Every particle of matter in the universe attracts every other particle with a gravitational force $F_g$ acting along line joining the two particles

$$F_g \propto \frac{m_1 m_2}{r^2}$$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$G=$Universal Gravitational Constant
Superposition Of Gravitational Forces

\[ \vec{F}_{sat} = \sum_i \vec{F}_{i \rightarrow sat} \]

\[ \vec{F}_{i \rightarrow sat} = \frac{GM_i m_{sat}}{r_i^2} \]
Superposition Of Gravitational Forces

Gravitational forces combine vectorially

A satellite is to be sent to position $x$ between earth & moon where there is no NET gravitational force due to these bodies. Find $x$.

\[ F = \vec{F}_{E} + \vec{F}_{M} \]

\[ 0 = -G \frac{M_{E}m}{x^{2}} + G \frac{M_{M}m}{(d-x)^{2}} \]

\[ (d-x)^{2} = x^{2} \left( \frac{M_{M}}{M_{E}} \right) \]

\[ x = 0.9d = \text{equilibrium point} \]
Relating Little $g$ and Big $G$

Revised Definition: Weight of a body is the total gravitational force exerted on that body by all other bodies in the universe!

When body is near earth, influence of all other objects is negligible (far far away) ⇒ Weight = Earth’s grav. attraction

Weight force on a body of mass $m$ at earth's surface

$$w = mg = F_g = G \frac{M_E m}{R_E^2} \quad \Rightarrow \quad g = \frac{GM_E}{R_E^2}$$
Weight Force On A Body Near Earth

\[ w = F_g = G \frac{M_E m}{r^2} \]

Earth, mass \( m_E \)

\( r = R_E = 6.38 \times 10^6 \text{ m} \)

Astronaut, mass \( m \)

- \( w = \) astronaut’s weight = \( G m_E m / r^2 \)
- \( r = \) astronaut’s distance from the center of the earth
- \( r - R_E = \) astronaut’s distance from the surface of the earth

As \( r \rightarrow \infty \Rightarrow w \rightarrow 0 \)
Measuring Weight Force

Gravity is not a force that one measures directly. If you hold a spring scale with some mass \( m \) hanging on it, the spring scale applies tension force \( F \) to hanging body and reading on the scale is \( |F| \). If you are unaware of earth’s rotation, you would think that scale reading = weight of body since the spring is in equilibrium \( \vec{F} = -\vec{w} \); \( w = \text{apparent weight} \).

But if bodies are rotating with Earth then they are not exactly in equilibrium. Apparent weight \( w \neq w_0 \) (true weight).

\[
 w_0 = \frac{GM_E m}{R_E^2} \quad \text{and is measured at the North pole (no rotation \( \Rightarrow \) Inertial system)}
\]

At Equator, body moving in circle of radius \( R_E \) \( \Rightarrow w_0 - F = \frac{mv^2}{R_E} \) (Centripetal Force).

So apparent weight which is = magnitude of \( F \) \( \Rightarrow w = w_0 - \frac{mv^2}{R_E} \).

\[
 \Rightarrow g_{\text{equator}} = g_0 - \left(\frac{v^2}{R_E}\right) \text{ and } \Delta g = \frac{v^2}{R_E} = \frac{(468 \text{ m/s})^2}{6.38 \times 10^6 \text{ m}} = 0.0337 \text{ m/s}^2
\]
True Weight Vs Apparent Weight: Redux

At the north or south pole: apparent weight is the same as true weight.

- $w_0$ = true weight of object of mass $m$
- $\vec{F}$ = force exerted by spring scale on object of mass $m$
- $\vec{F} + \vec{w}_0$ = net force on object of mass $m$; due to earth’s rotation, this is not zero (except at the poles)
- $\vec{w}$ = apparent weight = opposite of $\vec{F}$

Away from the poles: due to the earth’s rotation, apparent weight is not equal to true weight.

$\vec{W} = w_0 - m\vec{a}_{rad}$
(Apparent) Weightlessness In Space

Bodies in orbiting spacecraft are not weightless! Earth's gravity continues to attract them as though they were at rest w.r.t earth.

**Apparent weight** of a body in orbiting craft: \( \vec{w}' = \vec{w}_0 - m\vec{a}_{rad} = m(\vec{g}_0 - \vec{a}_{rad}) \)

Only force acting on craft is the Earth's gravity

\( \Rightarrow \vec{a}_{rad} \) towards earth's center

= value of acc. due to gravity at that point \( \Rightarrow \vec{g}_0 = \vec{a}_{rad} \) \( \Rightarrow \) app. weight \( w' = 0 \! \) !

