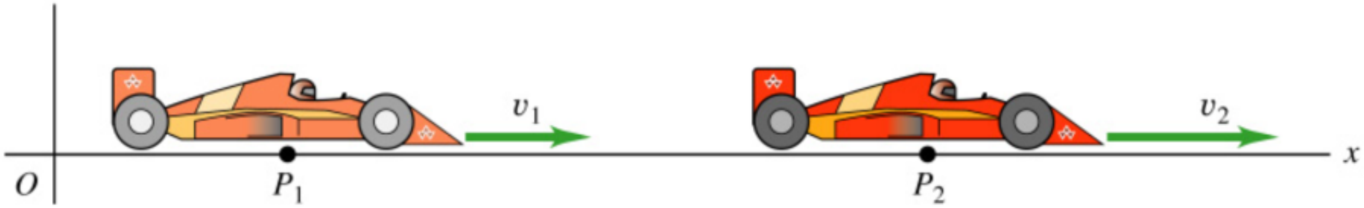
Physics 4A
Lecture 2: Jan. 8, 2015

Sunil Sinha

UCSD Physics

Average & Instant. Acceleration



$$\text{Average Acceleration } a_{av\text{-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

Instant acceleration = limit of average accelaration

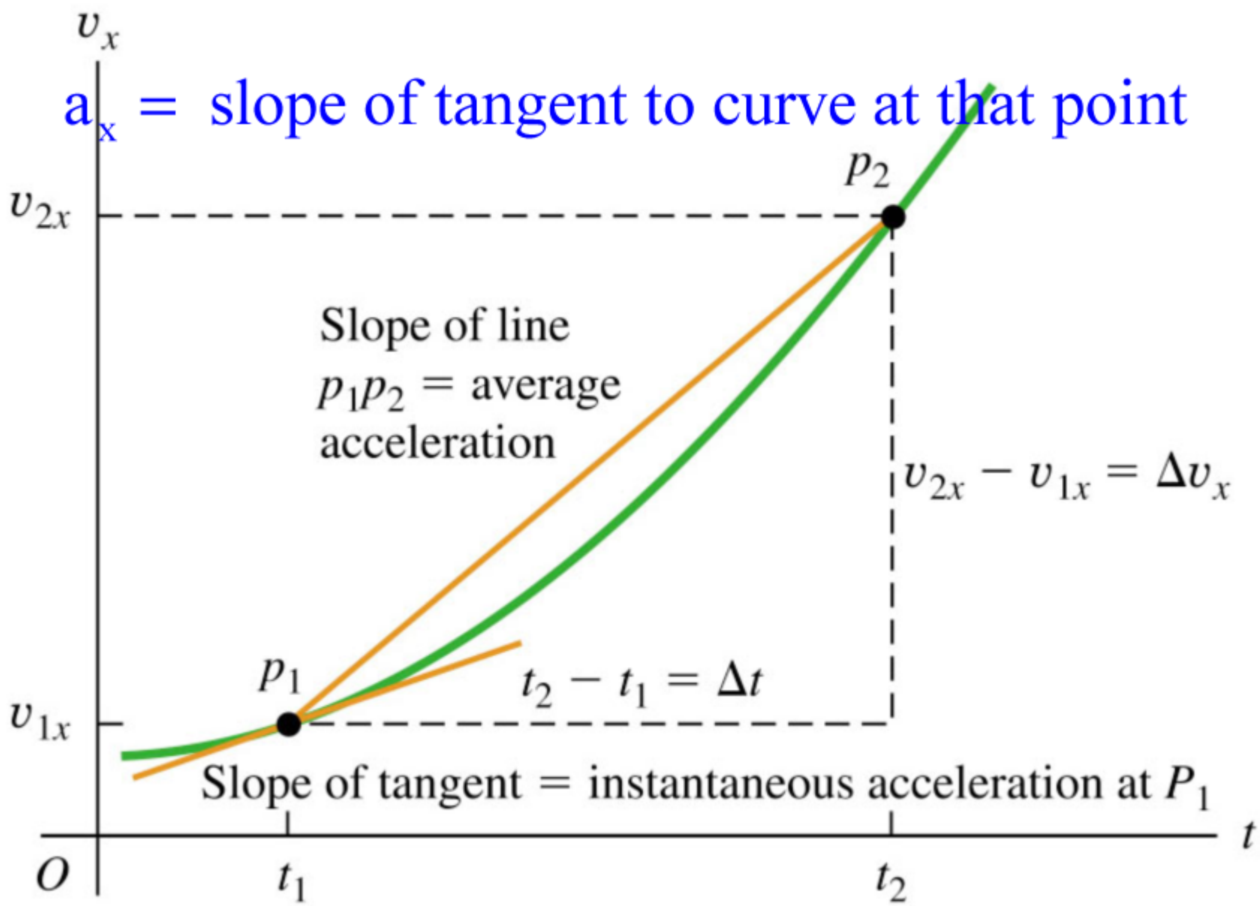
when time $\Delta t \to 0$. $$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

Acceleration has units of (m/s^2)

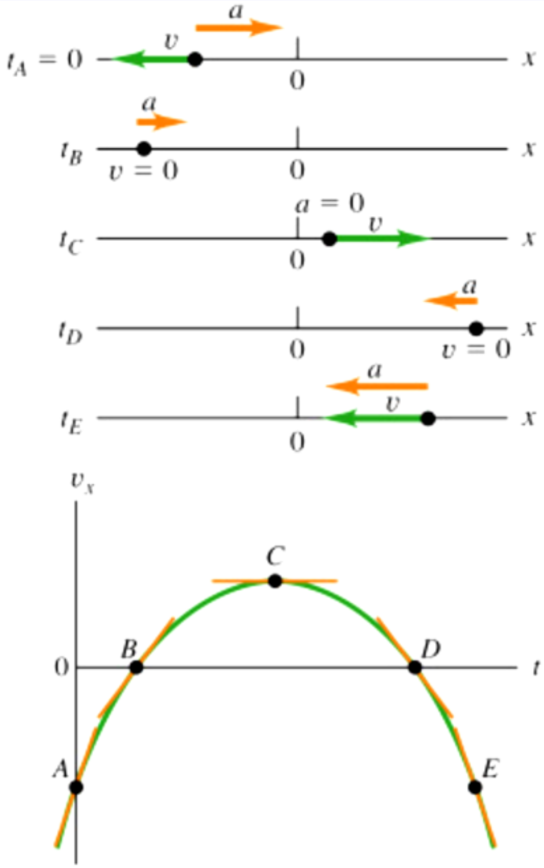
Now on, use acceleration to mean instant acceleration

Acceleration A_x On A v_x- t Graph

a_x = slope of tangent to curve at that point

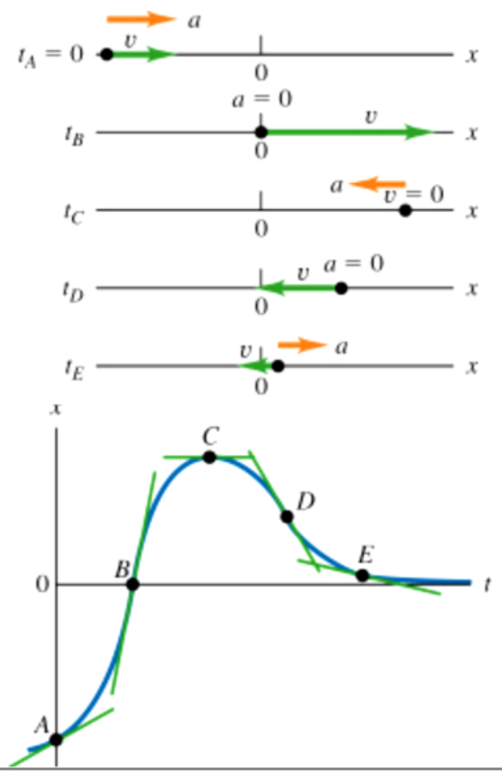


Examining a v_x – t Graph



	v_x-t graph	Motion of particle
A	$v_x < 0$; positive slope, so $a_x > 0$	moving in $-x$-direction, slowing down
B	$v_x = 0$; positive slope, so $a_x > 0$	instantaneously at rest, about to move in $+x$-direction
C	$v_x > 0$; zero slope, so $a_x = 0$	moving in $+x$-direction at maximum speed
D	$v_x = 0$; negative slope, so $a_x < 0$	instantaneously at rest, about to move in $-x$-direction
E	$v_x < 0$; negative slope, so $a_x < 0$	moving in $-x$-direction, speeding up

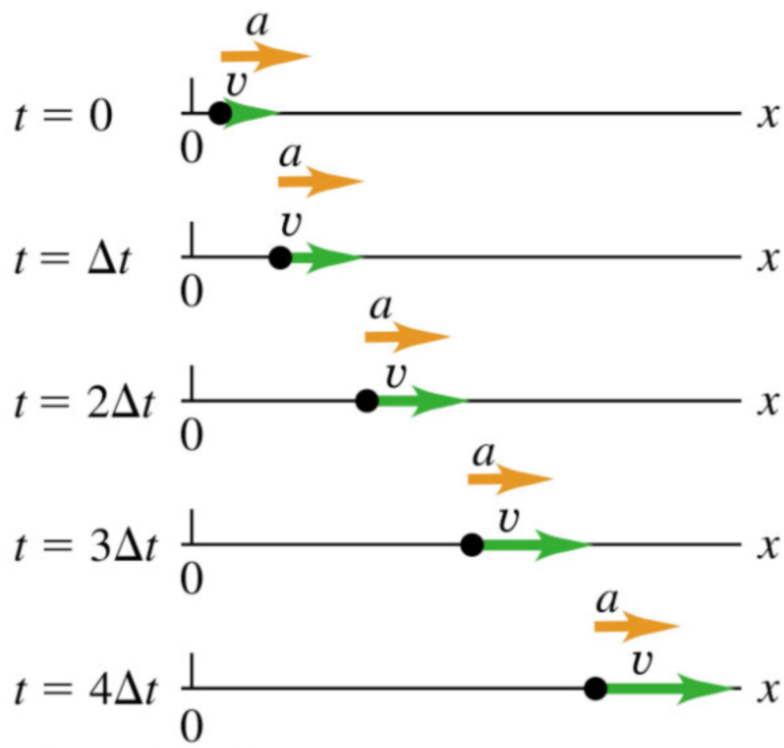
The x-t Graph For Same Motion



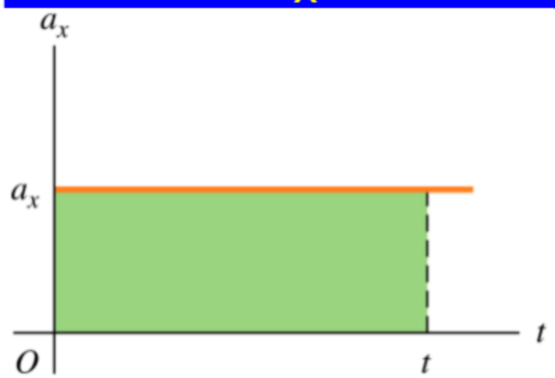
	x-t graph	Motion of particle
A	positive slope, upward curvature, so $v_x > 0$, $a_x > 0$	moving in $+x$-direction, speeding up
B	positive slope, zero curvature, so $v_x > 0$, $a_x = 0$	moving in $+x$-direction, speed not changing
C	zero slope, downward curvature, so $v_x = 0$, $a_x < 0$	instantaneously at rest, velocity changing from + to −
D	negative slope, zero curvature, so $v_x < 0$, $a_x = 0$	moving in $-x$-direction, speed not changing
E	negative slope, upward curvature, so $v_x < 0$, $a_x > 0$	moving in $-x$-direction, slowing down

