

Experiment 2, Physics 2BL

Nondestructive Deduction of the Mass Distribution and Manufacturing Variability of Racketballs.

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Preparation

Before this experiment, we recommend you review or familiarize yourself with the following:

- Chapters 4-6 in Taylor
- Moment of Inertia
- *t*-score

1. PHYSICS

1.1. Moments of inertia

The moment of inertia needed for this experiment is that of a hollow sphere with outer radius R and inner radius r :

$$I_{\text{hollow}} = \frac{2}{5}M \frac{(R^5 - r^5)}{(R^3 - r^3)}$$

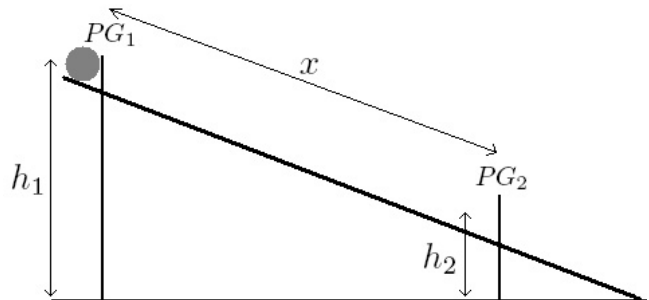
The tricky part is that not all the variables will be directly measurable. In this experiment, you will not be able to break apart the ball in order to measure the inner radius, r . You will use a clever nondestructive experimental method to deduce the moment of inertia and the fractional error in the thickness of a racketball.

Questions

1. The moment of inertia of a solid ball is $I = \frac{2}{5}MR^2$. Write out the moment of inertia of a solid ball of radius r and density ρ_{in} that has an outer coating of radius R (meaning a thickness of $R - r$) that has a density ρ_{out} , where ρ_{in} and ρ_{out} are constant.

1.2. Conservation of energy, rolling w/o slipping, rolling radius

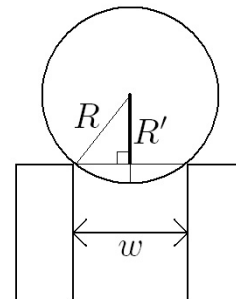
For the first part of this experiment we will be calculating the moment of inertia of a ball by rolling it down a ramp. The ball will start from rest at a height, h_1 , at the first photogate, PG_1 . We will measure the time, t , that it takes for the ball to get to travel a distance, x , when it arrives at a second photogate, PG_2 , which is at a height, h_2 . As long as the ball starts at rest, the change in potential energy due to the change in height,



$h = h_1 - h_2$, is converted into a final rotational and a final translational kinetic energy.

$$mgh = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2 \quad (1)$$

In order for the experiment to work it is important for the ball to be rolling without slipping during its journey down the ramp. This means that the ramp must not be too steep or too slippery. Even though there is a constant force of friction on the ball, it does no work on the ball because the frictional force does not act over a distance. In other words, the frictional force acts over zero displacement.



In our experiment there is a slight complication because the ball is not rolling on a flat surface but rather a square beam that is open on the top (See a cross of the ball sitting in this beam above.). This means that the rolling radius of the ball is smaller than its actual radius. We can calculate the rolling radius, R' by measuring the width of the beam and using the Pythagorean Theorem, or by measuring the length of a full rotation of the ball and dividing by 2π . Using the former method, we find:

$$R' = \sqrt{R^2 - \left(\frac{w}{2}\right)^2}$$

Questions

2. If we rolled the ball down a ramp without a groove would it roll faster or slower than the ball that is rolling down a square beam open on the top side (A cross section of the ball sitting in such a beam can be seen in the diagram on page 1.)? In this case we are not concerned with resistance or whether the ball will follow a straight line but merely the effect of rolling radius on the speed of the ball. Explain your reasoning.

Rolling without slipping gives us the condition $\omega = \frac{v}{R'}$ which can be plugged into the equation for conservation of energy in equation (1) and solved for I . Recall that R' is the rolling radius and not the actual radius.

$$I = M(R')^2 \left[\frac{2gh}{v_f^2} - 1 \right]$$

Instead of measuring the velocity at the bottom of the ramp we will use the following formula that comes from motion under constant acceleration. The final velocity, v_f , ends up being twice the average velocity:

$$v_f = \frac{2x}{t} \quad (2)$$

Which can be plugged into the expression for I :

$$I = M(R')^2 \left[\frac{ght^2}{2x^2} - 1 \right] \quad (3)$$

Questions

3. What experimental condition no longer holds if you increase the ramp angle by a large amount or coat the ramp in oil? Briefly explain why.

