Problem 1 (10 pts)
A simple pendulum consists of a small aluminum ball of mass 11 g and volume $4 \mathrm{~cm}^{3}$ suspended from a nearly massless cord. It oscillates with frequency 1 Hz .
(a) What is its frequency of oscillation if it is submerged in water, assuming there is no friction? density of water $=1 \mathrm{~g} / \mathrm{cm}^{3}$
(b) In fact there is friction, and the pendulum in water is critically damped. How long does it take for the angle that the cord makes with the vertical to change from initial value 10 degrees (where the velocity is zero) to final value 0.1 degrees?
(c) Estimate the change in the entropy of the universe in the process (b) if the water is at $20^{\circ} \mathrm{C}$.

Problem 2 (10 pts)


A thin glass tube of length $\ell=80 \mathrm{~cm}$ is put vertically into a container with mercury $(\mathrm{Hg})$, so that half its length is below the surface. All the air that was in the glass tube initially remains in it. Ignore the vapor pressure of Hg . Find the height h of the mercury inside the tube. Assume air is an ideal gas and everything is at the same temperature at all times. Atmospheric pressure is $101,300 \mathrm{~N} / \mathrm{m}^{2}$, and the density of Hg is $13.6 \mathrm{~g} / \mathrm{cm}^{3}$.

Problem 3 (10 pts)
A container has helium gas (monoatomic, atomic weight=4) at atmospheric pressure and temperature $0^{\circ} \mathrm{C}$. Assume He behaves as an ideal gas. Gas constant $\mathrm{R}=8.314 \mathrm{~J} / \mathrm{mol} \mathrm{K}$
(a) Find the speed of sound propagating through this gas assuming sound propagation is an isothermal process. Give your answer in $\mathrm{m} / \mathrm{s}$. Hint: bulk modulus is $B=-V \partial P / \partial V$ (b) In reality, sound propagation is an adiabatic process (when the He is compressed/rarified it temperature slightly increases/decreases). Find the speed of sound propagating through He under this assumption.
(c) An approximate expression for the speed of sound through He at another temperature T not too far from $0^{\circ} \mathrm{C}$ is $\mathrm{v}=(\mathrm{a}+\mathrm{bT}) \mathrm{m} / \mathrm{s}$
with T expressed in ${ }^{\circ} \mathrm{C}$ and a and b constants. The value of a is the value you found in (b). Find the value of $b$.

## Problem 4 (10 pts)

The passenger in a moving car $(\mathrm{M})$ at speed $20 \mathrm{~m} / \mathrm{s}$ is playing a guitar as it approaches a person on the ground (G) playing an identical guitar. When they play the same string, G hears 25 beats per second (i.e. the beat frequency is 25 Hz ). Speed of sound $=343 \mathrm{~m} / \mathrm{s}$.
(a) How many beats per second does M hear?
(b) If the string being played has mass 0.5 g and is under 100 N tension, what is its length in cm ? Assume the beats originate in the fundamental frequency of the string.
(c) What is the wavelength of the fundamental string vibration, and what is the wavelength of the sound wave produced, all in cm ?

Problem 5 (10 pts)


There are 10 mol of liquid water in equilibrium with steam in the closed container shown in the figure, of volume $1.5 \mathrm{~m}^{3}$, at temperature $80^{\circ} \mathrm{C}$. Assume the steam behaves as an ideal gas, and the volume occupied by the liquid can be ignored since it is very small. The saturated vapor pressure of water at $80^{\circ} \mathrm{C}$ is $4.73 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}$, at $90^{\circ} \mathrm{C}$ it is $7.01 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}$. The latent heat of vaporization of water is $2.26 \times 10^{6} \mathrm{~J} / \mathrm{kg}$. Assume the specific heat of water and steam in this temperature range is the same, $3000 \mathrm{~J} / \mathrm{kg}{ }^{\circ} \mathrm{C}$. The gas constant is $\mathrm{R}=8.314 \mathrm{~J} / \mathrm{mol} \mathrm{K}$. Molecular weight of water is 18 .
(a) How many mol of vapor (steam) are there in the container at $80^{\circ} \mathrm{C}$ ?
(b) The temperature of the system is raised to $90^{\circ} \mathrm{C}$. After a long time, does all the liquid evaporate or only part? Justify your answer.
(c) Find the pressure in the container at $90^{\circ} \mathrm{C}$ after a long time, in $\mathrm{N} / \mathrm{m}^{2}$.
(d) Find the change in entropy of this system in raising its temperature from $80^{\circ} \mathrm{C}$ to $90^{\circ} \mathrm{C}$. Your answer may be approximate, within $10 \%$.

