Chapter 30

1. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

2. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d\left(BA\right)}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(\pi r^2\right)}{dt} = -2\pi rB\frac{dr}{dt} \\ &= -2\pi\left(0.12\,\text{m}\right)\left(0.800\,\text{T}\right)\left(-0.750\,\text{m/s}\right) \\ &= 0.452\,\text{V}.\end{aligned}$$

3. The total induced emf is given by

$$\begin{aligned}\varepsilon &= -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt} \\ &= -(120)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(22000/\text{m})\,\pi\left(0.016\,\text{m}\right)^2\left(\frac{1.5\ \text{A}}{0.025\ \text{s}}\right) \\ &= 0.16\,\text{V}.\end{aligned}$$

Ohm's law then yields $i = |\varepsilon|/R = 0.016\ \text{V}/5.3\Omega = 0.030\ \text{A}$.

4. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2\frac{dB}{dt} = -\pi\left(0.12\,\text{m}\right)^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times10^{-2}\ \text{V}.$$

(b) For 2.0 s $< t <$ 4.0 s: $\varepsilon \propto dB/dt = 0$.

(c) For 4.0 s $< t <$ 6.0 s:

$$\varepsilon = -\pi r^2\frac{dB}{dt} = -\pi\left(0.12\,\text{m}\right)^2\left(\frac{-0.5\,\text{T}}{6.0\,\text{s} - 4.0\,\text{s}}\right) = 1.1\times10^{-2}\ \text{V}.$$

5. The field (due to the current in the straight wire) is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

6. From the datum at $t = 0$ in Fig. 30-35(b) we see 0.0015 A = V_{battery}/R, which implies that the resistance is

$$R = (6.00\ \mu\text{V})/(0.0015\ \text{A}) = 0.0040\ \Omega.$$

Now, the value of the current during 10 s < t < 20 s leads us to equate

$$(V_{\text{battery}} + \varepsilon_{\text{induced}})/R = 0.00050\ \text{A}.$$

This shows that the induced emf is $\varepsilon_{\text{induced}} = -4.0\ \mu\text{V}$. Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -A\,a\ .$$

Plugging in $\varepsilon = -4.0\times10^{-6}$ V and $A = 5.0\times10^{-4}\ \text{m}^2$, we obtain $a = 0.0080$ T/s.

7. (a) The magnitude of the emf is

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12(2.0) + 7.0 = 31\,\text{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through R.

8. The resistance of the loop is

$$R = \rho\frac{L}{A} = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{\pi\left(0.10\,\text{m}\right)}{\pi\left(2.5\times10^{-3}\ \text{m}\right)^2/4} = 1.1\times10^{-3}\,\Omega.$$

We use $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\text{A})\left(1.1\times10^{-3}\,\Omega\right)}{\pi\left(0.05\,\text{m}\right)^2} = 1.4\ \text{T/s}.$$

9. The amplitude of the induced emf in the loop is

$$\begin{aligned}\varepsilon_m &= A\mu_0 n i_0\omega = (6.8\times10^{-6}\,\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\ \text{A})(212\ \text{rad/s})\\ &= 1.98\times10^{-4}\,\text{V}.\end{aligned}$$

10. (a) The magnetic flux Φ_B through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)\left(\cos 45°\right) = \pi r^2 B/\sqrt{2}\ .$$

Thus,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) = -\frac{\pi\left(3.7\times10^{-2}\,\text{m}\right)^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\text{T}}{4.5\times10^{-3}\,\text{s}}\right)$$
$$= 5.1\times10^{-2}\,\text{V}.$$

(a) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

11. (a) It should be emphasized that the result, given in terms of sin($2\pi ft$), could as easily be given in terms of cos($2\pi ft$) or even cos($2\pi ft + \phi$) where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f\,NabB$.

(b) We solve

$$\varepsilon_0 = 150\text{ V} = 2\pi f\,NabB$$

when f = 60.0 rev/s and B = 0.500 T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\text{ m}^2$.

12. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For $\vec{B} = (4.00\times10^{-2}\,\text{T/m})y\hat{\text{k}}$, $dB/dt = 0$ and hence $\varepsilon = 0$.

(b) None.

(c) For $\vec{B} = (6.00\times10^{-2}\,\text{T/s})t\hat{\text{k}}$,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(0.400\text{ m}\times0.250\text{ m})(0.0600\text{ T/s}) = -6.00\text{ mV},$$

or $|\varepsilon| = 6.00$ mV.

(d) Clockwise.

(e) For $\vec{B} = (8.00\times10^{-2}\,\text{T/m}\cdot\text{s})yt\,\hat{\text{k}}$,

$$\Phi_B = (0.400)(0.0800t)\int y\,dy \;=\; 1.00\times10^{-3}t\,,$$

in SI units. The induced emf is $\varepsilon = -d\Phi B/dt = -1.00$ mV, or $|\varepsilon| = 1.00$ mV.

(f) Clockwise.

(g) $\Phi_B = 0 \;\;\Rightarrow\;\; \varepsilon = 0$.

(h) None.

(i) $\Phi_B = 0 \;\;\Rightarrow\;\; \varepsilon = 0$.

(j) None.

13. The amount of charge is

$$q(t) = \frac{1}{R}[\Phi_B(0) - \Phi_B(t)] = \frac{A}{R}[B(0) - B(t)] = \frac{1.20\times10^{-3}\,\text{m}^2}{13.0\,\Omega}[1.60\,\text{T} - (-1.60\,\text{T})]$$
$$= 2.95\times10^{-2}\text{C}\ .$$

14. Figure 30-40(b) demonstrates that dB/dt (the slope of that line) is 0.003 T/s. Thus, in absolute value, Faraday's law becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}$$

where $A = 8\times10^{-4}$ m^2. We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-40(c) to be $i = dq/dt =$ 0.002 A (the slope of that line). Therefore, the resistance of the loop is

$$R = \frac{|\varepsilon|}{i} = \frac{A\,|dB/dt|}{i} = \frac{(8.0\times10^{-4}\ \text{m}^2)(0.0030\ \text{T/s})}{0.0020\ \text{A}} = 0.0012\,\Omega\,.$$

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\text{ m})^2}{2}(0.870\text{ T/s}) = 1.74\text{ V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\text{ V} + 1.74\text{ V} = 21.7\text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since the flux arises from a dot product of vectors, the result of one sign for B_1 and B_2 and of the opposite sign for B_3 (we choose the minus sign for the flux from B_1 and B_2, and therefore a plus sign for the flux from B_3). The induced emf is

$$\begin{aligned}\varepsilon &= -\sum\frac{d\Phi_B}{dt} = A\left(\frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt}\right)\\ &= (0.10\text{ m})(0.20\text{ m})\left(2.0\times10^{-6}\text{ T/s} + 1.0\times10^{-6}\text{ T/s} - 5.0\times10^{-6}\text{ T/s}\right)\\ &= -4.0\times10^{-8}\text{ V}.\end{aligned}$$

The minus sign means that the effect is dominated by the changes in B_3. Its magnitude (using Ohm's law) is $|\varepsilon|/R = 8.0\ \mu$A.

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

17. Equation 29-10 gives the field at the center of the large loop with R = 1.00 m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00\times10^{-4}$ m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where i_0 = 200 A and

$$k = (-200\text{ A} - 200\text{ A})/1.00\text{ s} = -400\text{ A/s},$$

we find

(a) $$B(t=0) = \frac{\mu_0 i_0}{2R} = \frac{\left(4\pi\times10^{-7}\text{ H/m}\right)\left(200\text{A}\right)}{2\left(1.00\text{m}\right)} = 1.26\times10^{-4}\text{ T},$$