Motion With Constant Acceleration

Motion diagram showing position, velocity & acceleration of an object



The a_x– t Graph For Constant a_x



$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \text{constant}$$

Start clock at $t_1 = 0$, observe again at time $t_2 = t$

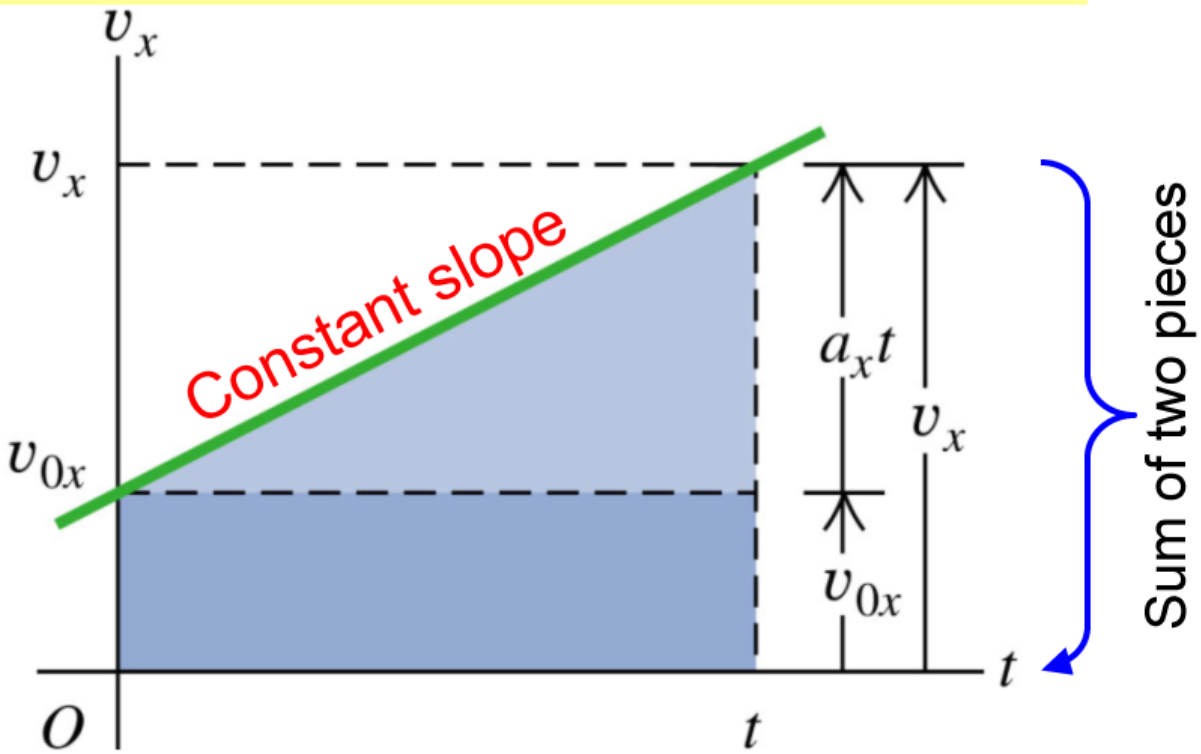
Call v_{0x} the velocity at $t_1 = 0, v_x$ the velocity at $t_2 = t$

$$a_x = \frac{v_x - v_{0x}}{t - 0}$$

$$\Rightarrow \quad \boxed{v_x = v_{0x} + a_x t} \;\; ; \; a_x = \text{constant}$$

The v_x-t Graph For Constant a_x

$$v_x = v_{0x} + a_x t \quad ; \; a_x = \text{constant}$$



Evolution of x vs t when a_x=Constant

At time $t_1 = 0$, object at $x = x_0$, has $v_{x1} = v_{0x}$

At time $t_2 = t$, object at $x = x$, has $v_{x2} = v_x$

then $\boxed{v_{av\text{-}x} = \dfrac{x - x_0}{t}}$

When a_x = constant, velocity changes at const rate

so for time interval $0 \rightarrow t$, $v_{av\text{-}x} = \dfrac{v_{0x} + v_x}{2}$

But since $v_x = v_{0x} + a_x t \Rightarrow v_{av\text{-}x} = \dfrac{1}{2}\left(v_{0x} + v_{0x} + a_x t\right)$

$$= \boxed{v_{0x} + \frac{1}{2} a_x t}$$

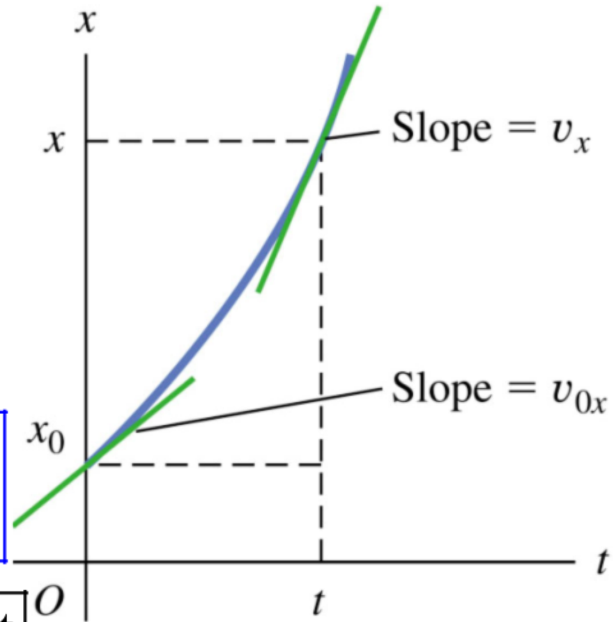
Evolution of x vs t when a_x=Constant

$$v_{av\text{-}x} = \frac{x - x_0}{t}$$

$$= v_{0x} + \frac{1}{2} a_x t$$

$$\Rightarrow x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$\Rightarrow$ Parabolic curve in x-t



Relating x, v_x & a_x (without time t)

write $t = \frac{v_x - v_{0x}}{a_x}$

substitute in $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$$\Rightarrow x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

$$\Rightarrow (x - x_0)2a_x = \boxed{2v_{0x}v_x} - 2v_{0x}^2 + v_x^2 \boxed{-2v_{0x}v_x} + v_{0x}^2$$

$$\Rightarrow \boxed{v_x^2 = v_{0x}^2 + 2a_x(x - x_0)}$$

An Expression Without a_x

Since $v_{av\text{-}x} = \dfrac{x - x_0}{t}$ and $v_{av\text{-}x} = \dfrac{v_{0x} + v_x}{2}$

$$\Rightarrow \frac{x - x_0}{t} = \frac{v_{0x} + v_x}{2}$$

$$\Rightarrow \quad x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t$$

This is a useful expression to have

when a_x = constant but unknown

Using Calculus

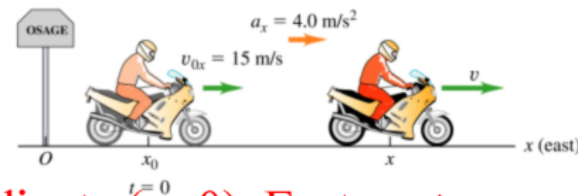
$$v = dx / dt$$

$$a = dv / dt = d^2x / dt^2$$

$$v = \int a\,dt = at + C = v_0 + at$$

$$x = \int v\,dt = \int (v_0 + at)dt = v_0 t + \frac{1}{2}at^2 + C = x_0 + v_0 t + \frac{1}{2}at^2$$

Motorcyclist going east, accelerates after passing signpost. He accelerates at $4.0 m/s^2$. At t=0, he is 5.0m east of signpost, moving east 15 m/s. (a) find his position and velocity at t=2.0s. Where is motorcyclist when his velocity is 25 m/s ?