2. METHODS FOR STATISTICAL ANALYSIS

2.1. Standard Deviation and Standard Deviation of the Mean

Many times you will be making a series of measurements in order to get a mean value. It is important to keep track of the difference between the error on one of the measurements, which is the standard deviation, σ_x , and the error (or standard deviation) of the mean, $\sigma_{\bar{x}}$. The more measurements you make, the smaller the error on the mean becomes. Let's make sure we understand where this comes from.

The equation for the mean of N values labeled x_i is a very simple function of N variables:

$$\bar{x}(x_1, x_2, \dots, x_N) = \frac{1}{N}(x_1 + x_2 + \dots + x_N)$$

Each one of the measurements, x_i , has an error, which is known as the standard deviation:

$$\sigma_x = \sqrt{\frac{\sum (\bar{x} - x_i)^2}{N - 1}}$$

In order to propagate the error from each of x_i to \bar{x} we have to use the error propagation equation:

$$\sigma_{\bar{x}} = \sqrt{\left[\left(\frac{\partial \bar{x}}{\partial x_1} \right) \sigma_x \right]^2 + \left[\left(\frac{\partial \bar{x}}{\partial x_2} \right) \sigma_x \right]^2 + \dots + \left[\left(\frac{\partial \bar{x}}{\partial x_N} \right) \sigma_x \right]^2}$$

Looking back on the equation for \bar{x} we see that each of the x_i 's have the same partial derivative.

$$\frac{\partial \bar{x}}{\partial x_i} = \frac{1}{N}$$

Plugging everything in we get the following:

$$\sigma_{\bar{x}} = \sqrt{\left(\frac{\sigma_x}{N} \right)^2 + \left(\frac{\sigma_x}{N} \right)^2 + \dots + \left(\frac{\sigma_x}{N} \right)^2} = \sqrt{N \left(\frac{\sigma_x}{N} \right)^2}$$

Which reduces to a simple expression.

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \quad (4)$$

Questions

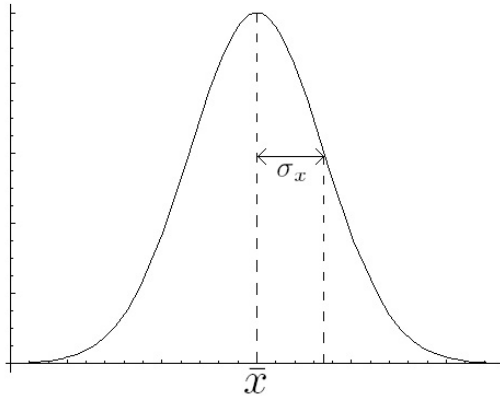
4. Suppose you are trying to measure the mass of an M&M and you measure 50 of them individually and obtain a mean value of 1.05g and a standard deviation of .15g. What is the standard deviation of the mean, $\sigma_{\bar{m}}$? How many M&M's should be massed in order to get a $\sigma_{\bar{m}} \leq 0.01$? Assume the standard deviation doesn't change for more or less mass measurements. In other words, use $\sigma = .15g$ as the standard deviation for the second question.

For extra practice see Taylor #4.18 and 4.26

2.2. Normal Distributions

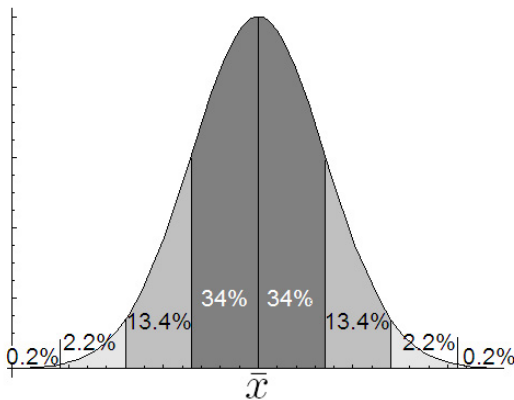
It is a fact of nature that many things that seem to have a uniform value, such as the width of the hairs on your head or the size of the grains of sand at the beach,

when probed with precise measuring instruments actually tend to follow what we call a Gaussian, or Normal Distribution. That means that if we plot a histogram of these values most will be centered about a mean, with fewer and fewer at the extreme regions. The resolution of this curve improves if we decrease the bin size and increase the sample size, but in general these curves have only two parameters. The mean, \bar{x} , and the standard deviation, σ_x .



