These solutions are from the 3rd edition of the book, but with the problem number from the 4th edition. Some of the equations that are referred to in the solutions may not correspond to the same equation in the 4th edition of the book.

9.20

(a) Use the binomial expansion for the square root in Eq. 9.126:

$$\kappa \cong \omega\sqrt{\frac{\epsilon\mu}{2}}\left[1+\frac{1}{2}\left(\frac{\sigma}{\epsilon\omega}\right)^2-1\right]^{1/2} = \omega\sqrt{\frac{\epsilon\mu}{2}}\frac{1}{\sqrt{2}}\frac{\sigma}{\epsilon\omega} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}.$$

So (Eq. 9.128) $d = \frac{1}{\kappa} \cong \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$. qed

For pure water, $\begin{cases} \epsilon = \epsilon_r\epsilon_0 = 80.1\,\epsilon_0 \quad \text{(Table 4.2)}, \\ \mu = \mu_0(1+\chi_m) = \mu_0(1-9.0\times 10^{-6}) \cong \mu_0 \quad \text{(Table 6.1)}, \\ \sigma = 1/(2.5\times 10^5) \quad \text{(Table 7.1)}. \end{cases}$

So $d = (2)(2.5\times 10^5)\sqrt{\frac{(80.1)(8.85\times 10^{-12})}{4\pi\times 10^{-7}}} = \boxed{1.19\times 10^4\text{ m.}}$

(b) In this case $(\sigma/\epsilon\omega)^2$ dominates, so (Eq. 9.126) $k \cong \kappa$, and hence (Eqs. 9.128 and 9.129) $\lambda = \frac{2\pi}{k} \cong \frac{2\pi}{\kappa} = 2\pi d$, or $d = \frac{\lambda}{2\pi}$. qed

Meanwhile $\kappa \cong \omega\sqrt{\frac{\epsilon\mu}{2}}\sqrt{\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{(10^{15})(4\pi\times 10^{-7})(10^7)}{2}} = 8\times 10^7$; $d = \frac{1}{\kappa} = \frac{1}{8\times 10^7} = 1.3\times 10^{-8} = \boxed{13\text{ nm.}}$ So the fields do not penetrate far into a metal—which is what accounts for their opacity.

(c) Since $k \cong \kappa$, as we found in (b), Eq. 9.134 says $\phi = \tan^{-1}(1) = 45°$. qed

Meanwhile, Eq. 9.137 says $\frac{B_0}{E_0} \cong \sqrt{\epsilon\mu\frac{\sigma}{\epsilon\omega}} = \boxed{\sqrt{\frac{\sigma\mu}{\omega}}.}$ For a typical metal, then, $\frac{B_0}{E_0} = \sqrt{\frac{(10^7)(4\pi\times 10^{-7})}{10^{15}}} = \boxed{10^{-7}\text{ s/m.}}$ (In vacuum, the ratio is $1/c = 1/(3\times 10^8) = 3\times 10^{-9}$ s/m, so the magnetic field is comparatively about 100 times larger in a metal.)

9.21

(a) $u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) = \frac{1}{2}e^{-2\kappa z}\left[\epsilon E_0^2\cos^2(kz-\omega t+\delta_E) + \frac{1}{\mu}B_0^2\cos^2(kz-\omega t+\delta_E+\phi)\right]$. Averaging over a full cycle, using $\langle\cos^2\rangle = \frac{1}{2}$ and Eq. 9.137:

$$\langle u\rangle = \frac{1}{2}e^{-2\kappa z}\left[\frac{\epsilon}{2}E_0^2 + \frac{1}{2\mu}B_0^2\right] = \frac{1}{4}e^{-2\kappa z}\left[\epsilon E_0^2 + \frac{1}{\mu}E_0^2\epsilon\mu\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right] = \frac{1}{4}e^{-2\kappa z}\epsilon E_0^2\left[1+\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right].$$

