther
Examine motion of the hockey puck under a similar push (force) on an **air-hockey table**

Puck now travels even farther before it stops

What’s different is that the surface has even smaller **friction**

In absence of friction, puck would never slow down and we **won't** need a force to keep it moving
Newton’s First Law Of Motion

- Actually first noticed by Galileo in his experiments with smooth surfaces (technology!)
- Later generalized by Newton:

When there is no net force acting on an object, the object maintains its motion with a constant velocity *(zero acceleration)*

Conversely:
When an object moves with constant velocity, the net force acting on it must be zero

Tendency of an object to maintain its state of rest or uniform motion is called *inertia*
Newton’s 2\textsuperscript{nd} Law Of Motion

\[ \overrightarrow{F} \propto \overrightarrow{a} \]
\[ \overrightarrow{F} = m\overrightarrow{a} \quad \text{or} \quad \overrightarrow{a} = \frac{\overrightarrow{F}}{m} \]

\( m = \text{inertial mass} = \text{measure of an object's resistance to change in its motion (The "Inertia" of 1st law)} \)
Mass Of An Object

\[ \frac{\sum \vec{F}}{\vec{a}} \]

is property of the object & is constant regardless of the magnitude of the applied force

The larger the mass of an object, harder it is to change its velocity. e.g.: Truck Vs Baseball

Definition of inertial mass: \( m = \frac{\sum \vec{F}}{\vec{a}} \); SI unit = kg

The definition tells us how to measure mass

*No matter where you measure it, Earth, Mars or Pluto, mass of an object is always the same!*
Newton’s 2\textsuperscript{nd} Law Of Motion

\[ \vec{F} \propto \vec{a} \]

At all points in circular motion, \( \vec{a} \) & \( \sum \vec{F} \) are in same direction: towards the center
Measuring Mass

• To measure mass, “you” have to “mess” with object
  measure change in its motion due to applied force

• Masses add like scalars

\[ \vec{a}_1 = \sum \vec{F} / m_1 \]

\[ \vec{a}_2 = \sum \vec{F} / m_2 \]

\[ \vec{a}_3 = \sum \vec{F} / (m_1 + m_2) \]
More on Force

\[ \vec{F} = m\vec{a} \] is NOT the DEFINITION of Force!
Equation shows the reaction of the object to a force

SI Unit of force is the NEWTON

1 Newton is the amount of net force that gives an acceleration \( a = 1 \text{ m/s}^2 \) to an object of mass \( m = 1.0 \text{ kg} \)

\[ 1 \text{ N} = 1\text{kg.m/s}^2 \]

In CGS units: 1 dyne = 1 g.cm/s\(^2\) = 10\(^{-5}\) Newton

In British units: 1 Pound=1 slug.ft/s\(^2\) = 4.45N
The Force Vector

• Force is a *push or pull* on some object
• Gives quantitative description of interaction between an object(s) & its environment
  – i.e: pulling a box

• Force is a vector:
  – has direction and magnitude
  – SI unit of magnitude is the Newton (N)
  – Force of gravitational attraction that earth exerts on your body is your weight
Two Kinds Of Forces: Contact & Field

- **Contact forces**: (a) Hand pushing against a wall, (c) Kicking a football.
- **Field forces**: (d) Magnetic force on a charged particle, (e) Electric force between charges, (f) Magnetic force on a magnet.

