PHYS 4A WINTER '15

HW #6

6.) $\alpha = \frac{\omega_f - \omega_i}{t} = \frac{3600 - 1800 \text{ rpm}}{1.4 \text{ s}} = 1285.7 \frac{\text{rpm}}{\text{s}}$.

$\Rightarrow \alpha = 1285.7 \frac{\times 2\pi}{60} \text{ rad}/\text{s}^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$

$\Rightarrow \theta = \frac{3600^2 - 1800^2}{2 \times 1285.7} \frac{(\text{rpm})^2}{(\text{rpm}/\text{s})}$

$\Rightarrow \theta = 3780 \cdot \text{rpm} \times \text{s}$

$\Rightarrow \theta = 3780 \frac{\times 2\pi}{60 \cancel{\text{s}}} \text{ rad } \cancel{\text{s}}$

$\Rightarrow \theta = 395.85 \text{ rad}$.

28.) a) $d = 92 \text{ cm} \Rightarrow r = 46 \text{ cm}$

$I = 7.8 \text{ kgm}^2$

$$I = \int r^2 \, dm$$

For a given I, minimum mass would mean maximum r, i.e. the mass is spread/distributed furthest from the axis.

$\therefore$ in case of ~~a~~ wheel, all the mass is at a distance $R = 46 \text{ cm}$

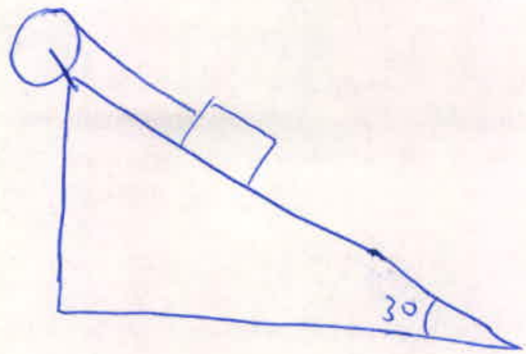
$$\therefore I = MR^2$$

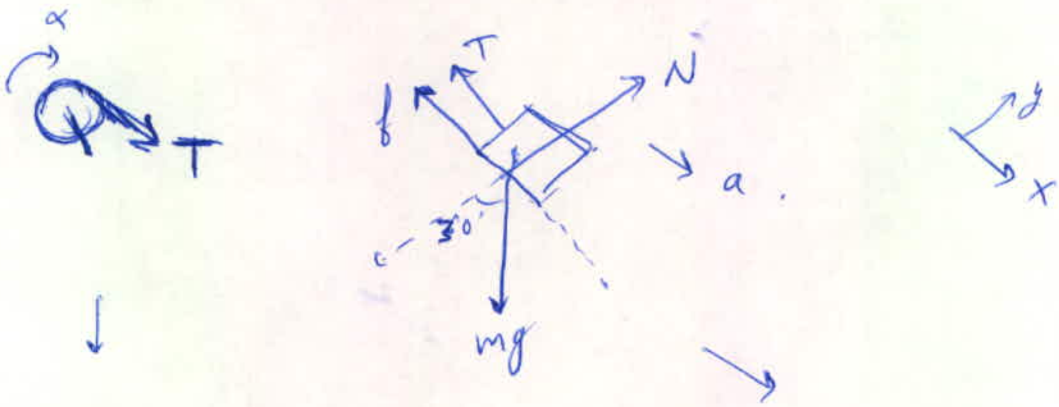
$$\Rightarrow M = \frac{7.8}{(0.46)^2}$$

$$\Rightarrow M = 36.86 \text{ kg}$$

b) It could have more mass, if there is mass present inside the wheel, like in the form of spokes.

42.)





$\tau = I\alpha$ — (1)

& $\tau = TR$ — (2)

From (1) & (2).