(b) $$B(t=0.500\text{s}) = \frac{\left(4\pi\times10^{-7}\text{ H/m}\right)\left[200\text{A} - \left(400\text{A/s}\right)\left(0.500\text{s}\right)\right]}{2\left(1.00\text{m}\right)} = 0,\text{ and}$$

be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and f = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is A = (0.500 m) × (0.300 m) = 0.150 m^2, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBA2\pi f\ \sin(2\pi ft)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi\, fNAB = 2\pi\left(16.7\,\text{rev/s}\right)\left(100\,\text{turns}\right)\left(0.15\,\text{m}^2\right)\left(3.5\,\text{T}\right) = 5.50\times10^3\ \text{V}\ .$$

20. We note that 1 gauss = 10^{-4} T. The amount of charge is

$$\begin{aligned} q(t) &= \frac{N}{R}[BA\cos 20° - (-BA\cos 20°)] = \frac{2NBA\cos 20°}{R} \\ &= \frac{2(1000)(0.590\times10^{-4}\,\text{T})\pi(0.100\,\text{m})^2(\cos 20°)}{85.0\,\Omega + 140\,\Omega} = 1.55\times10^{-5}\,\text{C}\ . \end{aligned}$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

21. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40\ \text{rev/s})(2\pi\ \text{rad/rev})}{2\pi} = 40\ \text{Hz}\ .$$

(b) First, we define angle relative to the plane of Fig. 30-44, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0, which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$, then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\Phi_B = \int d\Phi_B = \int_0^{\ell} \left(4t^2 y\ell\right) dy = 2t^2\ell^3 \ .$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 \ .$$

At t = 2.5 s, the magnitude of the induced emf is 8.0×10^{-5} V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

28. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 ia}{2\pi} \ln\left(\frac{r+b/2}{r-b/2}\right).$$

When $r = 1.5b$, we have

$$|\Phi_B| = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(4.7\mathrm{A})(0.022\mathrm{m})}{2\pi}\ln(2.0) = 1.4\times10^{-8}\,\mathrm{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$\begin{aligned} i_{\text{loop}} &= \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 ia}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+b/2}{r-b/2}\right)\right| = \frac{\mu_0 iabv}{2\pi R[r^2-(b/2)^2]} \\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(4.7\mathrm{A})(0.022\mathrm{m})(0.0080\mathrm{m})(3.2\times10^{-3}\,\mathrm{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\mathrm{m})^2]} \\ &= 1.0\times10^{-5}\,\mathrm{A}. \end{aligned}$$

29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350\ \mathrm{T})(0.250\ \mathrm{m})(0.55\ \mathrm{m/s}) = 0.0481\ \mathrm{V}\ .$$

(b) By Ohm's law, the induced current is

$$i = 0.0481\ \mathrm{V}/18.0\ \Omega = 0.00267\ \mathrm{A}.$$

By Lenz's law, the current is clockwise in Fig. 30-50.

(c) Equation 26-27 leads to $P = i^2R = 0.000129$ W.

30. Equation 26-28 gives ε^2/R as the rate of energy transfer into thermal forms (dE_{th}/dt, which, from Fig. 30-51(c), is roughly 40 nJ/s). Interpreting ε as the induced emf (in absolute value) in the single-turn loop ($N = 1$) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Equation 29-23 gives $B = \mu_0 ni$ for the solenoid (and note that the field is zero outside of the solenoid, which implies that $A = A_{coil}$), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = A\frac{dB}{dt} = A_{coil}\frac{d}{dt}\left(\mu_0 n i_{coil}\right) = \mu_0 n A_{coil}\frac{di_{coil}}{dt}.$$

where Fig. 30-51(b) suggests that $di_{coil}/dt = 0.5$ A/s. With $n = 8000$ (in SI units) and $A_{coil} = \pi(0.02)^2$ (note that the loop radius does not come into the computations of this problem, just the coil's), we find V = 6.3 μV. Returning to our earlier observations, we can now solve for the resistance:

$$R = \varepsilon^2/(dE_{th}/dt) = 1.0 \text{ m}\Omega.$$

31. Thermal energy is generated at the rate $P = \varepsilon^2/R$ (see Eq. 26-28). Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is 1.69×10^{-8} Ω·m (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire ($d = 0.00100$ m is the wire thickness). The area *enclosed* by the loop is

