University of California at San Diego – Department of Physics – TA: Shauna Kravec

Quantum Mechanics B (Physics 130B) Fall 2014
Worksheet 4

Announcements

- The 130B web site is:

  http://physics.ucsd.edu/students/courses/fall2014/physics130b/ .

  Please check it regularly! It contains relevant course information!

- Greetings everyone! This week we're going to discover why bra-ket notation is useful and do perturbation theory for a spin.

Problems

1. **Don't Give In**

   Suppose you're walking down the street and a man approaches you with a well-prepared quantum state of the form:

   $$\psi(\theta, \phi) = 2\sqrt{\frac{15}{16\pi}} \cos\theta \sin\theta \cos\phi \tag{1}$$

   He then asks you to predict the average value of various angular momentum quantities. Snickering, he offers only one piece of advice:

   $$Y_{2,\pm 1}(\theta, \phi) \equiv \mp\sqrt{\frac{15}{8\pi}} \cos\theta \sin\theta e^{\pm \mathbf{i}\phi} \tag{2}$$

   Can you figure out the answers to these questions without doing any integrals?

   (a) Calculate $\langle L_z \rangle$ for the state 1. If you need $L_z = -\mathbf{i}\partial_\phi$

   (b) Calculate $\langle L^2 \rangle$ again for 1. If you need $L^2 = -\nabla^2$ restricted to the 2-sphere.

2. **Sanity Check**

   Consider a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B} = \{B_x, 0, B_z\}$

   Generically the Hamiltonian to describe such a situation is:

   $$\hat{H} = -\mu_B \vec{B} \cdot \vec{\boldsymbol{\sigma}} \tag{3}$$

   where μ_B is the Bohr magneton and $\vec{\boldsymbol{\sigma}} = \{\sigma_x, \sigma_y, \sigma_z\}$ is a vector of Pauli matrices.

(a) Suppose $B_x = 0$, find the eigenstates and energies associated with 3

(b) Now suppose $B_z \gg B_x \neq 0$ and compute the first and second order corrections to the energy using perturbation theory.

(c) Now it turns out 3 is exactly solvable. Compute the energies of the exact eigenstates by direct diagonalization. Show by second order Taylor expansion this agrees with the above.