Take signpost as origin of coordinate (x=0), East → +x

At $t=0, x_0=5.0m, v_{0x}=15m/s; a_x=4.0m/s^2$

(a) what is x, v_x at t=2.0s, (b) x when $v_x=25m/s$

(a) Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

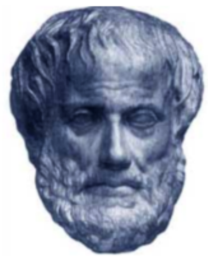
$$= 5.0m + (15m/s)(2.0s) + \frac{1}{2}(4.0m/s^2)(2.0s)^2 = 43m$$

Velocity at x=43m: $v_x = v_{0x} + a_x t = 23m/s$

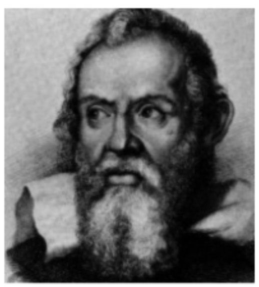
(b) no t given ! so use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$$\Rightarrow x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x} = 5.0m + \frac{(25m/s)^2 - (15m/s)^2}{2a_x} = 55m$$

Motion With Constant Acceleration: Freely Falling Bodies



Aristote (4 BC) believed (didn't check!) that heavier objects fall faster through a medium than lighter ones

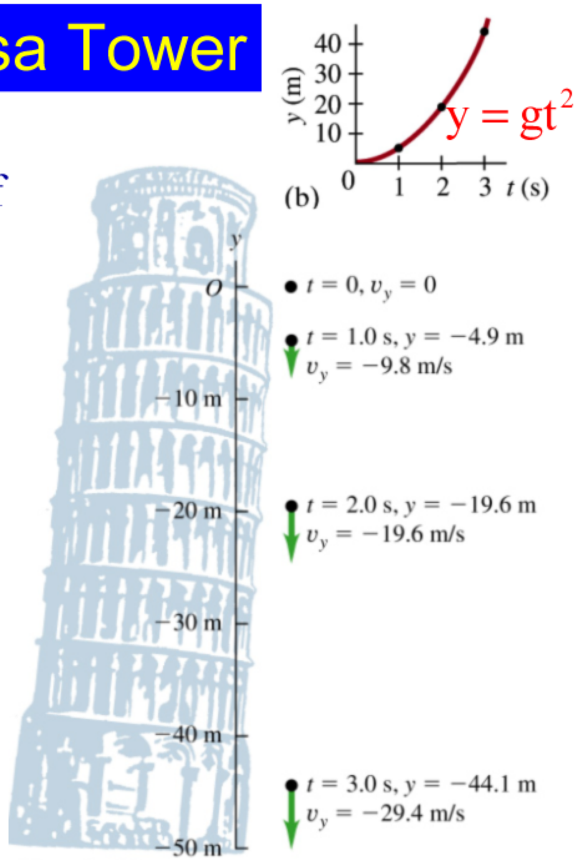


19 centuries later, Galileo did some experiments, disproved this by asserting that all objects falling freely experience a downward acceleration that is constant and independent of object's weight

Go to the wikipedia site on this, http://en.wikipedia.org/wiki/Galileo's_Leaning_Tower_of_Pisa_experiment and there is an embedded video of this experiment being done by a US astronaut on the surface of the moon!!

Free Fall From Pisa Tower

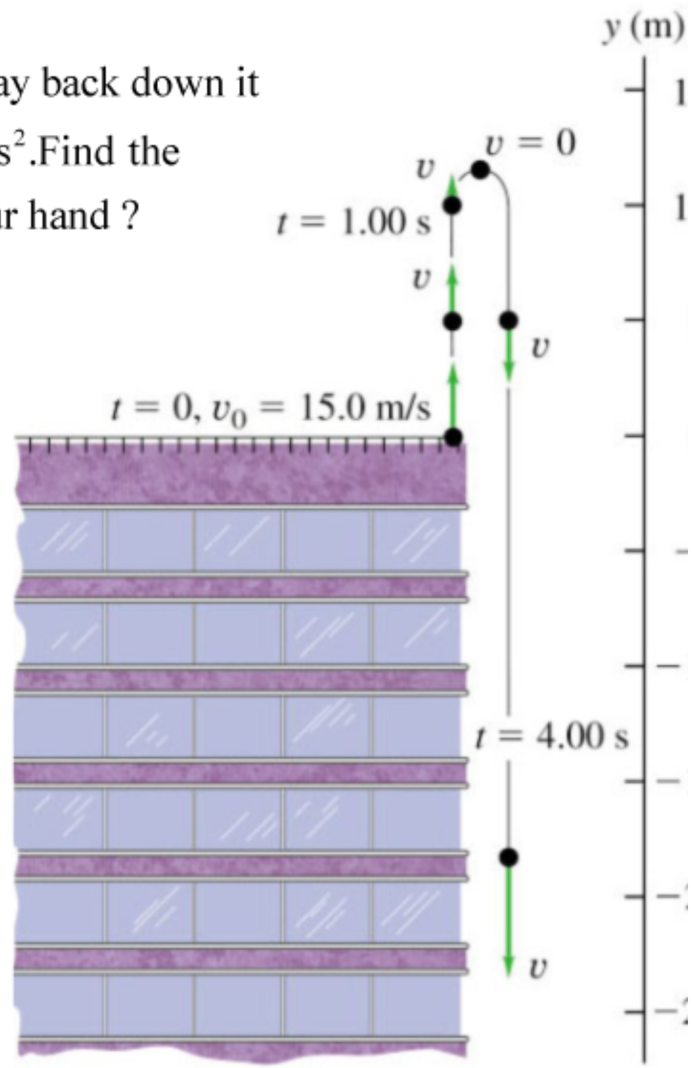
- Examine a falling object
- Free fall: An idealization of the motion where one ignores "small" effects like
  - Air
  - Earth's rotation
  - Altitude at location etc
- Free fall is motion with constant acceleration
  - Down or up
- Acceleration $g = -9.8\ m/s^2$ on earth, $-1.6 m/s^2$ on moon & $-270\ m/s^2$ on the sun



Inclined Plane Demo By Galileo

Giuseppe Bezzuoli, Tribuna di Galileo, Firenze

You throw a ball vertically upwards from roof of a building. Ball leaves your hand at point even with the roof railing with an upward speed of 15.0m/s; ball is then in free fall. On its way back down it just misses the railing. Acceleration due to gravity g =9.80m/s^2.Find the position and velocity of ball 1.00s and 4.00s after leaving your hand ?



Motion is in straight line but vertical (y axis).

y=0 is at the roof and +y direction is upwards

$$\text{Initial position } y_0 = 0,$$

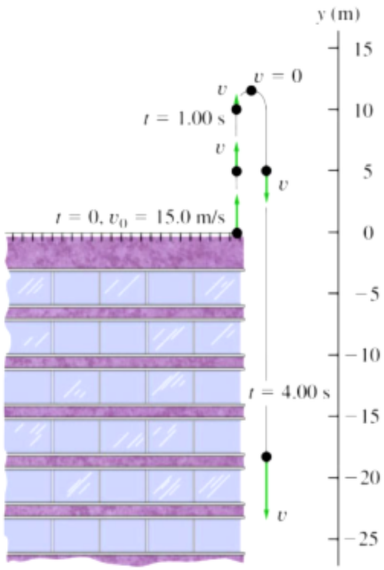
$$v_{0y} = +15.0 m/s,$$

$$a_y = g = -9.80 m/s^2 \text{ down}$$

Find x & v at t=1.00s,4.00s

Position y and velocity v_y after ball leaves hand obtained

from $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2; v_y = v_{0y} + a_y t$ $[a_y = g = -9.80 m/s^2]$



$$y(t = 1s) = 0 + (15.0 m/s)(1s) + \frac{1}{2}(-9.80 m/s^2)(1)^2$$
$$= +10.1m \text{ (above roof)}$$
$$v_y(t = 1s) = 15.0 m/s + (-9.80 m/s^2)(1s)$$
$$= +5.2 m/s \text{ (going upwards)}$$

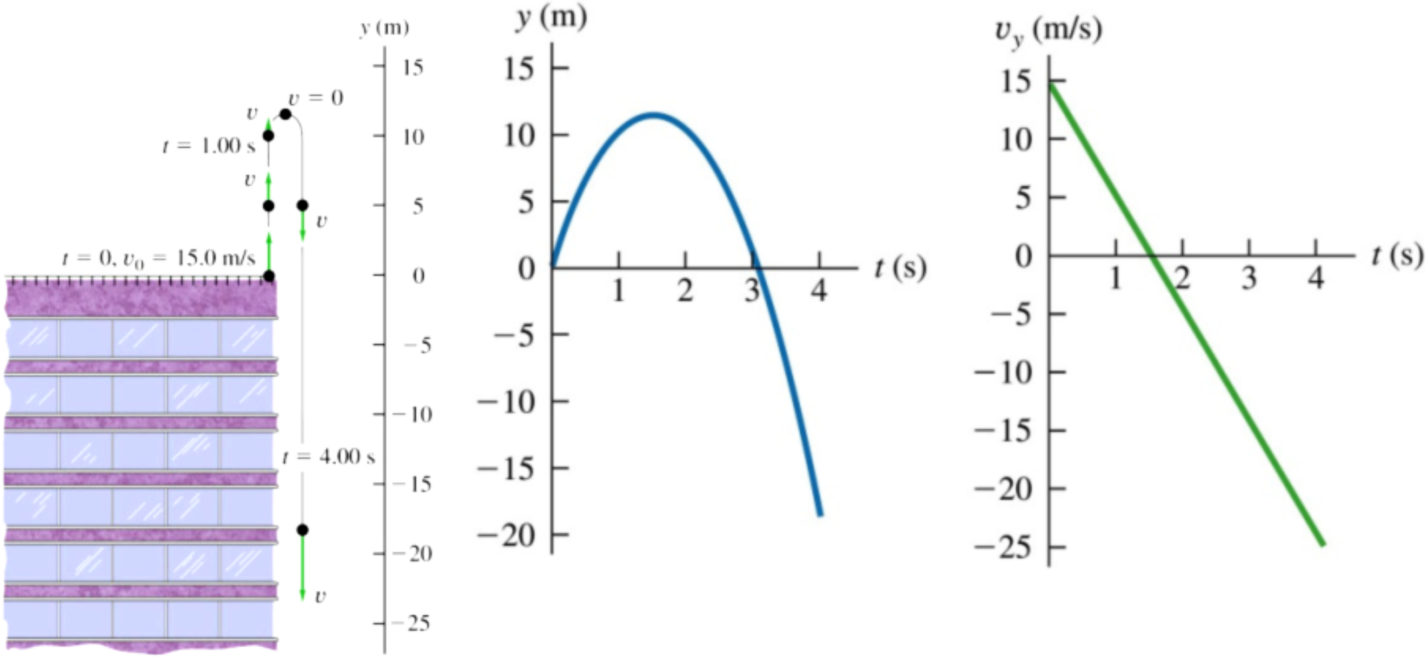
Similarly :

$y(t = 4s) = -18.4m$ (below roof)

$v_y(t = 4s) = -24.2 m/s$ (going downwards)

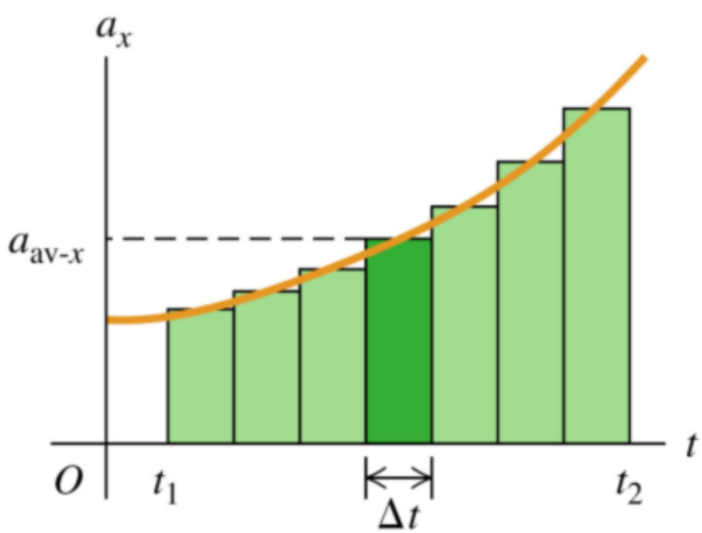
This is due to the pull of gravity !

