Physics 176/276 - Quantitative Molecular Biology
Problem Set #3
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1 Flickering Channel

Consider N ion channels which can be either opened or closed. The state of a channel changes from open to closed at rate k_{OC} and from closed to open at rate k_{CO}.

a) Write down the master equation for the probability, p_n, that n channels are open. Find the steady state solution for the probability distribution.

b) For large N, *i.e.* $Np \gg 1$ and $N(1-p) \gg 1$, the distribution for the number of open channels obtained in **a)** can be approximated by a Gaussian. Calculate this Gaussian.

c) Write down the Langevin equation describing the fraction of open channels $x = n/N$.

d) Find the steady state solution for x. Write down the Langevin equation for a small perturbations from the mean $\delta x = x - \langle x \rangle$.

e) Find the correlation function $\langle \delta x(t) \delta x(t + \tau) \rangle$

2 Linear birth process

Let us consider the linear birth process which is characterized by the following reaction,

$$A \xrightarrow{\lambda} A + A \tag{1}$$

At time $t = 0$ there are $n_0 = 1$ individuals present.

a) Write down the master equation for the probability, $p_n(t)$, to find n individuals at time t. Introduce the probability generating function $G(z, t) = \sum_{n=0}^{\infty} p_n(t) z^n$ to derive a partial differential equation (pde).

b) Solve the pde by applying the method of characteristics.

c) Find an expression for the probabilities $p_n(t)$.