$$A_{loop} = \pi r_{loop}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire ($L = 0.500$ m) is the circumference of the loop. This enclosed area is used in Faraday's law (where we ignore minus signs in the interest of finding the magnitudes of the quantities):

$$\varepsilon = \frac{d\Phi_B}{dt} = A_{loop}\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

where the rate of change of the field is $dB/dt = 0.0100$ T/s. Consequently, we obtain

$$P = \frac{\varepsilon^2}{R} = \frac{(L^2/4\pi)^2(dB/dt)^2}{\rho L/(\pi d^2/4)} = \frac{d^2L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(1.00\times10^{-3}\text{ m})^2(0.500\text{ m})^3}{64\pi(1.69\times10^{-8}\Omega\cdot\text{m})}\left(0.0100\text{ T/s}\right)^2$$
$$= 3.68\times10^{-6}\text{ W}.$$

32. Noting that $|\Delta B| = B$, we find the thermal energy is

38. From the “kink” in the graph of Fig. 30-55, we conclude that the radius of the circular region is 2.0 cm. For values of r less than that, we have (from the absolute value of Eq. 30-20)

$$E(2\pi r) = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi r^2 a$$

which means that $E/r = a/2$. This corresponds to the slope of that graph (the linear portion for small values of r) which we estimate to be 0.015 (in SI units). Thus, $a = 0.030$ T/s.

39. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where B_0 = (30.0 T + 29.6 T)/2 = 29.8 T and B_1 = (30.0 T – 29.6 T)/2 = 0.200 T. Then from Eq. 30-25

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}\left[B_0 + B_1 \sin(\omega t + \phi_0)\right] = \frac{1}{2}B_1\omega r\cos(\omega t + \phi_0).$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6\times10^{-2}\ \mathrm{m}) = 0.15\ \mathrm{V/m}.$$

40. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0\times10^{-3}\ \mathrm{H})(5.0\times10^{-3}\ \mathrm{A})}{400} = 1.0\times10^{-7}\,\mathrm{Wb}.$$

41. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60\times10^{-3}\ \mathrm{T})(\pi)(0.100\,\mathrm{m})^2 = 2.45\times10^{-3}\,\mathrm{Wb}.$$

(b) Equation 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\ \mathrm{Wb}}{3.80\,\mathrm{A}} = 6.45\times10^{-4}\ \mathrm{H}.$$

42. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

or $|di/dt| = 5.0\,\text{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then *i* must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times10^{-4}\ \text{H}.$$

46. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes $|\varepsilon| = L|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

(a) For 0 < t < 2 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(7.0\ \text{A} - 0)}{2.0\times10^{-3}\ \text{s}} = 1.6\times10^{4}\ \text{V}.$$

(b) For 2 ms < t < 5 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(5.0\,\text{A} - 7.0\,\text{A})}{(5.0 - 2.0)10^{-3}\ \text{s}} = 3.1\times10^{3}\ \text{V}.$$

(c) For 5 ms < t < 6 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(0 - 5.0\,\text{A})}{(6.0 - 5.0)10^{-3}\ \text{s}} = 2.3\times10^{4}\ \text{V}.$$

47. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must add, $L_{\text{eq}} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{\text{eq}} = \sum_{n=1}^{N} L_n$.

48. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt}+\frac{di_2(t)}{dt}=\frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also apply to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that the field of one inductor not to have significant influence (or "coupling'') in the next.

(b) Just as with resistors, $\frac{1}{L_{\text{eq}}}=\sum_{n=1}^{N}\frac{1}{L_n}$.