Description With y-t and v-t Graphs



Now: Case when $a = a(t) \neq$ constant

Graph of acceleration Vs time



Use Calculus, divide interval between t_1 & t_2 in slices of Δt

Change in velocity $\boxed{\Delta v_x = a_{av-x}\Delta t}$

= area of shaded strip with height a_{av-x} & width Δt

Total velocity change from $t_1 \to t_2$

= total area under $a_x - t$ curve between vertical lines t_1 & t_2

In Calculus parlance, as $\Delta t \to 0$:

Velocity change = integral of a_x with t

$$\boxed{v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x dt}$$

Case when $a = a(t) \neq$ constant

Similarly since $v_x = \frac{dx}{dt} \Rightarrow dx = v_x dt$

The change in position $$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x dt$$

In Conclusion:

$$v_x = v_{0x} + \int_{t=0}^{t=t} a_x dt$$ and $$x = x_0 + \int_{t=0}^{t=t} v_x dt$$

Sally driving along a straight highway. At t=0, Sally is moving at 10m/s in +x dir when she passes signpost at x=50m

Her acceleration is a= $a(t) = 2.0\text{m/s}^2 - (0.10\text{m/s}^3)t$

Find (a) expression for v & x vs t (b) when is v largest & how much is it ? (c) where is car when it reaces this max. v ?

Use $v_x = v_{0x} + \int_{t=0}^{t=t} a_x dt$ & $x = x_0 + \int_{t=0}^{t=t} v_x dt$

At t=0, $x_0 = 50\text{m}, v_{0x} = 10\text{m/s}$, find $v_x = v_x(t)$

$$v_x = 10\text{m/s} + \int [2.0\text{m/s}^2 - (0.10\text{m/s}^3)t]dt$$

Use $\int t^n dt = \frac{t^{n+1}}{n+1}$ $\Rightarrow v_x = 10\text{m/s} + (2.0\text{m/s}^2)t - \frac{1}{2}(0.10\text{m/s}^3)t^2$

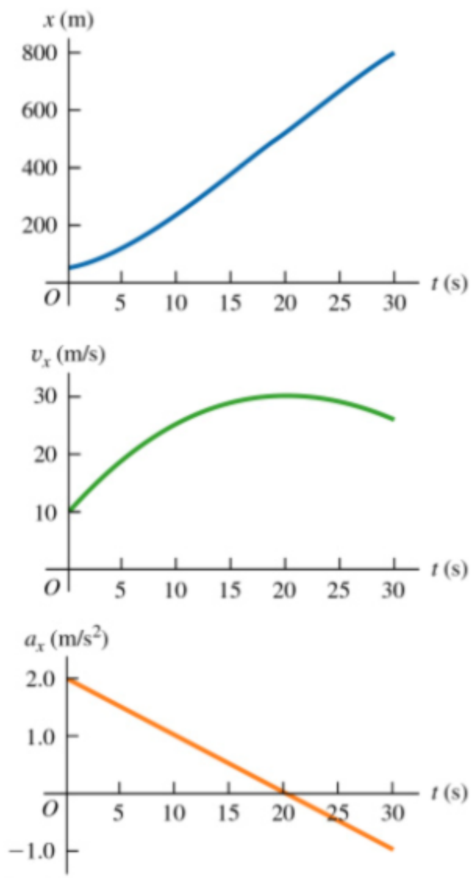
$$\text{and } x = 50\text{m} + \int [10\text{m/s} + (2.0\text{m/s}^2)t - \frac{1}{2}(0.10\text{m/s}^3)t^2]dt$$

$$\Rightarrow x = 50\text{m} + (10\text{m/s})t + \frac{1}{2}(2.0\text{m/s}^2)t^2 - \frac{1}{2\times 3}(0.10\text{m/s}^3)t^3$$

Maximum value of v_x when $\frac{dv_x}{dt} = a_x = 0$, Using v_x expression

$$\Rightarrow a_x = 0 = 2.0\text{m/s}^2 - (0.10\text{m/s}^3)t \Rightarrow \boxed{t = 20\text{s}}$$

Graph of x, v_x & a_x



$$v_{x-max} = v_x(t = 20s) = 10m/s + (2.0m/s^2)(20s) + \frac{1}{2}(0.10m/s^3)(20s)^2$$

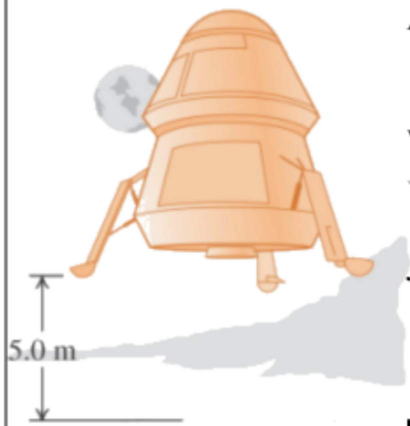
$$= 30m/s$$

To get position x at t=20s when v_x=maximum

Input t = 20s in $x(t) = 50m + (10m/s)t + \frac{1}{2}(2.0m/s^2)t^2 - \frac{1}{6}(0.10m/s^3)t^3$

$$=517m$$

Touchdown On The Moon



A lunar lander is making its descent to moon base. The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0m above surface and has a downward speed of 0.8m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches surface. $g_{moon} = 1.6 m/s^2$

Apply constant acceleration equations to the motion of the lander

Let downward be positive. Lander is in freefall $\Rightarrow a_y = g_{moon}$

What we know: $v_{0y} = +0.8 m/s$, $y - y_0 = 5.0 m$, $a_y = 1.6 m/s^2$, no idea about t!

$$\text{Use } v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\Rightarrow v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8 m/s)^2 + 2(1.6 m/s^2)(5.0 m)}$$

$$= 4.1 m/s$$

The same descent on Earth would have led to $v_y = 9.9 m/s$ due to the stronger acceleration due to gravity g.

Spiderman steps from the top of a tall building. He falls freely from rest to the ground a distance of h. He falls a distance of h/4 in the last 1.0s of his fall. What is the height h of the building?

Which equation to use? depends on what we know?

Divide Spidey's motion in 2 segments: $y = 0 \rightarrow y = 3/4h$ and $y = 3/4h \rightarrow h$

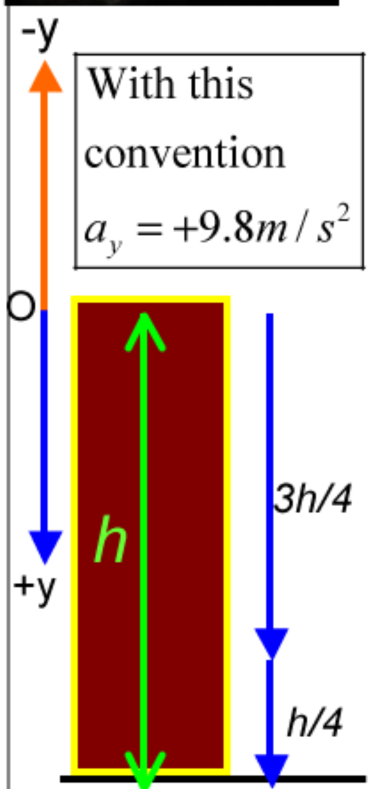
Motion from roof to h/4 above ground $\Rightarrow y - y_0 = 3/4h, v_0 = 0, a_y = g$

$$\text{Use } v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

So we get: $v_y^2 = 0 + a_y(3/4h) \Rightarrow v_y = \sqrt{2\, a_y(3/4h)} = 3.834\sqrt{h}\ \sqrt{\text{m}}/\text{s}$

Spiderman's speed after he has fallen for 3/4h is $\boxed{v_y = 3.83\sqrt{h}\sqrt{\ \text{m}}/\text{s}}$

With this convention $a_y = +9.8 m/s^2$



In the next segment, $y - y_0 = h/4$, $v_{0y} = 3.83\sqrt{h}\sqrt{\ \text{m}}/\text{s}$, $a_y = g$, $t = 1\text{s}$

clearly we should use: $y = y_0 + v_0 t + (1/2)a_y t^2$

$\Rightarrow (h/4) = 3.83\sqrt{h}\sqrt{\ \text{m}} + 4.90m.$ Now solve for h...but how ?