But Eq. 9.126 $\Rightarrow 1+\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} = \frac{2}{\epsilon\mu}\frac{k^2}{\omega^2}$, so $\langle u\rangle = \frac{1}{4}e^{-2\kappa z}\epsilon E_0^2\frac{2}{\epsilon\mu}\frac{k^2}{\omega^2} = \boxed{\frac{k^2}{2\mu\omega^2}E_0^2 e^{-2\kappa z}.}$ So the ratio of the magnetic contribution to the electric contribution is

$$\frac{\langle u_{\text{mag}}\rangle}{\langle u_{\text{elec}}\rangle} = \frac{B_0^2/\mu}{E_0^2\epsilon} = \frac{1}{\mu\epsilon}\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} = \sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} > 1. \quad \text{qed}$$

(b) $\mathbf{S} = \frac{1}{\mu}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu}E_0B_0e^{-2\kappa z}\cos(kz-\omega t+\delta_E)\cos(kz-\omega t+\delta_E+\phi)\,\hat{\mathbf{z}}$; $\langle\mathbf{S}\rangle = \frac{1}{2\mu}E_0B_0e^{-2\kappa z}\cos\phi\,\hat{\mathbf{z}}$. [The average of the product of the cosines is $(1/2\pi)\int_0^{2\pi}\cos\theta\cos(\theta+\phi)\,d\theta = (1/2)\cos\phi$.] So $I = \frac{1}{2\mu}E_0B_0e^{-2\kappa z}\cos\phi = \frac{1}{2\mu}E_0^2e^{-2\kappa z}\left(\frac{K}{\omega}\cos\phi\right)$, while, from Eqs. 9.133 and 9.134, $K\cos\phi = k$, so $\boxed{I = \frac{k}{2\mu\omega}E_0^2e^{-2\kappa z}.}$ qed

9.23

(a) We are told that $v = \alpha\sqrt{\lambda}$, where α is a constant. But $\lambda = 2\pi/k$ and $v = \omega/k$, so $\omega = \alpha k\sqrt{2\pi/k} = \alpha\sqrt{2\pi k}$. From Eq. 9.150, $v_g = \frac{d\omega}{dk} = \alpha\sqrt{2\pi}\frac{1}{2\sqrt{k}} = \frac{1}{2}\alpha\sqrt{\frac{2\pi}{k}} = \frac{1}{2}\alpha\sqrt{\lambda} = \frac{1}{2}v$, or $\boxed{v = 2v_g.}$

(b) $\frac{i(px - Et)}{\hbar} = i(kx - \omega t) \Rightarrow k = \frac{p}{\hbar},\ \omega = \frac{E}{\hbar} = \frac{p^2}{2m\hbar} = \frac{\hbar k^2}{2m}$. Therefore $\boxed{v = \frac{\omega}{k} = \frac{E}{p} = \frac{p}{2m} = \frac{\hbar k}{2m}};$ $v_g = \frac{d\omega}{dk} = \frac{2\hbar k}{2m} = \frac{\hbar k}{m} = \boxed{\frac{p}{m}.}$ So $\boxed{v = \frac{1}{2}v_g.}$ Since $p = mv_c$ (where v_c is the classical speed of the particle), it follows that $\boxed{v_g \text{ (not } v\text{) corresponds to the classical veloctity.}}$

9.25

Equation 9.170 $\Rightarrow n = 1 + \frac{Nq^2}{2m\epsilon_0}\frac{(\omega_0^2 - \omega^2)}{[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]}$. Let the denominator $\equiv D$. Then

$$\frac{dn}{d\omega} = \frac{Nq^2}{2m\epsilon_0}\left\{\frac{-2\omega}{D} - \frac{(\omega_0^2 - \omega^2)}{D^2}\left[2(\omega_0^2 - \omega^2)(-2\omega) + \gamma^2 2\omega\right]\right\} = 0 \Rightarrow 2\omega D = (\omega_0^2 - \omega^2)\left[2(\omega_0^2 - \omega^2) - \gamma^2\right]2\omega;$$

$(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2 = 2(\omega_0^2 - \omega^2)^2 - \gamma^2(\omega_0^2 - \omega^2)$, or $(\omega_0^2 - \omega^2)^2 = \gamma^2(\omega^2 + \omega_0^2 - \omega^2) = \gamma^2\omega_0^2 \Rightarrow (\omega_0^2 - \omega^2) = \pm\omega_0\gamma;$

$\omega^2 = \omega_0^2 \mp \omega_0\gamma,\ \omega = \omega_0\sqrt{1 \mp \gamma/\omega_0} \cong \omega_0(1 \mp \gamma/2\omega_0) = \omega_0 \mp \gamma/2$. So $\omega_2 = \omega_0 + \gamma/2,\ \omega_1 = \omega_0 - \gamma/2$, and the width of the anomalous region is $\boxed{\Delta\omega = \omega_2 - \omega_1 = \gamma.}$

From Eq. 9.171, $\alpha = \frac{Nq^2\omega^2}{m\epsilon_0 c}\frac{\gamma}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$, so at the maximum ($\omega = \omega_0$), $\alpha_{\max} = \frac{Nq^2}{m\epsilon_0 c\gamma}$.

