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Quantum Mechanics B (Physics 130B) Fall 2014
Worksheet 6

Announcements

- The 130B web site is:

  http://physics.ucsd.edu/students/courses/fall2014/physics130b/ .

  Please check it regularly! It contains relevant course information!

- Greetings everyone! This week we're going to kick the harmonic oscillator and talk about spontaneous emission.

Problems

1. **Give it a Kick**

   Consider the $D = 1$ simple harmonic oscillator in its groundstate. Suppose something kicks the system imparting an additional momentum p_0. What's the probability the system remains in the ground state?

   (a) What's the new Hamiltonian for the system? Express this in terms of the usual ladder operators $\hat{a}$ and $\hat{a}^\dagger$

   (b) Define a new operator $\hat{A} \equiv \hat{a} - \beta$ where $\beta \equiv \frac{1}{\mathrm{i}\omega}\frac{p_0}{m}\sqrt{\frac{m\omega}{2}}$.
   Show that the $\hat{A}$ are ladder operators: $[\hat{A}, \hat{A}^\dagger] = 1$

   (c) Rewrite the new Hamiltonian in terms of these operators, what do you find?

   (d) Relate the original groundstate $|0\rangle$ to the new groundstate $|\beta\rangle$

   (e) Using $|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$ compute $P = |\langle 0|\beta\rangle|^2$
   Hint: Insert identity and use the relation above.

2. **Multipole transitions**

   Consider an electric field of the form:

$$\vec{E}(r,t) = E_0(\cos\omega t + (k \cdot r)\sin\omega t)\hat{n} \quad (1)$$

which is coupling to a particle of charge q. Recall from lecture that the interaction Hamiltonian is: $H' = -qE(r,t)\hat{n} \cdot r$ and that the spatially independent term produces a spontaneous decay rate of:

$$R_{f \to i} = \frac{\omega^3 q^2 |\langle f|(\hat{n} \cdot r)|i\rangle}{\pi \epsilon_0 \hbar c^3} \tag{2}$$

(a) Write the expression analogous to 2 for the spatially varying piece

(b) Consider this problem where the particle is in a $D = 1$ oscillator potential with frequency Ω. Calculate the transition rate from n to $n - 2$; don't calculate the averaging over $\hat{n}$ or $\hat{k}$