Write as $\frac{1}{4}u^2 - 3.83u\sqrt{m} - 4.90m = 0$, solve for u (quadratic eq)

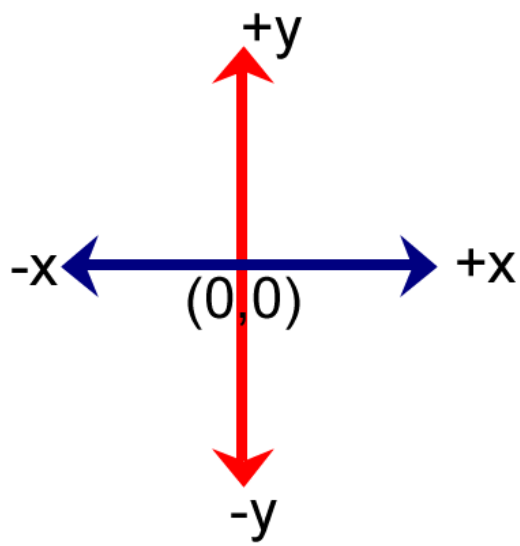
$$\text{if } au^2 + bu + c = 0 \Rightarrow u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Taking the positive root $\Rightarrow u = 16.52\sqrt{\text{m}} \Rightarrow \boxed{h = u^2 = 273m}$

Describing Physical Quantities

- Scalars → Quantities such as time, temperature, mass, speed can be described by just one number with an appropriate unit
  - math is simple: 2kg +3kg = 5kg (always!)
- Vectors→ Quantities with direction associated with them such as those quantifying motion (displacement, velocity)
  - needs a magnitude (how large or small)
  - needs a pointing direction (which way?)
  - math for these objects is more complicated

Need to Define A Reference Frame First

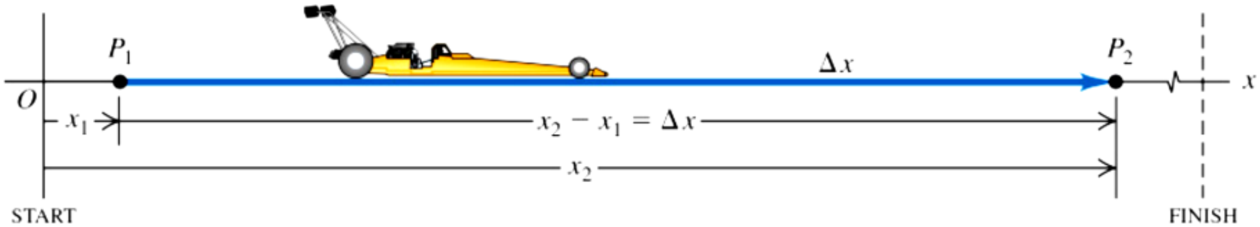


Defines for a displacement vector
which way is positive and which way is negative

Displacement Vector $\vec{x}$

Describe race car's motion by the that of a representative point on car→middle

Need a coordinate system to describe car's change in position
Choose x axis of coord. system to lie along car's straight line path



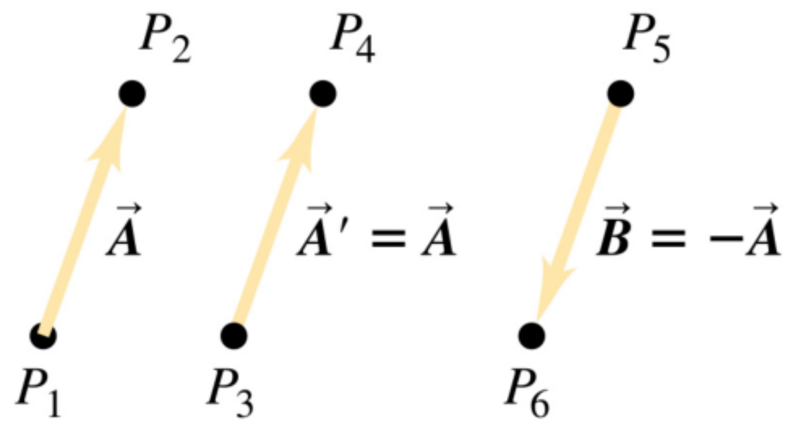
Displacement $\Delta x = x_2 - x_1$

Equal, Parallel & Anti Parallel Vectors

length of a vector A = its magnitude = | A |

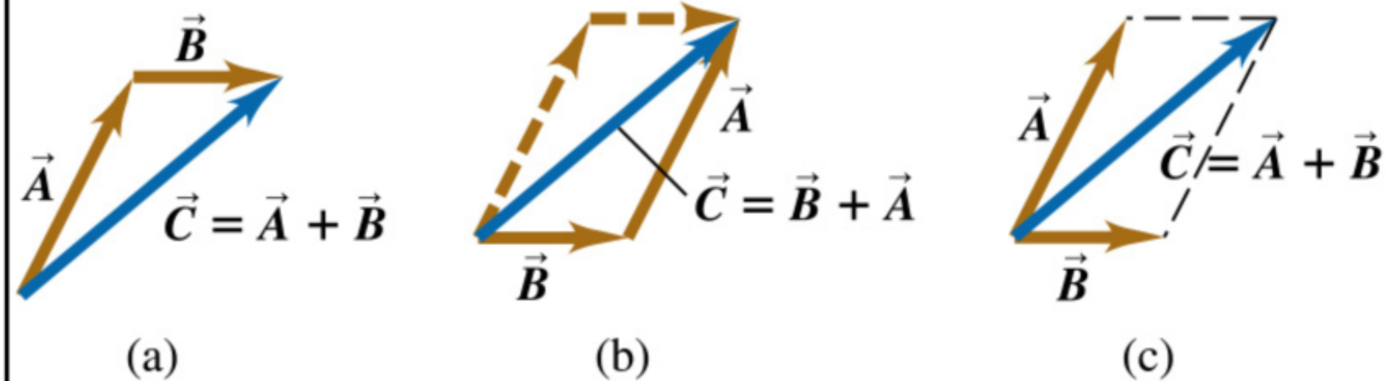
If two vectors have the same direction (but same or different magnitude then they are parallel (↑↑)

If two vectors have the opposite direction (but same or different magnitude) then they are anti-parallel (↑↓)



Vector Addition Is Commutative

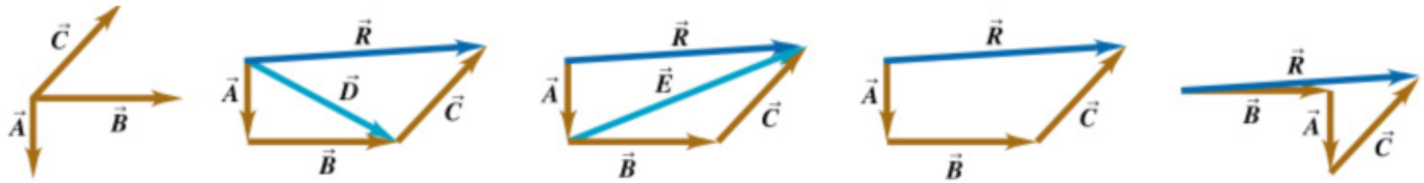
- Imagine a particle goes thru two consecutive displacements. Where is the particle at now ?



Vector addition is **commutative**, order of addition does not matter

Many-Vector Addition/Subtraction

To find the sum of many vectors, first find vector sum of any two, add the resultant vector to the next one vectorially and keep going



(a) (b) (c) (d) (e)

$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

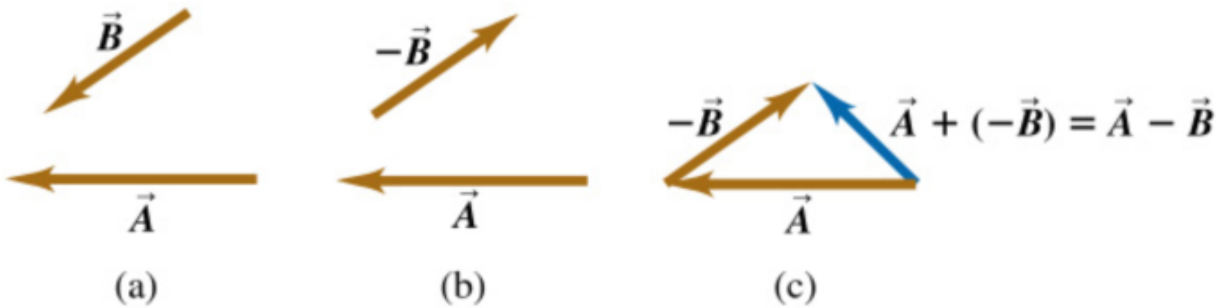
$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

Many ways to get to the same answer, as you could have guessed

Difference Of Two Vectors

Effectively an addition of two vectors: $\vec{A}$ and $-\vec{B}$

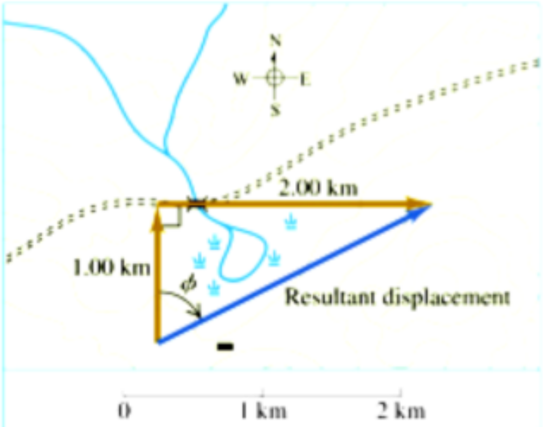
Just put tail of $-\vec{B}$ at the head of A



Check: $(\vec{A} - \vec{B}) + \vec{B} = \vec{A}$

Vector Addition: Using Scale Diagram

A skier skies 1.00km north then 2.00 km east on a horizontal ski field
(a) how far and in what direction is she from the starting point ?
(b) what is the magnitude and direction of her net displacement ?