At ω_1 and ω_2, $\omega^2 = \omega_0^2 \mp \omega_0\gamma$, so $\alpha = \frac{Nq^2\omega^2}{m\epsilon_0 c}\frac{\gamma}{\gamma^2\omega_0^2 + \gamma^2\omega^2} = \alpha_{\max}\left(\frac{\omega^2}{\omega^2 + \omega_0^2}\right)$. But

$$\frac{\omega^2}{\omega^2 + \omega_0^2} = \frac{\omega_0^2 \mp \omega_0\gamma}{2\omega_0^2 \mp \omega_0\gamma} = \frac{1}{2}\frac{(1 \mp \gamma/\omega_0)}{(1 \mp \gamma/2\omega_0)} \cong \frac{1}{2}\left(1 \mp \frac{\gamma}{\omega_0}\right)\left(1 \pm \frac{\gamma}{2\omega_0}\right) \cong \frac{1}{2}\left(1 \mp \frac{\gamma}{2\omega_0}\right) \cong \frac{1}{2}.$$

So $\alpha \cong \frac{1}{2}\alpha_{\max}$ at ω_1 and ω_2. qed

9.27

(a) From Eqs. 9.176 and 9.177, $\nabla\times\tilde{\mathbf{E}} = -\frac{\partial\tilde{\mathbf{B}}}{\partial t} = i\omega\tilde{\mathbf{B}}_0 e^{i(kz-\omega t)}$; $\nabla\times\tilde{\mathbf{B}} = \frac{1}{c^2}\frac{\partial\tilde{\mathbf{E}}}{\partial t} = -\frac{i\omega}{c^2}\tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}$.
In the terminology of Eq. 9.178:

$$(\nabla\times\tilde{\mathbf{E}})_x = \frac{\partial\tilde{E}_z}{\partial y} - \frac{\partial\tilde{E}_y}{\partial z} = \left(\frac{\partial\tilde{E}_{0_z}}{\partial y} - ik\tilde{E}_{0_y}\right)e^{i(kz-\omega t)}. \text{ So (ii) } \frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x.$$

$$(\nabla\times\tilde{\mathbf{E}})_y = \frac{\partial\tilde{E}_x}{\partial z} - \frac{\partial\tilde{E}_z}{\partial x} = \left(ik\tilde{E}_{0_x} - \frac{\partial\tilde{E}_{0_z}}{\partial x}\right)e^{i(kz-\omega t)}. \text{ So (iii) } ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y.$$

$$(\nabla\times\tilde{\mathbf{E}})_z = \frac{\partial\tilde{E}_y}{\partial x} - \frac{\partial\tilde{E}_x}{\partial y} = \left(\frac{\partial\tilde{E}_{0_y}}{\partial x} - \frac{\partial\tilde{E}_{0_x}}{\partial y}\right)e^{i(kz-\omega t)}. \text{ So (i) } \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z.$$

$$(\nabla\times\tilde{\mathbf{B}})_x = \frac{\partial\tilde{B}_z}{\partial y} - \frac{\partial\tilde{B}_y}{\partial z} = \left(\frac{\partial\tilde{B}_{0_z}}{\partial y} - ik\tilde{B}_{0_y}\right)e^{i(kz-\omega t)}. \text{ So (v) } \frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2}E_x.$$

$$(\nabla\times\tilde{\mathbf{B}})_y = \frac{\partial\tilde{B}_x}{\partial z} - \frac{\partial\tilde{B}_z}{\partial x} = \left(ik\tilde{B}_{0_x} - \frac{\partial\tilde{B}_{0_z}}{\partial x}\right)e^{i(kz-\omega t)}. \text{ So (vi) } ikB_x - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{c^2}E_y.$$

$$(\nabla\times\tilde{\mathbf{B}})_z = \frac{\partial\tilde{B}_y}{\partial x} - \frac{\partial\tilde{B}_x}{\partial y} = \left(\frac{\partial\tilde{B}_{0_y}}{\partial x} - \frac{\partial\tilde{B}_{0_x}}{\partial y}\right)e^{i(kz-\omega t)}. \text{ So (iv) } \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{c^2}E_z.$$