Symbols: 
- $m$: Mass
- $M$: Mass
- $q$: Charge
- $Q$: Charge
- Iron: Magnetic material
- N, S: North and South poles of a magnet.
<table>
<thead>
<tr>
<th>Description</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun’s gravitational force on the earth</td>
<td>$3.5 \times 10^{22}$</td>
</tr>
<tr>
<td>Thrust of a space shuttle during launch</td>
<td>$3.1 \times 10^7$</td>
</tr>
<tr>
<td>Weight of a large blue whale</td>
<td>$1.9 \times 10^6$</td>
</tr>
<tr>
<td>Maximum pulling force of a locomotive</td>
<td>$8.9 \times 10^5$</td>
</tr>
<tr>
<td>Weight of a 250-lb linebacker</td>
<td>$1.1 \times 10^3$</td>
</tr>
<tr>
<td>Weight of a medium apple</td>
<td>1</td>
</tr>
<tr>
<td>Weight of smallest insect eggs</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>Electric attraction between the proton and the electron</td>
<td>$8.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>in a hydrogen atom</td>
<td></td>
</tr>
<tr>
<td>Weight of a very small bacterium</td>
<td>$1 \times 10^{-18}$</td>
</tr>
<tr>
<td>Weight of a hydrogen atom</td>
<td>$1.6 \times 10^{-26}$</td>
</tr>
<tr>
<td>Weight of an electron</td>
<td>$8.9 \times 10^{-30}$</td>
</tr>
<tr>
<td>Gravitational attraction between the proton and the electron</td>
<td>$3.6 \times 10^{-47}$</td>
</tr>
<tr>
<td>in a hydrogen atom</td>
<td></td>
</tr>
</tbody>
</table>
Instrument For Measuring Forces

Spring balance is coil spring attached to a pointer. When force applied, spring stretches some. Amount of stretch directly proportional to force.
Superposition of Forces

When two forces $\vec{F}_1$ & $\vec{F}_2$ act simultaneously on an object, experiments show that the net effect on object's motion is same as effect of single force: $\vec{F}_{net} = \vec{R} = \vec{F}_1 + \vec{F}_2$

Generally true for any number $n$ of forces: $\vec{F}_{net} = \vec{R} = \sum_{n} \vec{F}_n$
Equilibrium Condition For Motion

When an object is acted on by no force or by several forces such that net force=\(0\), the object is said to be in equilibrium. In this state body is either at rest or moving in a straight line with constant velocity.

\[
\sum \vec{F} = 0 \quad \text{(body in equilibrium)}
\]

\[
\Rightarrow \quad \sum \vec{F}_x = 0 = \sum \vec{F}_y
\]
Net Force $\vec{F}_{\text{net}} = \vec{R} = \sum_{n} \vec{F}_n$

Net force is the vector sum of the three forces tugging at the bear in the horizontal plane

$\vec{F}_1 = (-5\text{N})\hat{i} \quad ; \quad \vec{F}_2 = (5\text{N})\hat{k} \quad ; \quad \vec{F}_3 = (-5\text{N})\hat{i} + (5\text{N})\hat{k}$

$\vec{F}_{\text{net}} = \vec{F} + \vec{F}_2 + \vec{F}_3 = 0!$
Components Of A Force Vector

Superposition of Force principle \( \Rightarrow \)

Any arbitrary force \( \vec{F} \) can be broken into components

\[
\vec{F} = \vec{F}_x + \vec{F}_y
\]
Pop Quiz:

• All bodies, regardless of mass, accelerate downwards at 9.8 m/s² at the surface of the earth. This means:

  • (A) Gravity’s force is independent of mass
  • (B) is proportional to mass
  • (C) goes as the square of the mass
Weight on Earth & Moon

Standard KG weighs 9.80N on earth

Standard KG weighs 1.6N on moon

Want to lose weight? Go to the moon!
Measuring Mass & Weight

On earth, an easy way to measure mass is by measuring its weight and comparing with the standard kg

\[ m_1a_1 = m_2a_2 \]

In Zero gravity you weigh nothing so to measure mass, use 2nd law

Apply a force & measure object's acceleration

\[ m = \frac{|\vec{F}|}{|\vec{a}|} \]
A Consequence of The 2\textsuperscript{nd} Law

Large $F$ implies large $a$ and vice-versa
Human body not capable of handling large $F$
To stop an accelerating body safely, need to decelerate slowly and over a "long" period of time.
This concept crucial for airbag design, fighter pilot ejector seats, bungee cords & even saving Lois Lane!
Ejection Seat In Fighter Planes

rocket to slowly accelerate pilot vertical to flight direction

used to be explosives, high acc. over short time = BAD