- Draw a picture of the situation, use vector addition
- Vectors form a right triangle, length and direction of the hypotenuse = resultant displacement vector

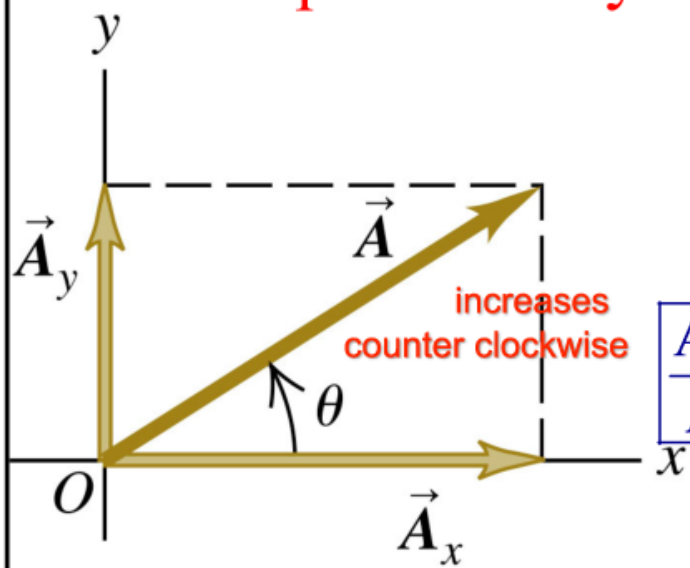
$$\text{Pythogoras Theorem} \Rightarrow \text{length} = \sqrt{(1.00\text{km})^2 + (2.00\text{km})^2} = 2.24\text{ km}$$

$$\tan\phi = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}} \Rightarrow \varphi = \tan^{-1}(2) = 63.4^{\circ}$$

Answer : 2.24km, 63.4° East of North or 26.6° North of East

Components Of A Vector

- In Cartesian coordinate system, you can represent any vector lying in x-y plane as sum of a vector parallel to x-axis and a vector parallel to y-axis



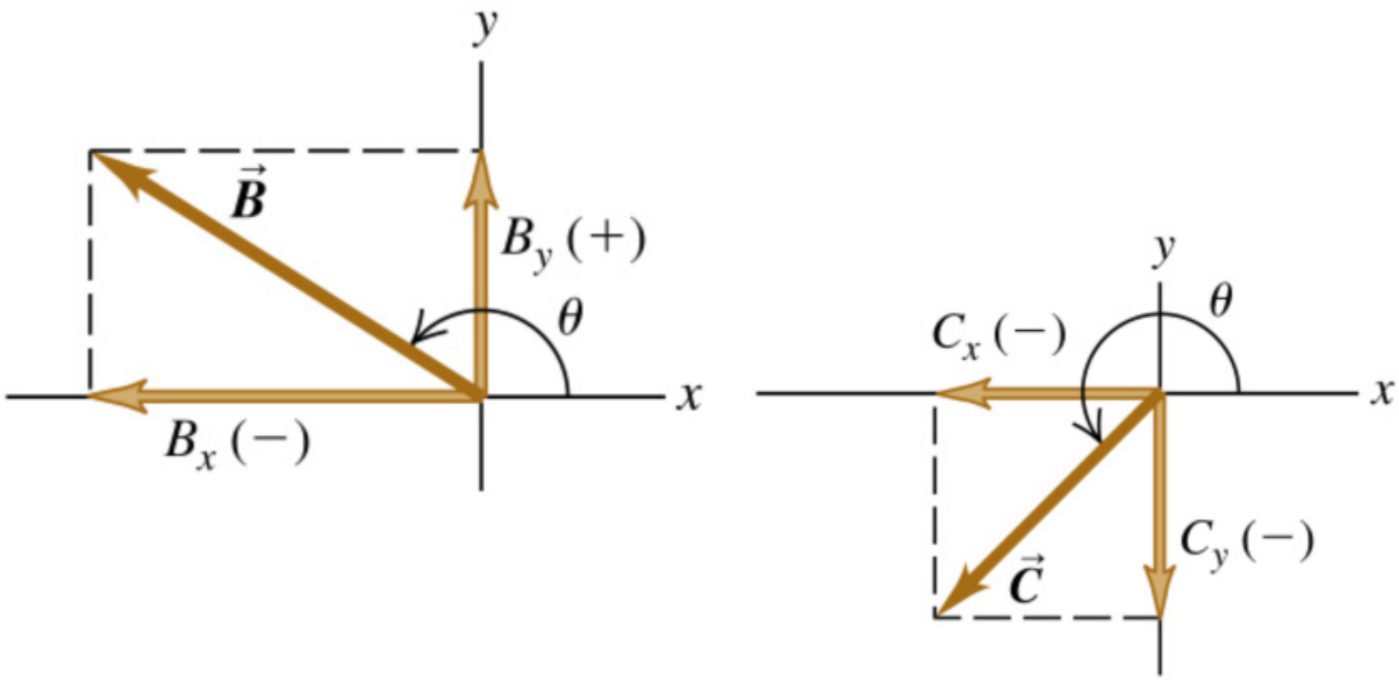
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Magnitudes A_x & A_y are components of $\vec{A}$

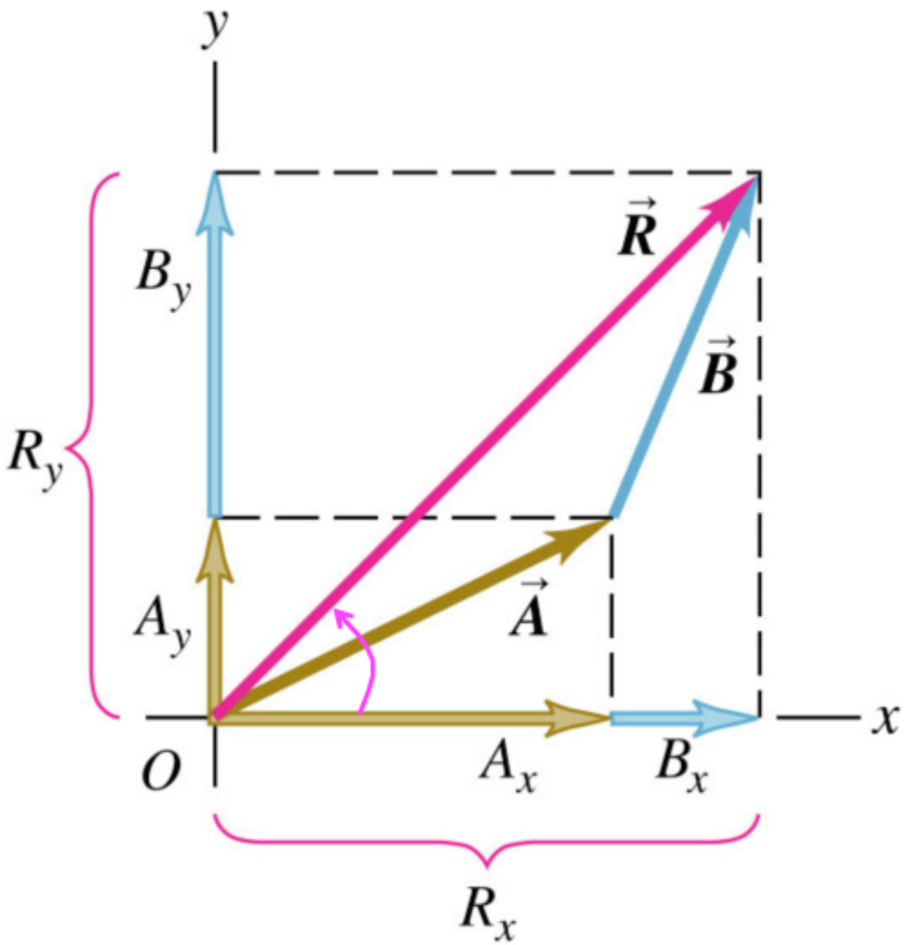
$$\frac{A_x}{A} = \cos\theta \qquad \frac{A_y}{A} = \sin\theta \qquad \frac{A_y}{A_x} = \tan\theta$$

$$A_x = A\cos\theta \; ; \; A_y = A\sin\theta$$

Vector components can be positive or negative depending on the vector orientation



Vector Addition Using Components



$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\boxed{\vec{R}_x = \vec{A}_x + \vec{B}_x}$$

$$\boxed{\vec{R}_y = \vec{A}_y + \vec{B}_y}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Unit Vectors

Unit vector is just a pointing vector

-describes a direction in space

-has magnitude of 1, with no unit

Unit Vector $\hat{i}$ *points in dir. of* +x

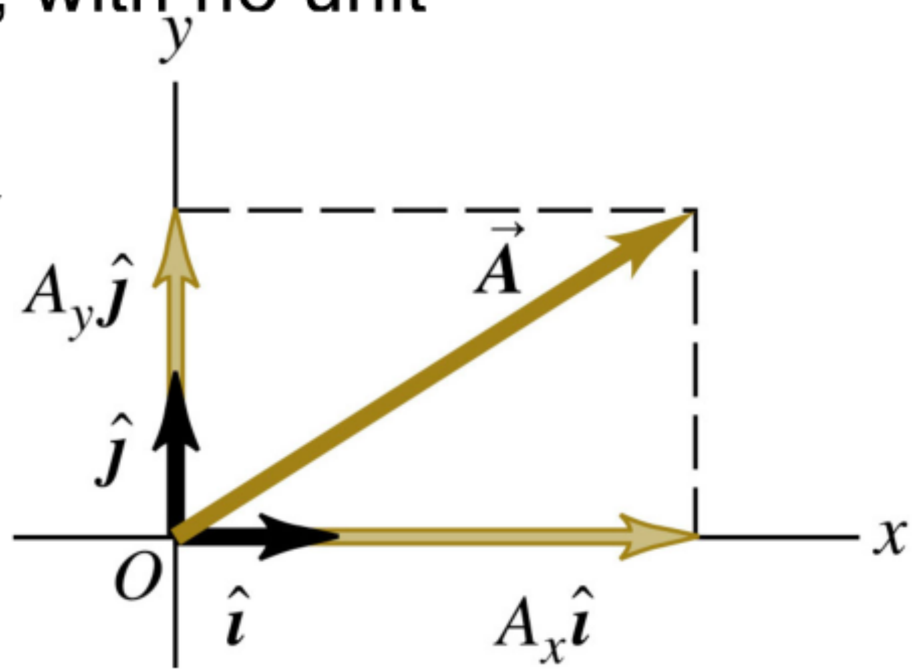
Unit Vector $\hat{j}$ *points in dir. of* +y