This confirms Eq. 9.179. Now multiply (iii) by k, (v) by ω, and subtract: $ik^2E_x - k\frac{\partial E_z}{\partial x} - \omega\frac{\partial B_z}{\partial y} + i\omega kB_y = ik\omega B_y + \frac{i\omega^2}{c^2}E_x \Rightarrow i\left(k^2 - \frac{\omega^2}{c^2}\right)E_x = k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}$, or (i) $E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$.

Multiply (ii) by k, (vi) by ω, and add: $k\frac{\partial E_z}{\partial y} - ik^2E_y + i\omega kB_x - \omega\frac{\partial B_z}{\partial x} = i\omega kB_x - \frac{i\omega^2}{c^2}E_y \Rightarrow i\left(\frac{\omega^2}{c^2} - k^2\right)E_y = -k\frac{\partial E_z}{\partial y} + \omega\frac{\partial B_z}{\partial x}$, or (ii) $E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$.

Multiply (ii) by ω/c^2, (vi) by k, and add: $\frac{\omega}{c^2}\frac{\partial E_z}{\partial y} - i\frac{\omega k}{c^2}E_y + ik^2B_x - k\frac{\partial B_z}{\partial x} = i\frac{\omega^2}{c^2}B_x - i\frac{\omega k}{c^2}E_y \Rightarrow i\left(k^2 - \frac{\omega^2}{c^2}\right)B_x = k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}$, or (iii) $B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$.

Multiply (iii) by ω/c^2, (v) by k, and subtract: $i\frac{\omega k}{c^2}E_x - \frac{\omega}{c^2}\frac{\partial E_z}{\partial x} - k\frac{\partial B_z}{\partial y} + ik^2B_y = i\frac{\omega^2}{c^2}B_y + \frac{i\omega k}{c^2}E_x \Rightarrow i\left(k^2 - \frac{\omega^2}{c^2}\right)B_y = \frac{\omega}{c^2}\frac{\partial E_z}{\partial x} + k\frac{\partial B_z}{\partial y}$, or (iv) $B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$.
This completes the confirmation of Eq. 9.180.

(b) $\nabla\cdot\tilde{\mathbf{E}} = \frac{\partial\tilde{E}_x}{\partial x} + \frac{\partial\tilde{E}_y}{\partial y} + \frac{\partial\tilde{E}_z}{\partial z} = \left(\frac{\partial\tilde{E}_{0_x}}{\partial x} + \frac{\partial\tilde{E}_{0_y}}{\partial y} + ik\tilde{E}_{0_z}\right)e^{i(kz-\omega t)} = 0 \Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + ikE_z = 0.$

Using Eq. 9.180, $\frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial^2 E_z}{\partial x^2} + \omega\frac{\partial^2 B_z}{\partial x\partial y}\right) + \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial^2 E_z}{\partial^2 y} - \omega\frac{\partial^2 B_z}{\partial x\partial y}\right) + ikE_z = 0,$
or $\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial^2 y} + \left[(\omega/c)^2 - k^2\right]E_z = 0.$

Likewise, $\nabla\cdot\tilde{\mathbf{B}} = 0 \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + ikB_z = 0 \Rightarrow$
$\frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial^2 B_z}{\partial x^2} - \frac{\omega}{c^2}\frac{\partial^2 E_z}{\partial x\partial y}\right) + \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial^2 B_z}{\partial y^2} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x\partial y}\right) + ikB_z = 0 \Rightarrow$
$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial^2 y} + \left[(\omega/c)^2 - k^2\right]B_z = 0.$

This confirms Eqs. 9.181. [You can also do it by putting Eq. 9.180 into Eq. 9.179 (i) and (iv).]