Please write in pen rather than in pencil. No regrading for finals written in pencil.
Write clearly and justify all steps.
Problem 1 (10 pts+2 pts extra credit)


A cylinder of height 10 cm and cross-sectional area $200 \mathrm{~cm}^{2}$ floats in water $\left(\rho=1 \mathrm{~g} / \mathrm{cm}^{3}\right)$ as shown in the figure to the left. A bird of mass 300 g is sitting on top of it. Half of the cylinder is submerged in water, and the system is in equilibrium. Temperature is $20^{\circ} \mathrm{C}$.
(a) Find the mass of the cylinder, in $g$.
(b) Then the bird flies away, as shown in the figure to the right. After a long time the cylinder stops moving at a new equilibrium position. What is the length of the cylinder submerged in water now?
(c) Immediately after the bird flies away, the cylinder starts oscillating up and down. Assuming negligible damping, find the frequency of oscillation, in rad/s.
(d) Find the maximum speed at which the cylinder moves after the bird flies away, in $\mathrm{cm} / \mathrm{s}$.
(e) When the cylinder has reached its new equilibrium position and stopped moving, has the entropy of the universe changed from the initial state? If it did, calculate the change in entropy, in J/K. If it didn't, explain why.

Problem 2 (10 pts)
A certain gas can exist both in monatomic and in diatomic forms. Find the ratio of the sound velocities in both forms, $\mathrm{v}_{\text {diatomic }} / \mathrm{v}_{\text {monoatomic }}$, for the gases at the same pressure and temperature. Assume they behave as ideal gases and assume adiabatic rather than isothermal compression/expansion when the sound wave propagates. Explain clearly your reasoning.

Please write in pen rather than in pencil. No regrading for finals written in pencil. Write clearly and justify all steps.

Problem 3 (10 pts+3 pts extra credit)


The two identical steel strings in the figure are tied to parallel bars at a distance 20 cm . The cross section of the strings is $0.1 \mathrm{~mm}^{2}$. The fundamental frequency of the upper string is 1000 Hz . The upper string is at temperature $25^{\circ} \mathrm{C}$, the lower string at $0^{\circ} \mathrm{C}$. Data: Density of steel: $8000 \mathrm{~kg} / \mathrm{m}^{3}$; Elastic modulus of steel: $200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}$; Linear thermal expansion coefficient of steel: $12 \times 10^{-6} /{ }^{\circ} \mathrm{C}$
(a) Find the tension in the upper string, in N .
(b) If the upper string was free to expand or contract under temperature changes, what would be its length (in cm ) at temperature $0^{\circ} \mathrm{C}$ under the tension calculated in (a)?
(c) What would be the extra tension (in N ) you would have to apply to prevent it from undergoing this length change?
(d) Find the fundamental frequency of the lower string, in Hz .
(e) When both strings are vibrating at their fundamental frequency, what is the time difference (in seconds) between two maxima in the sound intensity?

Problem 4 (10 pts+5 pts extra credit)
A person dressed in black is floating in outer space far from any star. You may approximate its body shape by a cylinder of height 1.5 m and radius 20 cm . Assume the person's specific heat is that of liquid water $\left(1 \mathrm{cal} / \mathrm{g}{ }^{\circ} \mathrm{C}\right)$ and its mass is 70 kg .
Data: $1 \mathrm{cal}=4.168 \mathrm{~J}$. $\quad$ Stefan-Boltzmann constant: $\sigma=5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}^{4}$.
(a) How many hamburgers does this person have to eat per day to keep its temperature at 36 C ? Each hamburger provides about 800 kcal , assume the work for chewing and digesting can be neglected.
(b) Assume the person stops eating. From the instant it finishes processing the last bit of food, assuming its temperature is $36^{\circ} \mathrm{C}$ at that time, how long does it take (in s) until its temperature drops to $34^{\circ} \mathrm{C}$ ? You may assume that the heat lost per second is constant during this process, because the temperature change is small.
(c) Find the temperature of this person after 1 day of fasting. You can't use the approximation used in (b) because the time interval is much longer.
Hint: find the differential equation describing this process and integrate it.