$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \bar{x})^2}{2\sigma^2}\right]$$

The useful thing about these curves is that when we normalize them (which means that we make the total area equal to 1) any portion of the area gives the probability of a value falling with that part of the curve. There is always a 68% probability of a value falling within + and - 1σ of the mean, 95.4% probability of a value falling within + and - 2σ of the mean, and 99.7% probability of a value falling within + and - 3σ of the mean. For example if i told you that the the average NBA basketball player was 75 ± 3 inches. If I found a sample of 100 players, I would expect 68 of them to be between 6' and 6'6". There should be about 16 players above 6'6" and 16 players under 6'. But what if we have a value that is




a generic $t\sigma$ away from the mean and we want to find the

likelihood of our getting that value? All we have to do is find the area under the curve that is within + and - $t\sigma$ from the mean. To save you time, these integrals are already calculated for you in a chart, copied from p.287 in Taylor. The arrows in the table below show you how to read the chart for when you want to find the probability that a value is within + and - 1.47σ from the mean. One tricky point to keep in mind is whether the probability (area under the curve) that you are interested in refers to both the left and right of the curve or just one side. *If you are only interested in one side of the curve you will have to divide these probabilities in half.*

t is known as the t-score and is defined as $t = \frac{x_i - \bar{x}}{\sigma_x}$, where x_i is the value of the measurement in question, \bar{x} is the mean of all the measurements, and σ_x is the standard deviation of all the measured values of x .

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{\bar{x}-t\sigma}^{\bar{x}+t\sigma} G_{\bar{x},\sigma}(x) dx$, as a function of t .



	$X-t\sigma$	X	$X+t\sigma$							
t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32

This chart, copied from p.287 in Taylor, gives the percentage probability that a given data point will be within $t\sigma$ of the mean.

Questions

5. Use the chart to find the probability that a given data point will be atleast 1.52σ away from the mean? Suppose, for some set of data, the mean is 5.3 and the standard deviation is 0.4. What is the probability of getting a value larger than 6.1? Or a value smaller than 4.9?

For extra practice see Taylor #4.14, 5.20, 5.36

2.3. Chauvenet's Criteria for Rejection of Data

If in your data collection and error was made during one measurement that you were consciously aware of, then you should throw out this data point. However, suppose you have one data point that is very far away from the mean and you are not aware of any error having been made during the measurement. Is it justifiable to throw this value out? In most cases it is better to not throw away data because you might be biased about the results of your data. *It is always better to take more data until the suspicious point does not contribute to the final result.* However, in some cases you do not have the ability to take additional data. In such cases you may use Chauvenet's rule-of-thumb for rejecting data.

$$t_{suspect} = \frac{|x_{suspect} - \bar{x}|}{\sigma_x}$$

$$n = N \times (1 - Prob(t_{suspect})) \leq 0.5$$

If we make N measurements and we want to challenge the validity of $x_{suspect}$ we can only toss it out if $n \leq 0.5$. Here, the $Prob(t_{suspect})$ is found in table A.

For extra practice see Taylor #6.4

WARNING: t is a variable commonly used to represent t-scores and time. Don't confuse them! When the variable t is used in these guidelines, it should be clear, based on the context, whether we are speaking of the t-score or time.

Questions

6. Briefly explain (no more than 2 sentences) what the value of $t_{suspect}$ represents.

2.4. Drawing a Histogram

It can be very useful to convert a set of data into a histogram. This allows you to see which points might be suspiciously off and whether the standard deviation looks reasonable and is not being skewed by faulty data points. Here is the procedure for drawing a histogram by hand.

1. Determine the range of your data by subtracting the smallest number from the largest one.
2. The number of bins should be approximately \sqrt{N} and the width of a bin should be the range divided by \sqrt{N} .
3. Make a list of the boundaries of each bin and determine which bin each of your data points should fall into.

4. Draw the histogram. The y axis should be the number of values that fall into each bin.

5. Sometimes this procedure will not produce a good histogram. If you make too many bins the histogram will be flat and too few bins will not show the curve on either side of the maximum. You might need to play around with the number of bins to produce a better histogram.

3. EXPERIMENTAL PROCEDURE, RACKETBALLS

3.1. Initial Measurements

Set up the ramp and photogates according to the diagram on the first page. It is extremely important that the setup not be disturbed at any point during the experiment. To help prevent potential disruption, it is best to snugly wedge the photogate with the digital display between the lower end of the ramp and the table. Then the external photogate that you position higher up the ramp should be clamped to the table. You can find a more complete diagram of the set up in the lecture slides. Your TAs should give you a short demonstration of the setup before you begin. You will need to measure the inner width of the ramp, w , the length of the ramp between the photogates, x , and the heights of the photogates, h_1 and h_2 . Also, you will need to choose a racketball. There may be different brands and colors of racketballs in the lab. Which ever brand/color you choose, you must use that same kind of racketball throughout the experiment! You also need to measure the mass of the ball, M and its radius R . You may use whatever tools you like to make these measurements, just be sure to record the associated uncertainties and units with each measurement. Finally, calculate the rolling radius, R' .

