PHYSICS 161 Prof: Kim Griest
HOMEWORK 2 DUE: Wed, 16 Apr, 2014

1. Use a spacetime diagram
a) to show one minute on a moving clock is measured longer than one minute in the lab frame.
b) to shade all the points in the past lightcone of an event that occurs at $(t, x) = (2, 1)$.
c) to shade all the points that an event that occured at $(t, x) = (2, 1)$ could influence in the future.

2. Assume that you are at the origin of a laboratory reference system at time t=0 when you start your clock (event A). Determine whether the following events are within the future lightcone or past lightcone of event A, or "elsewhere" (outside of the event A lightcone) (assume the speed of light is 3×10^8m/s and a year lasts 3.15×10^7s)
(a) A flashbulb goes off 12 km away at time t=1 s.
(b) A flashbulb goes off 12 km away at time t=0.
(c) A flashbulb goes off 120,000 km away at time t=3 s.
(d) A flashbulb goes off 120 km away at time t=3 s.
(e) A supernova explodes 181,000 ly away at time $t = -5.8 \times 10^{12}$ s.
(f) A supernova explodes 181,000 ly away at time $t = 5.8 \times 10^{12}$ s.
(g) A supernova explodes 181,000 ly away at time $t = -5.6 \times 10^{12}$ s.
(h) A supernova explodes 181,000 ly away at time $t = 5.6 \times 10^{12}$ s.
(i) For item (e), could an observer in another reference frame moving relative to yours measure that the supernova exploded *after* event A?

3. Assume event A is at the origin $(t, x) = (t', x') = (0, 0)$, and event B is at $(x, t) = (x_1, t_1)$, so the invariant interval between events A and B is $-t_1^2 + x_1^2$. Using the Lorentz transformation find (t', x'). Now plug these values into the invariant interval formula $\Delta s^2 = -\Delta t'^2 + \Delta x'^2$ to show the invariant interval is still $-t_1^2 + x_1^2$.

4. Because there is no such thing as absolute simultaneity, two observers in relative motion may disagree on which of two events A and B occurred first. Suppose, however, that an observer in reference frame O measures that event A occurred first and *caused* event B. For example, event A might be pushing a light switch, and event B might be a light bulb turning on. Prove that an observer in another frame O' cannot measure event B (the effect) occurring before event A (the cause). The temporal order of cause and effect are preserved by the Lorentz transformation equations. *Hint:* For event A to cause event B, information must have traveled from A to B, and the fastest that *anything* can travel is the speed of light.

5. In an inertial lab frame two events occur simultaneously at a distance of 3 m apart. In a moving frame, one event happens later than the other by 2×10^{-8} seconds. By what spatial distance are the two events separated in the moving frame? How fast is the frame moving? Solve this problem two ways: 1) find and use the Lorentz boost that connects the two frames, and 2) make use of the invariance of the spacelike interval between the two events.

6. (a) A satellite orbits the Earth in a circular orbit above the equator a distance of 650 km from the surface. Using only special relativity, calculate by how many seconds per day will a clock on such a satellite run slow compared to a clock on the Earth? *Hint:* Use good old Newton's laws for a spherical orbit to calculate the speed of the satellite. Mass of Earth is in back of book.
(b) Over one day, compared to ignoring special relativity, the time dilation you calculated above corresponds to how many meters difference in distance?
(c) Repeat the above calculation for GPS satellites. These move such they orbit the Earth every 12 hours.
(d) If the GPS clocks are reset twice per day, does special relativity need to be taken into account for GPS to give positional accuracy to 100 meters? To 10 meters? To 1 meter?

7. Lhasa, Tibet is at an altitude of 3.5 km above sea level. If a person lives there for 75 years, how much longer (in seconds) would gravitational time dilation have allowed that person to live if he or she had moved at birth from Lhasa to San Diego (which is at sea level). [*Hint:* use the time dilation equation twice and Taylor expand.] Are you glad you live in San Diego?

8. (a) O' frame moves at the speed of v relative to O frame. An observer in O' frame throws a ball at the speed of w. What is the speed of the ball for an observer in O frame? (*Hint:* Make use of inverse Lorentz transformation)
(b) Captain Jean-Luc Picard reports in his captain's log that he is chasing an alien vessel running away from the Earth. He measures the speed of the alien ship as 0.94c relative to his ship, and his starship, traveling in the same direction behind the alien ship, is moving at 0.98c away from Earth. What is the speed of the alien vessel for an observer on the Earth? (no warp drive)
(c) Can the alien vessel receive a radio message from Captain Picard?
(d) Can the alien vessel receive a radio message from the Earth?

9. The highest energy cosmic ray that has been observed has an energy of around 4×10^{20} eV. Assuming this particle is a proton, calculate the Lorentz gamma factor. How much (in grams) does this single proton weigh? (Note there are useful numbers and conversion factors in the back of the textbook; this is a problem to get you used to putting real numbers on things.)