Please write in pen rather than in pencil. No regrading for finals written in pencil.
Write clearly and justify all steps.

Problem 5 (10 pts)


The box in the figure contains 1 mol of He atoms. Initially $1 / 3$ are moving horizontally in the $+/-x$ direction at speed $300 \mathrm{~m} / \mathrm{s}$, and $2 / 3$ are moving in the $+/-z$ direction at speed 600 $\mathrm{m} / \mathrm{s}$. Assume the system is thermally insulated.
Data: Mass of He atom: $6.64 \times 10^{-27} \mathrm{~kg}$. Boltzmann constant: $\mathrm{k}_{\mathrm{B}}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$.
Gas constant: $\mathrm{R}=8.314 \mathrm{~J} / \mathrm{mol} \mathrm{K}$.
(a) After a long time the system reaches thermal equilibrium due to collisions between the atoms. Find the temperature in K.
(b) Find the force on the top wall after equilibrium is reached, in N .
(c) Find the root mean square velocity initially and after equilibrium is reached.
(d) Find the most probable speed initially and after equilibrium is reached.

Problem 6 (10 pts)


The engine shown in the figure absorbs 90 J per cycle from the reservoir at 300 K and delivers 40J of work per cycle.
(a) Assume it absorbs 30J per cycle from the reservoir at 200K. Find the change in entropy per cycle of the engine and of the universe. State whether or not this engine operating this way is consistent with the 2nd law of thermodynamics.
(b) Assume now the engine is operating in a reversible way, absorbing 90J per cycle from the reservoir at 300 K and delivering 40 J of work per cycle. Find the heat per cycle absorbed from or released to the heat reservoirs at 200 K and at 100 K .

Please write in pen rather than in pencil. No regrading for finals written in pencil.
Write clearly and justify all steps.

Problem 7 (10 pts)


3 mol of a monoatomic ideal gas evolve in a reversible way from points 1 to 2 to 3 in the diagram. At point 1 the pressure is P , the volume V , and the temperature $\mathrm{T}=300 \mathrm{~K}$.
Gas constant: $\mathrm{R}=8.314 \mathrm{~J} / \mathrm{mol} \mathrm{K}$.
(a) Find the temperature of the gas at point 2 and at point 3 , in K.
(b) Find the change in entropy of the gas between the initial state (1) and the final state
(3). Give your answer in $\mathrm{J} / \mathrm{K}$.
(c) Find the change in entropy of the environment between the initial state (1) and the final state (3). Give your answer in $\mathrm{J} / \mathrm{K}$.
(d) Find the total heat absorbed by the gas in going from 1 to 3 . Give your answer in J .

Problem 8 (10 pts)
Consider first as an example a system of 2 distinguishable coins. Each coin has 2 states, heads or tails (h or t). Assume the state h has energy 1J, the state thas energy 0J. The microstate of the system is specified by giving the state of each coin, so there are 4 microstates: tt , th, ht, hh. Assume they are all equally probable The macrostate for this system is specified by the total energy, which is either $0 \mathrm{~J}, 1 \mathrm{~J}$ or 2 J .

Assume now you have a system A with 3 such coins and a system B with 4 such coins.
(a) Give the number of microstates for each macrostate of system A (i.e. for each possible value of $E_{A}$ ). Same for $B$.
(b) Find the total number of microstates available to the combined system A and B if the energies can take any value.
(c) Assume now the total energy of the combined system A and B is 3J. Find the total number of microstates of the combined system $A$ and $B$ for that case.
(d) Assume the macrostate is specified by giving the energy of $A, E_{A}$, and that of $B, E_{B}$, and the total energy is $\mathrm{E}_{\mathrm{A}}+\mathrm{E}_{\mathrm{B}}=3 \mathrm{~J}$. What is the most probable macrostate in that case?
Give the values of $\mathrm{E}_{\mathrm{A}}$ and $\mathrm{E}_{\mathrm{B}}$ and the number of microstates for that particular macrostate.

