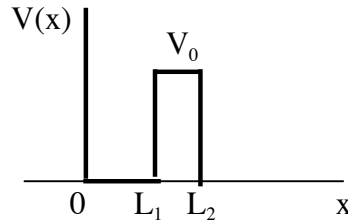


Use: $\hbar^2/2m_e = 3.81\text{eV}\text{\AA}^2$, m_e =electron mass

Problem 1 (10 pts)

$L_1=5\text{\AA}$, $L_2=8\text{\AA}$
 $V(x)=\infty$ for $x<0$
 $V(x)=0$ for $0<x<L_1$
 $V(x)=V_0$ for $L_1<x<L_2$
 $V(x)=0$ for $L_2<x$



An electron is in the region $0<x<L_1$ of the potential $V(x)$ shown in the figure in the lowest energy state. After approximately 1 second the electron escapes from this region and is in the region $x>L_2$.

- Estimate approximately the value of V_0 in eV. Explain all steps. Hint: V_0 is large
- For the value of V_0 found in (a) and the same L_1 , what should L_2 be for the electron to stay in the region $0<x<L_1$ for 10 seconds rather than 1 second?

Problem 2 (10 pts)

The wavefunction for an electron in a hydrogen-like ion with nuclear charge Ze is

$$\psi(r,\theta,\phi) = Cr^3 e^{-r/a_0} f(\theta)g(\phi)$$

where C is a constant and a_0 is the Bohr radius

- What is the energy of this electron in eV? Justify your answer.
- What is the value of Z ? Justify your answer.
- What are the possible values for the quantum numbers n , ℓ and m ? Justify your answer.
- What is the wavelength (in \AA) of the shortest wavelength photon that this electron can emit in making a transition from this state to another stationary state of this ion?

Problem 3 (10 pts)

For the electron described by the wavefunction in problem 2:

- Calculate the most probable r for this electron and explain how it is related to a Bohr orbit.
- Calculate $\langle 1/r \rangle$ for this electron in terms of a_0 (by doing integrals), and the average potential energy in eV.
- Calculate $\langle r \rangle$ for this electron in terms of a_0 .

$$\text{Use } \int_0^{\infty} dr r^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$$

Justify all your answers to all problems.