Foundations of Supersymmetry

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There is a large class of extensions to the Standard Model which propose a symmetry that relates bosons and fermions, known as supersymmetry. A theorem due to Coleman and Mandula proved the impossibility of combining space-time and internal symmetries in all but a trivial way, and the Haag-Lopuszanski-Sohnius theorem later generalized this to show supersymmetries are the only way of extending the Poincare algebra. Supersymmetric extensions of the Standard Model have been heavily studied for the last 50 years for their many theoretically attractive qualities, including fixing the divergent corrections to the Higgs mass and providing potential dark matter candidates.

INTRODUCTION

While the Standard Model (SM) has been experimentally very successful, we have theoretical reasons to believe it is incomplete. In the construction of beyond-SM theories, it is a natural question to ask – what possible symmetries can the theory contain? Suppose we are trying to describe the set of all possible symmetries of the S matrix of some scattering theory. S is Lorentz invariant if it contains a group of symmetries isomorphic to the Poincare group P. Symmetries which commute with P are called *internal symmetries*, and only act on particle-type indices. The question then arises – can we build theories with symmetries that cannot be expressed as a direct product of the Poincare group and internal symmetries?

Coleman and Mandula published a solution to this question in 1967 with a proof that for the symmetry group G of a large class of field theories (which we discuss later), G must be locally isomorphic to a direct product of the Poincare group and an internal symmetry group [1]. Haag, Lopuszanski, and Sohnius later generalized this theorem and showed that the only possible non-trivial symmetries (which are not Poincare or typical internal symmetries) are *supersymmetries*, those that exchange bosons with fermions (and vice versa). Supersymmetric (SUSY) extensions of the SM thus contain a supersymmetric partner particle (sparticle) for each SM particle.

SUSY theories offer very theoretically appealing beyond-SM candidates since they naturally solve many open questions. For example, we know corrections to the Higgs mass are quadratically divergent in the SM, and it fails to explain the nature of dark matter (for which we have an abundance of cosmological evidence). SUSY elegantly solves these problems, and so has become the dominant model for building beyond-SM theories [2, 3].

POSSIBLE SYMMETRIES OF THE S MATRIX

We begin with a discussion of Coleman and Mandula's 'no-go theorem' for combining space-time and internal symmetries [1]. Suppose G is the (continuous) symmetry group of the S matrix. Furthermore, suppose the following:

1. (Lorentz invariance) G contains a subgroup isomorphic to the Poincare group P.
2. (Particle finiteness) For any finite M, there are only finitely many particles with mass less than M.
3. (Scattering analycity) Elastic scattering amplitudes are analytic functions of center-of-mass energy (s) and momentum transfer (t), except at normal thresholds.
4. (Non-trivial scattering) For any two one-particle momentum eigenstates, scattering is non-trivial (except possibly at isolated values of s).
5. (A technical assumption) The generators of G have distributions for their kernels.

All these assumptions are physical, except possibly assumption (5), which Coleman-Mandula described as a necessary technical assumption "which, although weak, is ugly", but which they found necessary to manipulate infinitesimal generators in the course of the proof (though they hoped it could be weakened or removed completely in the future). The Coleman-Mandula theorem states that if assumptions (1)-(5) hold, then G is isomorphic to a direct product of the Poincare group and an internal symmetry group. In other words, the only possible symmetries of the S matrix factor into three distinct groups:

1. Poincare symmetries, since our theories are Lorentz invariant.
2. Internal symmetries, whose generators act on particle-type. Their commutators form a Lie algebra: $[A_i, A_j] = ic_{ij}^k A_k$
3. Discrete symmetries (i.e. C, P, T).

These results were greatly generalized by Haag, Lopuszanski, and Sohnius in 1975 when they proved an analogous version of the theorem for a weaker set of assumptions. By allowing anticommutating (as well as commutating) generators, they showed the Poincare algebra could be extended to allow for supersymmetries [4]. The generators of these supersymmetries act on particle states turning bosons (fermions) into fermions (bosons), with the anticommutation relations

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu$$

where P_μ is the total energy-momentum 4-vector [3]. Since the anticommuting generators transform as spinors, they cannot be internal symmetries (and in fact extend the Poincare symmetries) [2]. Other attempts at further weakening the assumptions of the Haag-Lopuszanski-Sohnius theorem have been made (e.g. [5]), but since they do not offer the same theoretical appeal of SUSY, it is widely believed that SUSY is the *only possible extension* of space-time symmetries in the SM.

CONSEQUENCES OF SUPERSYMMETRY

The inclusion of supersymmetry in beyond-SM theories has many favorable implications, both theoretically and phenomenologically.

Theoretically, SUSY solves many of the divergence problems in the SM. Since each bosonic particle has a fermionic partner of the same mass, they must have the same mass renormalization. But since fermionic mass terms can only be logarithmically divergent and bosonic mass terms can be quadratically divergent, by applying the supersymmetry it is clear the quadratic divergences (of scalars) must vanish at every order of perturbation theory [2, 3]. This automatically fixes problems such as the (quadratically) divergent Higgs mass corrections in the SM. Furthermore, since the vacuum state is supersymmetric (i.e. $Q_\alpha|0\rangle = Q^\dagger_\beta|0\rangle = 0$) the vacuum energy must be zero ($\langle 0|H|0\rangle = 0$), and in fact the bosonic and fermionic contributions to the vacuum energy exactly cancel at all orders in perturbation theory[3].

However, there is clearly no evidence for SUSY particles with the exact same mass as their SM partners, since we would have seen them already in our particle collliders. Thus if supersymmetry is realized in nature, it must not be an exact symmetry, but spontaneously broken. It is possible to have an approximate supersymmetry that still fixes the quadratic divergence of the Higgs mass at the right scales, so long as the mass differences are not too large [2]. Since such SUSY theories have a large spectrum of sparticles which are weakly coupled to the SM sector, they offer good candidates for dark matter particles to help explain cosmological observations.

A spontaneously broken supersymmetry is phenomenologically rich, and such sparticles should be within reach of the latest particle colliders. The most recent searches at the Large Hadron Collider (LHS) have yet to reveal experimental evidence for any SUSY theories. Nonetheless, search efforts preparing for the next LHC run at $\sqrt{s} = 13$ TeV are underway, and there is still parameter space to explore.

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