Relation between

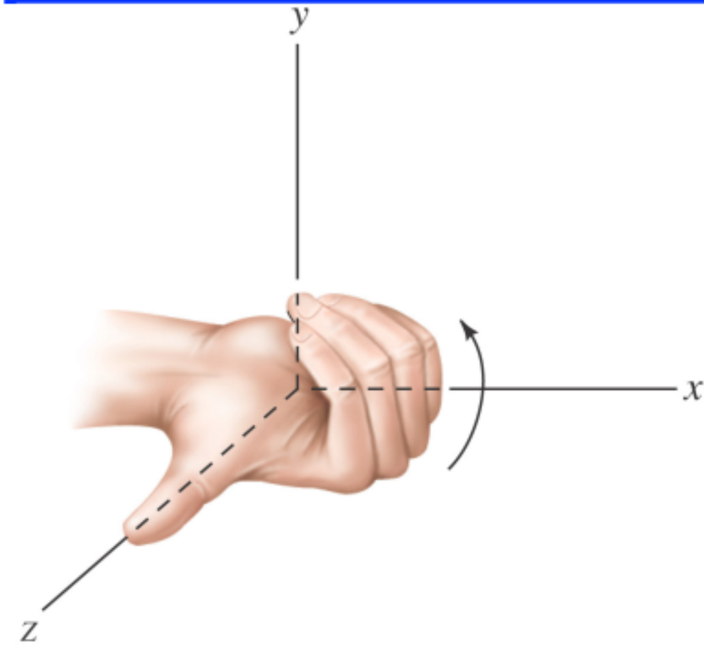
component vectors & component

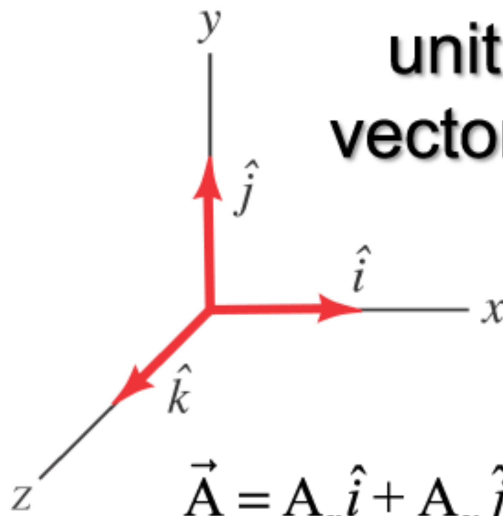
$\vec{A}_x = A_x \hat{i}$; $\vec{A}_y = A_y \hat{j}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$



Vectors in 3 Dimensional Space





unit vectors

Right hand rule specifies orientation of the 3 axes

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

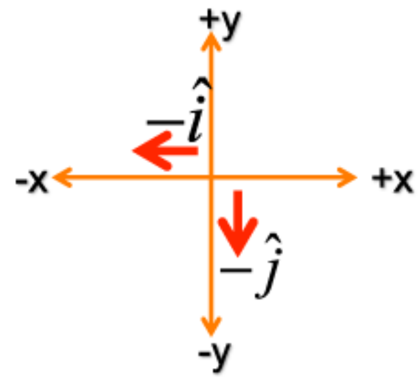
$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$\boxed{\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}}$$

More On Unit Vectors

$-\hat{i}$ is a unit vector in the direction of $-X$

$-\hat{j}$ is a unit vector in the direction of $-Y$



Pop Quiz

If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors then is the vector

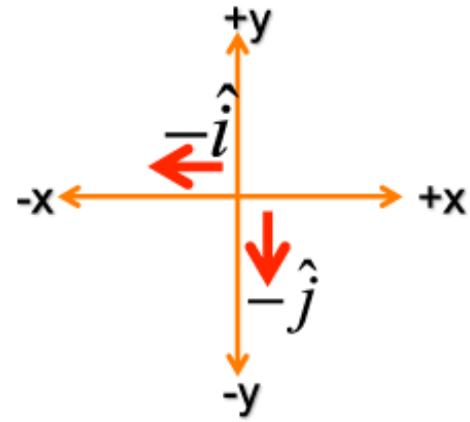
$\vec{r} = \hat{i} + \hat{j} + \hat{k}$ also a unit vector ?

A: Yes B: No

More On Unit Vectors

$-\hat{i}$ is a unit vector in the direction of $-X$

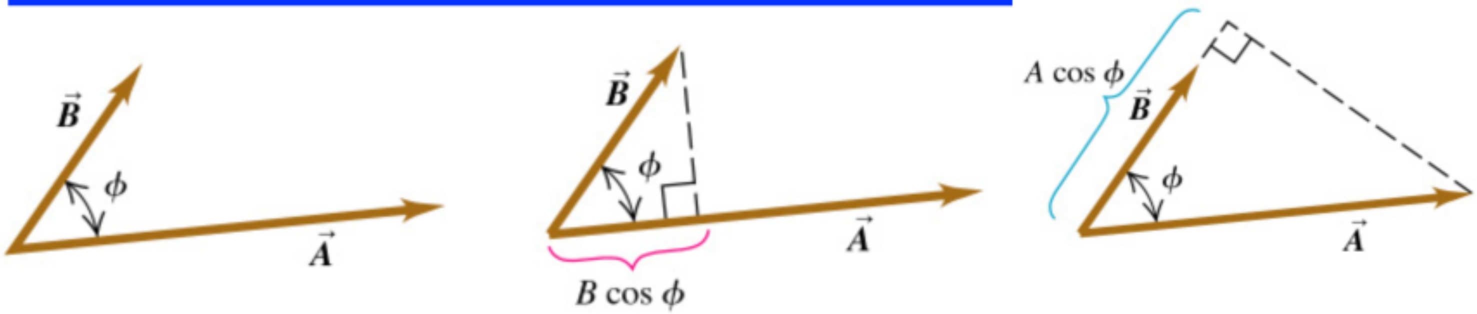
$-\hat{j}$ is a unit vector in the direction of $-Y$



Multiplying Vectors

- Scalar Product of 2 vectors
- Vector Product of 2 vectors

Scalar Product Of $\vec{A}$ & $\vec{B}$



Definition : $\vec{A} \bullet \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$

$= (B\cos\phi)A \quad = (A\cos\phi)B$

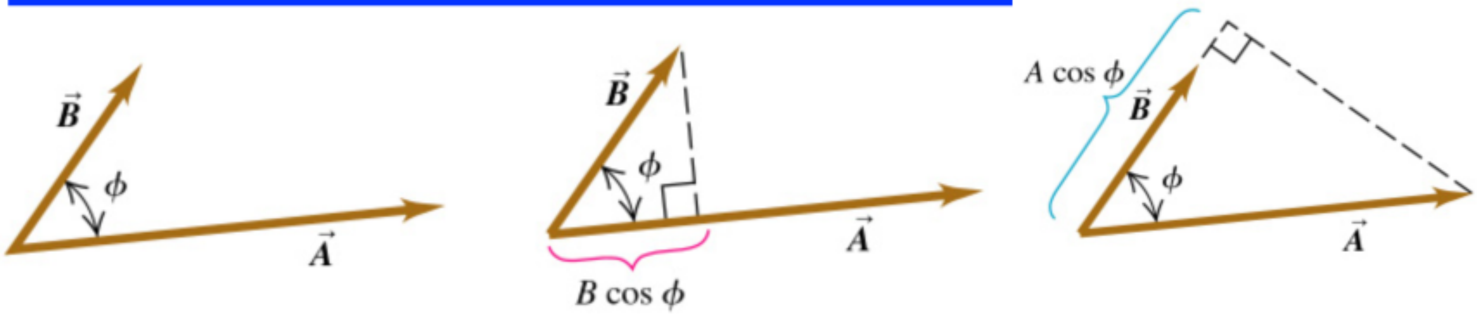
$\vec{A} \bullet \vec{B} =$ magnitude of B x proj of A on B

$\vec{A} \bullet \vec{B} =$ magnitude of A x proj of B on A

Largest when $\vec{A} \parallel \vec{B}$; Zero when $\vec{A} \perp \vec{B}$

$\hat{i} \bullet \hat{i} = 1 = \hat{j} \bullet \hat{j}$; $\hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{i} = 0$

Scalar Product Of $\vec{A}$ & $\vec{B}$



Definition : $\vec{A}\bullet\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$

$$= (B\cos\phi)A \quad = (A\cos\phi)B$$

$\vec{A}\bullet\vec{B}$ = magnitude of B x proj of A on B

$\vec{A}\bullet\vec{B}$ = magnitude of A x proj of B on A

Largest when $\vec{A} \parallel \vec{B}$; Zero when $\vec{A} \perp \vec{B}$

$\hat{i}\bullet\hat{i} = 1 = \hat{j}\bullet\hat{j}$; $\hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{i} = 0$

Scalar Product Of $\vec{A}$ & $\vec{B}$

$$\vec{A}\cdot\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{\jmath} + A_x\hat{\imath}\cdot B_z\hat{k}$$

$$+ A_y\hat{\jmath}\cdot B_x\hat{\imath} + A_y\hat{\jmath}\cdot B_y\hat{\jmath} + A_y\hat{\jmath}\cdot B_z\hat{k}$$

$$+ A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{\jmath} + A_z\hat{k}\cdot B_z\hat{k}$$

$$= A_xB_x\hat{\imath}\cdot\hat{\imath} + A_xB_y\hat{\imath}\cdot\hat{\jmath} + A_xB_z\hat{\imath}\cdot\hat{k}$$

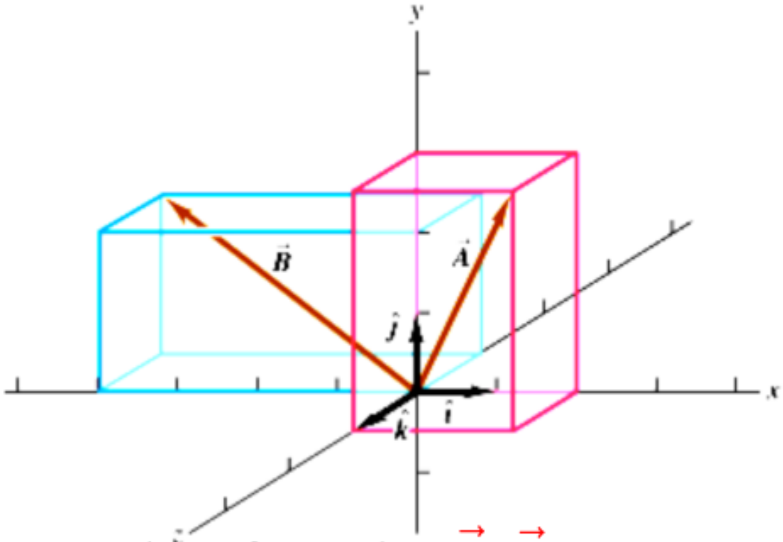
$$+ A_yB_x\hat{\jmath}\cdot\hat{\imath} + A_yB_y\hat{\jmath}\cdot\hat{\jmath} + A_yB_z\hat{\jmath}\cdot\hat{k}$$

$$+ A_zB_x\hat{k}\cdot\hat{\imath} + A_zB_y\hat{k}\cdot\hat{\jmath} + A_zB_z\hat{k}\cdot\hat{k}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Finding Angle Between Two Vectors

$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$; $\vec{B} = -4\hat{i} + 2\hat{j} - \hat{k}$, What's angle ϕ between them?