## Problem 6 (10 pts)

When a body at temperature $T_{1}$ is put in thermal contact with a heat reservoir at temperature $\mathrm{T}_{2}<\mathrm{T}_{1}$, an amount of heat $\mathrm{Q}_{0}$ is transferred from the body to the heat reservoir until the body attains temperature $\mathrm{T}_{2}$.
When we put a heat engine between the body initially at temperature $\mathrm{T}_{1}$ and the heat reservoir at temperature $\mathrm{T}_{2}$, in each cycle some work is done, some heat is absorbed from the body, some heat is exhausted to the heat reservoir, and the temperature of the body decreases by some amount. After a large number of cycles, the temperature of the body reaches $\mathrm{T}_{2}$ and no more work can be obtained. The temperature of the heat reservoir is always $\mathrm{T}_{2}$.
(a) What is the maximum fraction of $\mathrm{Q}_{0}$ that could have been converted into work by such a heat engine after a large number of cycles? Your answer should depend on $T_{1} / T_{2}$ only. The answer is not $1-T_{2} / T_{1}$.
(b) What is the range of possible values of $\mathrm{T}_{1} / \mathrm{T}_{2}$ if $0.5 \mathrm{Q}_{0}$ work is obtained in the process of cooling the body from $\mathrm{T}_{1}$ to $\mathrm{T}_{2}$ ? Your answer may be approximate, accurate to $10 \%$. Hint: Start by finding an expression for the heat capacity of the body expressed in terms of the quantities given.

Problem 7 (10 pts)


A monoatomic ideal gas ( n moles) undergoes the reversible cycle shown in the figure, from 1 to 2 to 3 and back to 1 .
1-2: from initial pressure $P_{1}$, initial volume $V_{1}$, doubling $P$ and $V$ so that $P / V$ is constant throughout the process, final pressure is $2 \mathrm{P}_{1}$, final volume $2 \mathrm{~V}_{1}$.
2-3: isothermal expansion until the pressure returns to the initial value $P_{1}$.
$3-1$ : isobaric compression until the volume returns to the initial value $\mathrm{V}_{1}$.
(a) Find the work done (with its sign) and the heat absorbed or released in process 1-2, in terms of the temperature at point $1, \mathrm{~T}_{1}$.
(b) Same for process 2-3, also in terms of $\mathrm{T}_{1}$.
(c) Same for process 3-1, also in terms of $\mathrm{T}_{1}$.
(d) Find the efficiency of this heat engine, defined as [total work done in cycle]/[total heat absorbed in cycle]
(e) Explain why the efficiency is so low.
(f) Assume that process 2-3 is a free expansion rather than reversible. Find the change in entropy of the universe in one cycle.

Problem 8 (10 pts)
System A has 2 particles and system B has 3 particles. There is a total of 3 energy units to be shared among the 2 systems. Each particle can only have an integer number of energy units (or zero of course).
(a) Find the total number of microstates of the combined system for each possible partition of the energy. I.e. for system A having 3 energy units and system B 0 units, for system A having 2 energy units and system B 1 unit, etc.
(b) Show that for the most likely partition of the energy the energy per particle is more similar than for more unlikely partitions.
(c) Assume initially system A has all the energy. Then A and B are allowed to interchange energy with each other, and the end result is the energy partition that gives the largest number of microstates. Find the change in entropy in this process expressed in terms of Boltzmann's constant k .