3.2. One ball N times

Set the photogates to PULSE mode, toggle switch to ON, and the black switch to 1ms. A crucial part of this experiment is releasing the ball consistently. The ball must begin with zero kinetic energy and from the same position for each trial. To do this, it is useful to tape a pen or pencil to the track. The pen/pencil should be perpendicular to the track and placed before the first photogate sensor such that, when the ball rests against the pen/pencil, the ball sits just before the photogate sensor (The photogate should *not* be triggered when the ball is resting against the pen/pencil, but it will be triggered immediately after you release the ball.).

Acquiring good data in this experiment is difficult and depends highly on your ability to release the ball consistently between trials. Before you begin your measurements, it is recommended that you practice releasing the

ball. You should do at least 20 practice trials. If you don't conduct practice trials, it is likely you will obtain an imaginary number for $\sigma_t(man)$ (defined below), which is incorrect.

After you complete your practice trials, begin measuring/recording the time t for the ball to roll down the ramp. Do this N times. Normally you would choose the number of trials N , however, in order to acquire good data, you need to do AS MANY TRIALS AS POSSIBLE. N will depend on the number of balls available. You should be able to do at least 30 trials, but if there are more than 30 balls, it is recommended that the number of trials you conduct equals the number of balls available. Now, calculate \bar{t} , $\sigma_t(meas)$, and $\sigma_{\bar{t}}(meas)$. Use equation (3) to calculate $I \pm \sigma_I$. You may take g in equation (3) to be a constant without an uncertainty (i.e. $g = 9.81 \frac{m}{s^2}$). Also remember, if you are using \bar{t} in your determination of I , then you must use the uncertainty associated with \bar{t} , i.e. $\sigma_{\bar{t}}$, when calculating σ_I . From the time data you've collected in this experiment, draw a histogram using the guidelines given in section 2.4.

Questions

7. Name at least one thing you must do or not do during your experiment.
8. In this section you are asked to create a histogram from the time data collected for one ball rolled down the ramp N times. What do you expect this histogram to look like? Why do you expect this?
9. Suppose you roll the ball down the ramp 5 times and measure the rolling times to be [3.092s, 3.101s, 3.098s, 3.095s, 4.056s]. According to Chauvenet's criterion, would you be justified in rejecting the time measurement $t = 4.056s$? Show your work.
10. You are conducting the racquetball rolling experiment. During one of the trials your lab partner bumps into the table. Should you keep the data from this trial? Explain your reasoning.

3.3. N balls one time

Now collect N balls and record the time for each to roll down the ramp. From this experiment you can determine the uncertainty in t that is due to the combined effects of measurement error and manufacturing error, which we call the total error $\sigma_t(total)$. This is simply the standard deviation of the time data collected in this experiment. Calculate \bar{t} and $\sigma_t(total)$.

3.4. Calculation of $\sigma_t(man)$, Percent error for d , and determination of N

Now we have what we need to determine the manufacturing variability in the racketballs. Use the following formula to calculate the error due to manufacturing:

$$\sigma_t(man) = \sqrt{\sigma_t(total)^2 - \sigma_t(meas)^2}$$

Now the percent variation in the thickness of the racketball, d , is given by the following relationship:

$$\frac{\sigma_d}{d} = 27 \frac{\sigma_t(man)}{t}$$

This relationship is obtained by using typical values for equations (3) and the moment of inertia of a hollow sphere and then making small deviations from those typical values to find how sensitive errors in d are to errors in I and how sensitive errors in I are to errors in t . See the lecture slides for a detailed derivation of this equation. The goal of this experiment is to determine $\frac{\sigma_d}{d}$ to an accuracy of 1%. Calculate the uncertainty in $\frac{\sigma_d}{d}$, which is $\frac{1}{\sqrt{N}} \frac{\sigma_d}{d}$ assuming the thicknesses d are following a normal distribution. How does it compare to the desired result of 1% or less? Finally, determine what the value of N should be so that the uncertainty in $\frac{\sigma_d}{d} \leq 0.01$.

4. T-SCORE ADDENDUM

Earlier we expressed t-score as $t = \frac{x_i - \bar{x}}{\sigma_x}$. However, this form of t-score assumes \bar{x} has zero uncertainty. A more general way of writing t-score would be the following

$$t = \frac{|x_1 - x_2|}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

where x_1 and x_2 are two quantities you are comparing and both have an uncertainty, σ_{x_1} and σ_{x_2} respectively. Note that this expression can be reduced to our previous expression if we assume one of the values has zero uncertainty. The important point is that the general t-score equation should be used when comparing two values that each have non-zero uncertainties.