Dot product formula : $\vec{A} \bullet \vec{B} = AB\cos\phi = A_x B_x + A_y B_y + A_z B_z$

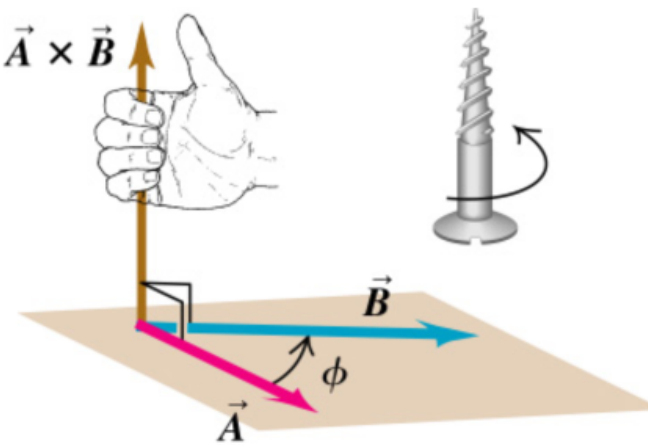
$$\Rightarrow \cos\phi = \frac{A_x B_x + A_y B_y + A_z B_z}{AB} = \frac{-3}{\sqrt{14}\sqrt{21}} = -0.175$$

$$\Rightarrow \boxed{\phi = \cos^{-1}(-0.175) = 100^{\circ}}$$

Vector Product Of $\vec{A}$ & $\vec{B}$: Definition

$$\vec{A}\times\vec{B}=\vec{C}=\text{Vector}\ \perp\ \text{to plane containing}\ \vec{A}\ \&\ \vec{B}$$

$$\text{with magnitude}\ |\vec{C}|=AB\sin\phi$$



Always two directions perpendicular to a plane, which one to choose ?

Follow right hand rule→ direction of thumb or advance of a right hand screw when vector **A** sweeps towards vector **B**

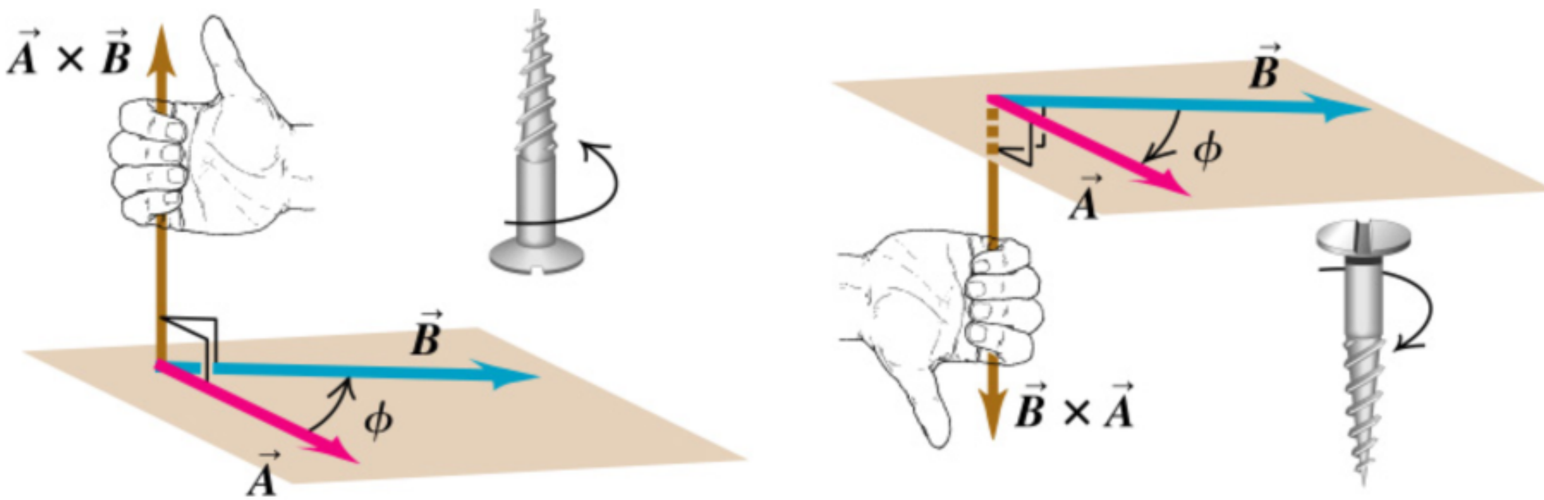
Right Hand Rule

Need to practice this until obvious !

Vector Product Of $\vec{A}$ & $\vec{B}$

As a Result of the Definition

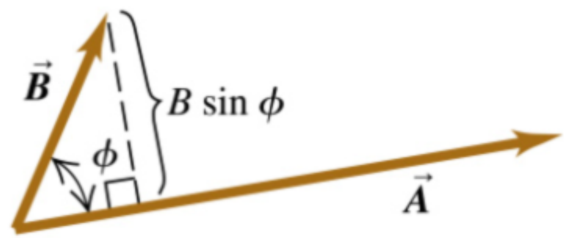
$$\vec{A} \times \vec{B} = \vec{C} = -(\vec{B} \times \vec{A})$$

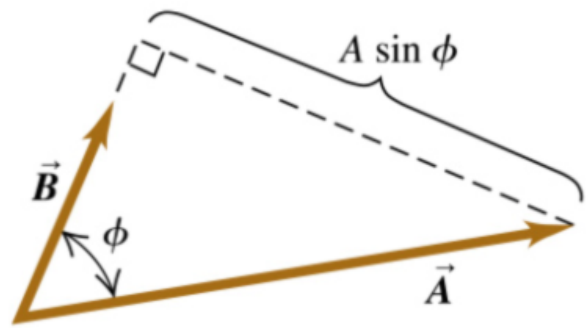


Vector Product Of $\vec{A}$ & $\vec{B}$: Definition

$\vec{A} \times \vec{B} = \vec{C} =$ Vector $\perp$ to plane containing $\vec{A}$ & $\vec{B}$

with magnitude $|\vec{C}| = AB \sin\phi$





Angle φ measured positive when from A turns towards B

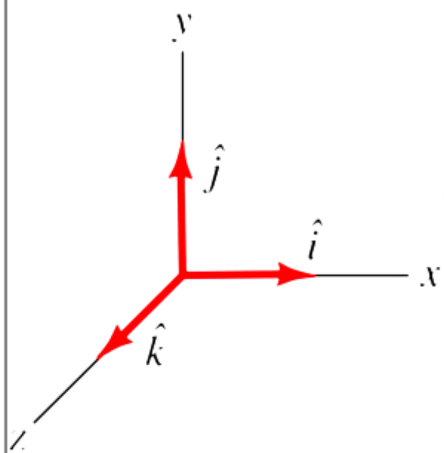
When $\vec{A} \perp \vec{B}$, angle $\varphi = 90^{o}$, magnitude maximum

When $\vec{A} \,||\, \vec{B}$, angle $\varphi = 0^{o}$, magnitude $= 0$

Vector Product of Unit Vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

verify now !



$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

Vector Product of Two 3-D Vectors

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k}$$

$$+ A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k}$$

$$+ A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k}$$

rewrite the individual terms as $A_x\hat{\imath} \times B_y\hat{\jmath} = (A_xB_y)\hat{\imath} \times \hat{\jmath}$, and so on.

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \quad C_y = A_zB_x - A_xB_z \quad C_z = A_xB_y - A_yB_x$$

Vector Product of Two 3-D Vectors

$$\text{If } \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

and

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

then

In the Determinant form

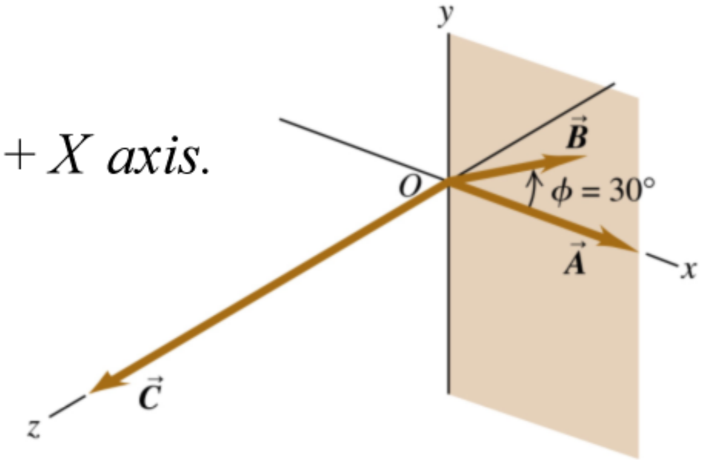
$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$C_x = A_yB_z - A_zB_y$, $C_y = A_zB_x - A_xB_z$, $C_z = A_xB_y - A_yB_x$

Calculating A Vector Product

$\vec{A} = 6\hat{i},\ \vec{B}$ *has magnitude 4 units and lies in x - y plane making an angle of* 30^o *w.r.t + X axis.*

Find $\vec{A}?\ \vec{B}$



$\vec{C} = \vec{A} \times \vec{B};$

$|\vec{C}| = AB\sin\phi = (6)(4)(\sin 30^o) = 12$

Use right hand rule

Direction of $\vec{A} \times \vec{B}$ is along +Z axis

$\Rightarrow \boxed{\vec{A} \times \vec{B} = 12\hat{k}}$

Pop quiz:
If I want the angle between 2 vectors, it is enough to know:

- A: Their Scalar Product
- B: Their Vector Product
- C: Either of the above
- D: Both of the above

Which of the following is a unit vector?

- A**: i.j**
- B: **i x i**
- C: **i x j**
- D: **i + j + k**