HW1 Solutions

1) (a) $L_x = yP_z - zP_y$

$L_y = zP_x - xP_z$

$$[L_x, L_y] = [yP_z - zP_y, zP_x - xP_z]$$

$$= [yP_z, zP_x] - \underset{\times}{[yP_z, xP_z]} - \underset{\times}{[zP_y, zP_x]} + [zP_y, xP_z]$$

$$= yP_z zP_x - zP_x yP_z + zP_y xP_z - xP_z zP_y$$

$$= \frac{\hbar}{i} yP_x - \frac{\hbar}{i} xP_y$$

$$= i\hbar (xP_y - yP_x)$$

$$= \boxed{i\hbar L_z} \checkmark$$

The commutation relations between L_x, L_y, L_z are essential for developing angular momentum in quantum mechanics. From here we can build up the L_+, L_-, L^2, and L_z operators

(b) $[A^2, B] = A^2B - BA^2$

$$A[A,B] + [A,B]A = A^2B - \cancel{ABA} + \cancel{ABA} - BA^2$$

$$= A^2B - BA^2 \checkmark$$

$$= [A^2, B]$$

This is a general property of communication relations that shows up a lot in Q.M.

1) ⓒ $[L^2, L_z] = \vec{L}[\vec{L}, L_z] + [\vec{L}, L_z]\vec{L}$

(HW 1 Solutions

$$[\vec{L}, L_z] = [L_x\hat{x} + L_y\hat{y} + \cancel{L_z\hat{z}}, L_z]$$

$$= -i\hbar L_y\hat{x} + i\hbar L_x\hat{y}$$

$$\vec{L}\cdot[\vec{L}, L_z] = -i\hbar L_x L_y + i\hbar L_y L_x$$

$$[L^2, L_z] = i\hbar(-L_xL_y + L_yL_x) + (-L_yL_x + L_xL_y)i\hbar$$

$$= 0 \checkmark$$

Since these two operators commute, we know that it's possible to find states that are eigenfunctions of both operators.

ⓓ physical significance given at the end of each section above

2) ⓐ $R = Cr^2 e^{-r/a}$

Griffiths 4.53: $-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[-\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Eu$ $\quad u = rR$

$$-\frac{\hbar^2}{2m}\, \frac{r(6a^2 - 6ar + r^2)}{a^2} + (-C_1 r^2 + C_2 r) = Er^3$$

$C_1 = \frac{-e^2}{4\pi\epsilon_0}$

$C_2 = \frac{\hbar^2 l(l+1)}{2m}$

grouping terms by their power in r

r^1) $-\frac{\hbar^2}{2m} 6r + C_2 r = 0$

$6 = l(l+1)$

$\boxed{l = 2}$

r^2) $\frac{\hbar^2 6r^2}{2ma} - C_1 r^2 = 0$

$\frac{6\hbar^2}{2ma} = \frac{e^2}{4\pi\epsilon_0}$

$\boxed{a = \frac{12\pi\epsilon_0\hbar^2}{me^2}}$

r^3) $-\frac{\hbar^2}{2ma^2} = E$

$\boxed{E = -\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{3^2}}$ so $n = 3$

ⓑ $L^2|\psi\rangle = \hbar^2 l(l+1)|\psi\rangle$

$= 6\hbar^2$

$|\vec{L}| = \sqrt{L^2} = \boxed{\hbar\sqrt{6}}$

ⓒ a is three times the Bohr radius, and E is the energy associated with the radial part of the wavefunction.