49. Using the results from Problems 30-47 and 30-48, the equivalent resistance is

$$L_{\text{eq}}=L_1+L_4+L_{23}=L_1+L_4+\frac{L_2L_3}{L_2+L_3}=30.0\text{ mH}+15.0\text{ mH}+\frac{(50.0\text{ mH})(20.0\text{ mH})}{50.0\text{ mH}+20.0\text{ mH}}$$
$$=59.3\text{ mH}.$$

50. The steady state value of the current is also its maximum value, ε/R, which we denote as i_m. We are told that $i = i_m/3$ at t_0 = 5.00 s. Equation 30-41 becomes $i=i_m\left(1-e^{-t_0/\tau_L}\right)$, which leads to

$$\tau_L=-\frac{t_0}{\ln(1-i/i_m)}=-\frac{5.00\text{ s}}{\ln(1-1/3)}=12.3\text{ s}.$$

51. The current in the circuit is given by $i=i_0e^{-t/\tau_L}$, where i_0 is the current at time t = 0 and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right)=-\frac{t}{\tau_L}.$$

This yields

$$\tau_L=-\frac{t}{\ln(i/i_0)}=-\frac{1.0\text{ s}}{\ln\left(\left(10\times10^{-3}\text{ A}\right)/\left(1.0\text{ A}\right)\right)}=0.217\text{ s}.$$

Therefore, $R = L/\tau_L$ = 10 H/0.217 s = 46 Ω.

52. (a) Immediately after the switch is closed, $\varepsilon - \varepsilon_L = iR$. But $i = 0$ at this instant, so $\varepsilon_L = \varepsilon$, or $\varepsilon_L/\varepsilon = 1.00$.

(b) $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon,$ or $\varepsilon_L/\varepsilon = 0.135$.

(c) From $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \quad \Rightarrow \quad t = \tau_L \ln 2 = 0.693\tau_L \quad \Rightarrow \quad t/\tau_L = 0.693.$$

53. (a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \;\Rightarrow\; e^{-t/\tau_L} = 0.200\,.$$

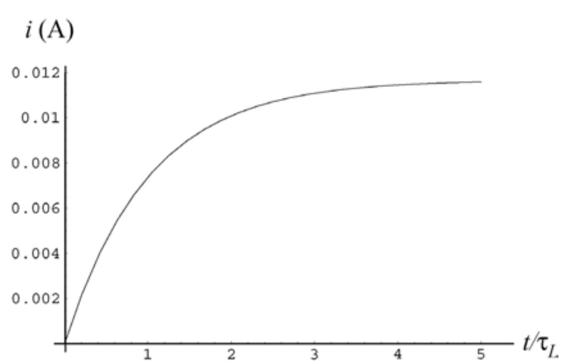
Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s}\,.$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A}\,.$$

The current as a function of t/τ_L is plotted below.

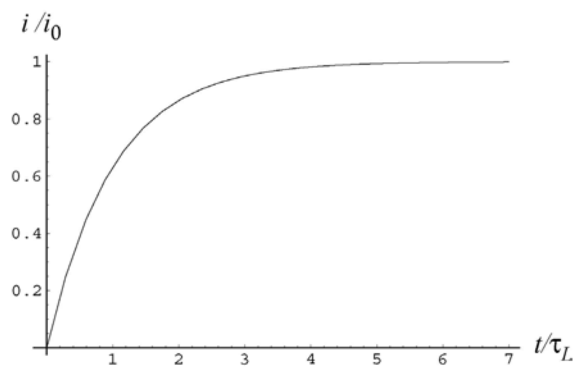


$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$ is the inductive time constant and ε is the battery emf. To calculate the time at which $i = 0.9990\varepsilon/R$, we solve for t:

$$0.990\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \Rightarrow \ln(0.0010) = -(t/\tau) \Rightarrow t/\tau_L = 6.91.$$

The current (in terms of i/i_0) as a function of t/τ_L is plotted below.