$TR = I\alpha$

$\Rightarrow T = \dfrac{I\alpha}{R}$ — (3)

$N = mg\cos 30^\circ$ — (4)

$f = \mu N$. — (5)

$mg\sin 30^\circ - f - T = ma$. — (6)

From (4), (5) & (6)

$mg\sin 30^\circ - \mu(mg\cos 30^\circ) - T = ma$

$\Rightarrow \mu = \dfrac{1}{mg\cos 30^\circ}\left[mg\sin 30^\circ - T - ma\right]$ — (7)

Also,

$\alpha = a/R$ — (8)

(Condition of no slipping between drum and string)

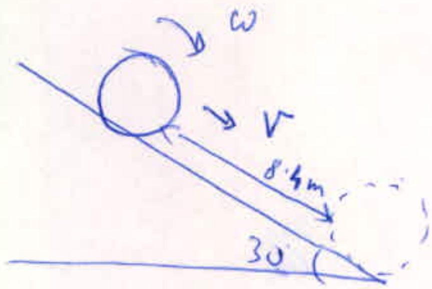
From ③, ⑦ & ⑧

$$\mu = \frac{1}{mg\cos 30^\circ}\left[mg\sin 30^\circ - \frac{I}{R}\left(\frac{a}{R}\right) - ma\right]$$

where $I = \frac{2}{5}MR^2$ for ~~a~~ solid drum

$\therefore \mu = 0.36$

54.)



Condition for Rolling without Slipping

$\rightarrow v = \omega R$

(at each time)

Conserving Energy,

$$\Delta U + \Delta K = W_{nc} = 0$$

$$\Rightarrow 0 - mgh + \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}_{K_f} - 0 = 0$$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\times\frac{2}{5}\cancel{m}\underbrace{R^2\omega^2}_{v^2} = \cancel{m}gh$$

$$\therefore \quad \frac{v^2}{2} + \frac{2}{10}v^2 = gh$$

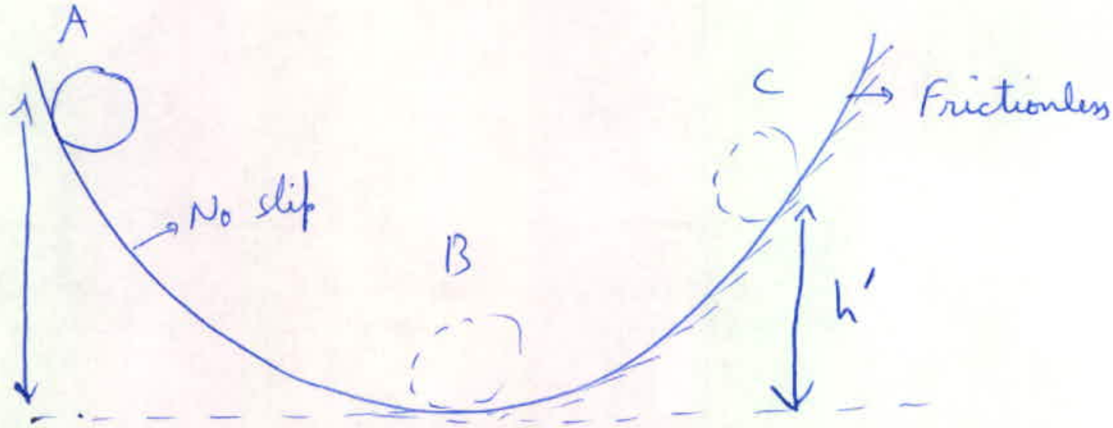
$$\Rightarrow \frac{7}{10}v^2 = gh$$

$$\Rightarrow v = \sqrt{\frac{10}{7}gh}$$

and $h = 8.4 \sin 30^\circ$

$$\therefore \quad v = 7.67 \text{ m/s}$$

60.)