\( \Rightarrow \) Apparent weightlessness of satellites
Apparent Weight in Elevator

\[ w - mg = ma \]
Gravitational Potential Energy

$W_{\text{grav}}$ done by grav. force when body moves directly

\[ r_1 \rightarrow r = r_2 \]

\[ W_{\text{grav}} = \int_{r_1}^{r_2} F_r \, dr = -G M_E m \int_{r_1}^{r_2} \frac{d}{r^2} = GM_E m \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \]

Change in grav. potential energy $U_1 - U_2 = W_{\text{grav}}$

If body moves away from earth, $r$ increases & gravity does negative work on object $\Rightarrow U$ increases

If body falls towards earth, $r$ decreases $\Rightarrow U$ decreases

Potential energy is measured relative to a ref. point

Work done in moving an object from $r = \infty$ to $r = r$

\[ W_{\text{grav}} = \frac{G M_E m}{r} \Rightarrow \Delta U = \left| U = -\frac{G M_E m}{r} \right| \]
Gravitational Potential Energy

Gravitational potential energy \( U = -\frac{Gm_Em}{r} \)

Earth, mass \( m_E \)

Astronaut, mass \( m \)

- \( U \) is always negative
- \( U \) becomes less negative with increasing radial distance \( r \)

\[ U = -\frac{G M_E m}{r} \]

As \( r \to \infty \Rightarrow U \to 0 \)
Getting Back Good Old $U_{\text{grav}} = mgh$ Relation

Above earth but close to it, work done in moving from $r_1 \to r_2$

$$W_{\text{grav}} = +GM_Em \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = GM_Em \frac{r_1 - r_2}{r_1 r_2}$$

If object stays close to earth $\Rightarrow r_1 \simeq R_E, r_2 \simeq R_E$

Rewrite $W_{\text{grav}} = GM_Em \left( \frac{r_1 - r_2}{R_E^2} \right)$ but since $g = \frac{GM_E}{R_E^2}$

$\Rightarrow W_{\text{grav}} = mg(r_1 - r_2)$

work done under constant acceleration due to gravity
Satellite Takes A Fall

A cannonball shot horizontally from mountaintop will fall to ground on a parabolic path. If shot with much higher speed it will go far enough that surface of spherical earth falls away beneath it. Ball will never catch up with the earth’s surface falling away and ball’s motion will be a circular orbit of “constant fall”

![Diagram of satellite orbit](image)
Satellite Motion: Closed & Open Orbits

Closed Orbit: Orbits (1) thru (5) close on themselves. All closed orbits are ellipses or segment of an ellipse. Trajectory (4) is a circular, a special case of an elliptical orbit.

Open Orbit: trajectories (6) & (7) are open orbits. For these paths the projectile never returns to earth but travels further away. NASA launches such probes to travel to “infinity & beyond” to probe properties of other planets like Jupiter & Saturn.
Circular Orbit Of Artificial Satellites

For satellite in circular orbit, (assume ≈ vaccum @ such heights) only force acting on it is the gravitational attraction of earth, directed towards center of earth ⇒ center of satellite orbit

Second law \( \Rightarrow \frac{GM_E m}{r^2} = \frac{mv_{sat}^2}{r} \Rightarrow v_{sat} = \sqrt{\frac{GM_E}{r}} \)

\( v_{sat} \) does not depend on \( m_{sat} \) & \( v_{sat} \) is fixed for a given orbit radius, making "catching up" with a satellite an interesting maneuver!
Q: What keeps a satellite rotating?
   A: Its speed!
   Too low and it crashes down
   Too high and it escapes earth’s grip!
Orbit Of Satellites

\[ v_{sat} = \sqrt{\frac{GM_E}{r}} \]

In circular motion

\[ \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} \]

\[ T_{sat} = 2\pi r \sqrt{\frac{r}{GM_E}} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \]

\[ E_{sat} = K_{sat} + U_{sat} = \frac{1}{2} m_{sat} v_{sat}^2 + \left( -\frac{GM_E m_{sat}}{r} \right) \]

\[ \Rightarrow E_{sat} = \frac{1}{2} m_{sat} \left( \frac{GM_E}{r} \right) - \frac{GM_E m_{sat}}{r} = -\frac{GM_E m_{sat}}{2r} \]

Total mech. energy is negative !⇒ system is a bound state

smaller the \( r \), lesser is the energy of the satellite-earth system

If \( r \) gets small enough, dissipative (drag) forces bring satellite down
**Projectile Velocity Needed To Escape Earth’s Gravity**

Gravity is a conservative force. Total energy of body of mass $m$, speed $v$ is $E = K + U = \frac{1}{2}mv^2 - \frac{GM_Em}{r}$

Escape speed $v_{esc}$ of a body launched from earth surface ($r = R_E$) is the minimum launch speed in order to escape earth's gravity ⇒ travel to $r = \infty$

Body with escape velocity will have $v=0$ at $r=\infty$

⇒ $K=0 \ & \ U=0 \Rightarrow [ E=0 ]$

Energy Conservation ⇒ $E=0$ at launch & all points after

At launch, $E = \frac{1}{2}mv_{esc}^2 - \frac{GM_Em}{R_E} = 0 \Rightarrow v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$

⇒ $v_{esc} = 11200 \text{m/s}$ !! Hence those booster rockets on shuttle
But You Can't Escape Everything

Meet Mr. Black Hole, The Cannibal of The Universe