56. From the graph we get $\Phi/i = 2 \times 10^{-4}$ in SI units. Therefore, with $N = 25$, we find the self-inductance is $L = N\Phi/i = 5 \times 10^{-3}$ H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol V to stand for the battery emf)

$$\frac{di}{dt} = \frac{V}{R}\frac{R}{L}\, e^{-t/\tau_L} = \frac{V}{L}\, e^{-t/\tau_L} = 7.1 \times 10^2 \text{ A/s} .$$

57. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\, di/dt = 0$. So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0$ A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5 \text{ s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

58. Applying the loop theorem,

$$\varepsilon - L\left(\frac{di}{dt}\right) = iR ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\tau_L = t/0.5108 = (5.00 \times 10^{-3}\ \text{s})/0.5108 = 9.79 \times 10^{-3}\ \text{s}$$

and the inductance is

$$L = \tau_L R = \left(9.79\times10^{-3}\,\text{s}\right)\left(10.0\times10^{3}\,\Omega\right) = 97.9\,\text{H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\text{H})\left(2.00\times10^{-3}\,\text{A}\right)^2 = 1.96\times10^{-4}\,\text{J}.$$

62. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0\ \text{H}/10\ \Omega = 0.20\ \text{s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9\times10^2\,\text{W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

63. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(Li^2/2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2 = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} \quad \Rightarrow \quad t = \tau_L \ln 2 = (37.0\,\text{ms})\ln 2 = 25.6\,\text{ms}.$$

64. Let $U_B(t) = \frac{1}{2}Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2}U_B(t\to\infty) = \frac{1}{4}Li_f^2$. This gives $i(t) = i_f\big/\sqrt{2}$. But $i(t) = i_f(1-e^{-t/\tau_L})$, so

$$1-e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{t}{\tau_L} = -\ln\left(1-\frac{1}{\sqrt{2}}\right) = 1.23.$$

65. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\begin{aligned}\int_0^t P_{\text{battery}}\,dt &= \int_0^t \frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\varepsilon^2}{R}\left[t+\frac{L}{R}\left(e^{-Rt/L}-1\right)\right]\\ &= \frac{(10.0\,\text{V})^2}{6.70\ \Omega}\left[2.00\ \text{s}+\frac{(5.50\,\text{H})\left(e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}-1\right)}{6.70\ \Omega}\right]\\ &= 18.7\ \text{J}.\end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$\begin{aligned}U_B &= \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2 = \frac{1}{2}(5.50\,\text{H})\left(\frac{10.0\,\text{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}\right]^2\\ &= 5.10\ \text{J}.\end{aligned}$$

(c) The difference of the previous two results gives the amount “lost” in the resistor:
18.7 J – 5.10 J = 13.6 J.

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{\left(4\pi\times10^{-7}\ \text{H/m}\right)(100\,\text{A})}{2\left(50\times10^{-3}\ \text{m}\right)} = 1.3\times10^{-3}\ \text{T}.$$

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\,\text{mH})(6.0\,\text{mA})}{200} = 90\,\text{nWb}\ .$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\,\frac{di_1}{dt} = (3.0\,\text{mH})(4.0\ \text{A/s}) = 12\,\text{mV}.$$

73. (a) Equation 30-65 yields

$$M = \frac{\varepsilon_1}{|di_2/dt|} = \frac{25.0\,\text{mV}}{15.0\,\text{A/s}} = 1.67\,\text{mH}\ .$$

(b) Equation 30-60 leads to

$$N_2\Phi_{21} = Mi_1 = (1.67\,\text{mH})(3.60\,\text{A}) = 6.00\,\text{mWb}\ .$$

74. We use $\varepsilon_2 = -M\,di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times10^3\ \text{V}}{6.0\,\text{A}/(2.5\times10^{-3}\ \text{s})} = 13\,\text{H}\ .$$

75. The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}}\,l\,dr = \int_a^{a+b} \frac{\mu_0 il}{2\pi r}\,dr = \frac{\mu_0 il}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

From the formula for M obtained above, we have

$$M = \frac{(100)(4\pi\times10^{-7}\ \text{H/m})(0.30\,\text{m})}{2\pi}\ln\left(1+\frac{8.0}{1.0}\right) = 1.3\times10^{-5}\ \text{H}\ .$$