Conserving energy between points A & B,

$$\Delta U + \Delta K = W_{nc} = 0$$

$$\Rightarrow 0 - mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - 0 = 0$$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2 + \frac{1}{2} \times \frac{2}{5}\cancel{M}\underbrace{R^2\omega^2}_{v^2} = \cancel{m}gh$$

(as $v = \omega R$) → No Slip Condition

$$\Rightarrow \frac{1}{2} \times \frac{7}{5} v^2 = gh$$

$$\Rightarrow v = \sqrt{\frac{10}{7} gh}$$

This is the velocity of centre of mass.

$\because$ The Path is frictionless from B to C, there ~~is~~ can't be any Net Torque acting on the ball after Point B

$$\tau_{mg} = mg \times 0 = 0$$

$$\tau_N = N \times R \times \sin 0 = 0$$

$\therefore$ Rotational energy of the ball cannot change, i.e. Ball will continue to rotate with angular velocity ω it attained after point B.

Now, Conserving Energy between points B & C,

$$\Delta U + \Delta K = W_{nc} = 0$$

$$\Rightarrow mgh' - 0 + \underbrace{\tfrac{1}{2}I\omega^2}_{K_f} - \underbrace{\left(\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2\right)}_{K_i} = 0$$

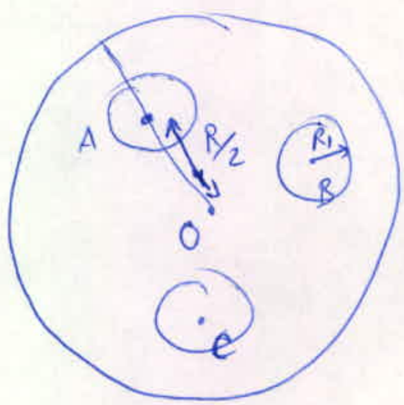
$$\Rightarrow mgh' = \frac{1}{2}Mv^2$$

$$\Rightarrow h' = \frac{v^2}{2g}$$

$$\Rightarrow h' = \frac{\cancel{10}^{5}}{7}\frac{\cancel{g}h}{\cancel{2g}}$$

$$\Rightarrow h' = \frac{5h}{7}$$

64.)



$\rho \rightarrow$ density

$\therefore \rho = M/\pi R^2$

$$M_1 = \frac{M}{\pi R^2} \times \pi R_1^2 = \frac{M R_1^2}{R^2}$$

$$I_{Lost} \text{ of 1 hole around its centre} = \frac{1}{2} M_1 R_1^2$$

$$= \frac{1}{2} \frac{M R_1^2}{R^2} \cdot R_1^2$$

$$= \frac{1}{2} \frac{M R_1^4}{R^2}$$

{ $M_1 \rightarrow$ Mass of a hole
$R_1 \rightarrow$ Radius of a hole }

By Parallel Axis Theorem,

$$\text{i.e. } I = I_{cm} + Mh^2$$

$$\left.\begin{array}{r} I_{Lost} \text{ of 1 hole around the centre} \\ \text{of the large wheel} \end{array}\right\} = \frac{1}{2} M \frac{R_1^4}{R^2} + M_1 \left(\frac{R}{2}\right)^2$$

$$= \frac{1}{2} M \frac{R_1^4}{R^2} + \left(\frac{M R_1^2}{R^2}\right) \times \frac{R^2}{4}$$

$$= \frac{1}{2} M \frac{R_1^4}{R^2} + M \frac{R_1^2}{4}$$

$\therefore$ Total Loss $= 3\, I_{Lost}$.

$$\therefore \quad 0 \cdot 15 = \frac{\text{Total loss}}{\text{Original } I}$$

$$\Rightarrow 0.15 = \frac{3\left(\frac{1}{2} M \frac{R_1^4}{R^2} + M \frac{R_1^2}{4}\right)}{\frac{MR^2}{2}}$$

$$\Rightarrow \quad 3\frac{R_1^4}{R^4} + \frac{3R_1^2}{2R^2} = 0.15$$

Let R_1^2/R^2 be x

$$\therefore \quad 3x^2 + 3x/2 = 0.15$$

$$\Rightarrow 20x^2 + 10x - 1 = 0$$

$$\therefore x = \frac{-10 \pm \sqrt{10^2 + 4.(20)}}{2.20}$$

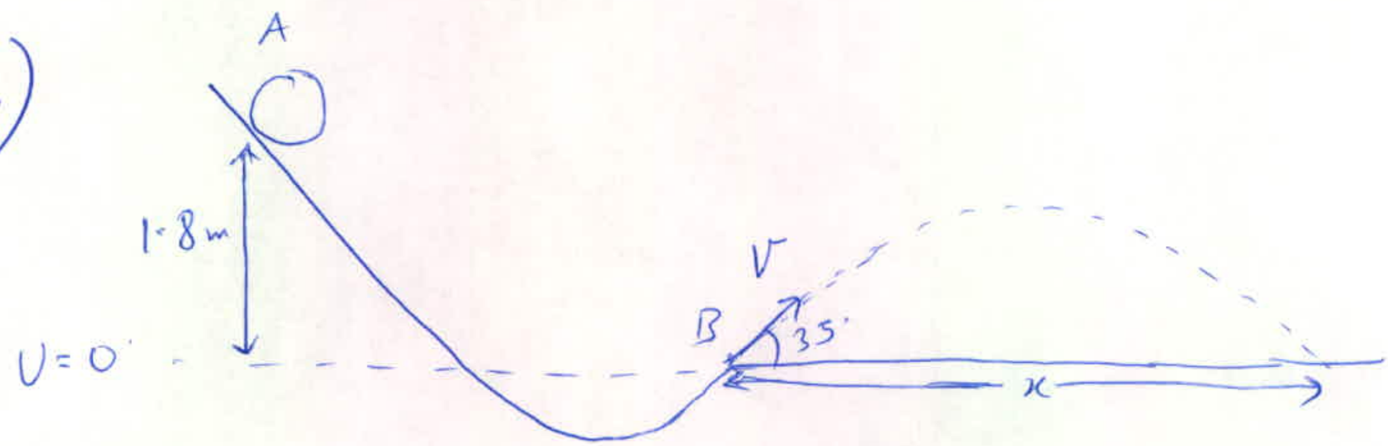
$$\Rightarrow x = 0.085$$

$$\Rightarrow \frac{R_1^2}{R^2} = 0.085$$

$$\Rightarrow R_1 = \sqrt{0.085}\, R$$

$$\Rightarrow R_1 = 0.29\, R$$

74.)



Let the $I_{ball} = \lambda M R^2$

where $\lambda = 2/5 \rightarrow$ for solid sphere

and $\lambda = 2/3 \rightarrow$ for hollow sphere

Conserving Energy between points A & B,

$$\Delta U + \Delta K = W_{nc} = 0$$

$$0 - mg(1\cdot 8) + \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2 = 0$$

where $\frac{1}{2} I \omega^2 = \frac{1}{2}(\lambda M R^2)\,\omega^2$

$$= \frac{\lambda}{2} M V^2 \quad \{V = \omega R\}.$$

$$\therefore\ \cancel{m}g(1\cdot 8) = \frac{1}{2}\cancel{m} V^2 + \frac{\lambda}{2}\cancel{m} V^2$$

$$\Rightarrow\ V = \sqrt{\frac{2g(1\cdot 8)}{\lambda+1}}$$

Now, the range of the projectile is given by,

$$x = \frac{V_0^{\,2}\sin 2\theta_0}{g}$$

$$\Rightarrow\ x = \frac{2\cancel{g}(1\cdot 8)}{(\lambda+1)\cancel{g}}\sin(2\times 35^\circ)$$

$$\Rightarrow\ x = \frac{3\cdot 6 \sin 70^\circ}{(\lambda+1)}$$

$\therefore$ For solid marble $\rightarrow$

$\lambda = 2/5$

$x = 2\cdot 42\ \text{m}$

For hollow tennis ball $\rightarrow$

$\lambda = 2/3$

$x = 2\cdot 03\ \text{m}$