76. (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n\pi R^2\right)}{i_s} = \mu_0\pi R^2 nN\ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

77. (a) We assume the current is changing at (nonzero) rate di/dt and calculate the total emf across both coils. First consider the coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus, the induced emf's are

$$\varepsilon_1 = -\left(L_1 + M\right)\frac{di}{dt} \text{ and } \varepsilon_2 = -\left(L_2 + M\right)\frac{di}{dt} .$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{eq} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of coil rather than the front (as pictured in the diagram). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -\left(L_1 - M\right)\frac{di}{dt} .$$

Similarly, the emf across coil 2 is

$$\varepsilon_2 = -\left(L_2 - M\right)\frac{di}{dt} .$$

The total emf across both coils is

$$\varepsilon = -\left(L_1 + L_2 - 2M\right)\frac{di}{dt} .$$

This is the same as the emf that would be produced by a single coil with inductance

$$L_{eq} = L_1 + L_2 - 2M.$$

78. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right] = \frac{\varepsilon}{R\tau_L}e^{-t/\tau_L} = \frac{\varepsilon}{L}e^{-t/\tau_L} .$$

With $\tau_L = L/R$ (Eq. 30-42), L = 0.023 H and ε = 12 V, t = 0.00015 s, and di/dt = 280 A/s, we obtain e^{-t/τ_L} = 0.537. Taking the natural log and rearranging leads to R = 95.4 Ω.

79. (a) When switch S is just closed, $V_1 = \varepsilon$ and $i_1 = \varepsilon/R_1$ = 10 V/5.0 Ω = 2.0 A.

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) $i_s = i_1 + i_2$ = 2.0 A + 0 = 2.0 A.

(d) Since $V_L = \varepsilon$, $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) $V_L = \varepsilon$ = 10 V.

(f) $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = \dfrac{\varepsilon}{L} = \dfrac{10\text{ V}}{5.0\text{ H}} = 2.0\text{ A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so i_1 = 2.0 A.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2$ = 10 V/10 Ω = 1.0 A.

(i) $i_s = i_1 + i_2$ = 2.0 A + 1.0 A = 3.0 A.

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon$ = 10 V.

(k) $V_L = 0$.

(l) $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = 0$.

80. Using Eq. 30-41: $i = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$, where τ_L = 2.0 ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0\text{ ns}.$$

81. Using Ohm’s law, we relate the induced current to the emf and (the absolute value of) Faraday’s law:

$$i = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right|.$$

As the loop is crossing the boundary between regions 1 and 2 (so that “x” amount of its length is in region 2 while “$D - x$” amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

95. (a) As the switch closes at t = 0, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf (ε_{L1}) of the L_1 = 0.30 H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt}=\frac{|\varepsilon_{L1}|}{L_1}=\frac{6.0}{0.30}=20\,\text{A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with R_1 = 8.0 Ω, we have

$$\varepsilon - i\,R_1 - |\varepsilon_{L1}| - |\varepsilon_{L2}| = 0 \;\;\Rightarrow\;\; i=\frac{6.0\,\text{V}}{R_1}=0.75\,\text{A}.$$

96. Since $A=\ell^2,$ we have $dA/dt = 2\ell d\ell/dt$. Thus, Faraday's law, with N = 1, becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -2\ell B\frac{d\ell}{dt}$$

which yields ε = 0.0029 V.

97. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt}=\frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right]=\frac{\varepsilon}{R\tau_L}e^{-t/\tau_L} = \frac{\varepsilon}{L}e^{-t/\tau_L}$$

With τ_L = 0.28 ms (by Eq. 30-42), L = 0.050 H, and ε = 45 V, we obtain di/dt = 12 A/s when t = 1.2 ms.

98. (a) From Eq. 30-35, we find L = (3.00 mV)/(5.00 A/s) = 0.600 mH.

(b) Since $N\Phi = iL$ (where Φ = 40.0 μWb and i = 8.00 A), we obtain N = 120.